Prediction of Lightweight Aggregate Concrete Compressive Strength

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ABSTRACT

Nowadays, the better performance of lightweight structures during earthquake has resulted in using lightweight concrete more than ever. However, determining the compressive strength of concrete used in these structures during their service through a non-destructive test is a popular and useful method. One of the main methods of non-destructive testing in the assessment of compressive strength of concrete in the service is ultrasonic pulse velocity test. The aim of this study is predicting the compressive strength of lightweight aggregate concrete by offering a suitable mathematical formulation. Many samples of lightweight aggregate concrete, made by expanded clay, have been produced and tested. After determining the actual compressive strength and indirect ultrasonic pulse velocity for each sample, a relationship was presented to predict the compressive strength through Gene Expression Programming (GEP). The results show the presented equation has high accuracy in estimating the compressive strength of samples and that experimental results are perfectly compatible with the test results.

1. Introduction

Structural lightweight aggregate concrete is an important and versatile material for use in modern construction. It has many and varied applications including multistory building frames and floors, bridges, offshore oil platforms, and prestressed or precast elements of all types. Many architects, engineers, and contractors recognize the inherent economies and advantages offered by this material as evidenced by the many impressive lightweight concrete structures found today throughout the world. For more than 80 years structural lightweight aggregate concrete has solved weight and durability problems in buildings and exposed structures. Lightweight concrete has strengths comparable to normal weight concrete, yet is typically 25% to 35% lighter. Structural lightweight concrete offers design flexibility and substantial cost savings by providing: less dead load, improved seismic structural response, longer spans, better fire
ratings, and thinner sections, decreased story height, smaller size structural members, less reinforcing steel, and lower foundation costs. Lightweight concrete precast elements offer reduced transportation and placement costs [1-4].

The use of lightweight structural concrete to reduce the weight of earthquake resistant buildings is useful material having many applications. Therefore, research on the properties of different types of lightweight concretes and the evaluation of the corresponding concrete strength has been considered by many researchers [5, 6].

In general, the main defect of non-destructive methods for evaluating the properties of concrete is that any bit of property of concrete being tested is influenced by several parameters and besides there is no meaningful physical relationship between what is expected from that characteristic of concrete and what the test shows. Therefore, using these methods in the evaluation of the desired characteristic of concrete (especially compressive strength) is faced with many errors and also, they need to be supplied with the Calibration graphs for every kind of concrete that doing this is costly and time-consuming. Manufacturers of these devices often find an empirical relationship for their tests. Many of these relationships are not suitable for all types of concrete require a different calibration comparing other types of concrete. Therefore, much research has been done on the combination of non-destructive methods to improve evaluation. But, the problems of preparing calibration graphs for every kind of concrete have been used still remains [7-9]. In this regard, considering mentioned problems, the use of mathematical and evolutionary models such as fuzzy logic, neural networks, artificial intelligence and genetic algorithm which can be managed based on empirical studies have been developed [10].

Up to the beginning of the 21st century, the most common test to determine the compressive strength of concrete was testing cylindrical and cubic samples [11]. But these samples cannot totally show the actual characteristics of the concrete used in the construction of actual structures due to the factors such as inattention to actual conditions, differences in the processing, how to select samples from the entire population, changes in the type and amount of materials from one scale to another and the difference in density. On one hand, today, with the occurrence of events such as an earthquake, fires, structural changes in the desired usage of the structure, uncertainty about the quality of concrete in structures due to a defect in construction, it is necessary to evaluate the concrete strength. On the other hand, because non-destructive tests are easy to conduct and low cost and time saving, it is considered by the engineers worldwide [7, 9, and 12].

Gene expression programming (GEP) is, like genetic algorithms (GAs) and genetic programming (GP), a genetic algorithm as it uses populations of individuals, selects them according to fitness and introduces genetic variation using one or more genetic operators [13]. The fundamental difference between the three algorithms resides in the nature of the individuals: in GAs the individuals are linear strings of fixed length (chromosomes); in GP
the individuals are non-linear entities of different sizes and shapes (parse trees); and in GEP the individuals are encoded as linear strings of fixed length (the genome or chromosomes) which are afterwards expressed as non-linear entities of different sizes and shapes (simple diagram representations or expression trees) [14]. It is necessary first to give feedback on the ultrasonic tests on concrete that may be the most important non-destructive test on the concrete. By conducting multiple tests on concrete samples made of different kinds of aggregates, Facaoaru has provided the pulse velocity ranges with considering the kind of the aggregates [15]. Malhotra has studied the Velocity of the pulses in samples of concrete with different water-cement ratios and different types of aggregates [16]. Gaydecki investigated the Propagation and attenuation of ultrasonic pulses in concrete [17]. By studying the pulses passing from concrete specimens made with limestone aggregates with different qualities, Romel proposed a model to assess the quality of concrete [18]. Through artificial neural networks and using ultrasonic pulse velocity, Manish et al. tried to predict the compressive strength of concrete and compared the results of neural networks and multiple variable regressions [9]. Through artificial neural networks and using ultrasonic pulse velocity Kewalramani and Gupta tried to predict the compressive strength of concrete and compared the results of neural networks and multiple variable regressions [19]. Through Programming in MATLAB programming environment and considering the number of parameters of concrete and with the method of neural networks, Trtnik et al. provided a model for concrete compressive strength Using ultrasonic pulse velocity [7, 12]. Also Mousavi et al. proposed a new model for predicting the compressive strength of high performance concrete using gene expression programming [20]. Fakharian et al. proposed a model for compressive strength prediction of FRP-Confined rectangular columns in terms of Genetic Expression Programming (GEP) [21]. Using GEP, Ebtehaj and Bonakdary presented a model to predict sediment transport in sewer [22]. In another study, Hadianfard and Jafari suggested some equations to predict compressive strength of lightweight aggregate concrete using the ultrasonic pulse velocity test through gene expression programming [7].

Given the increasing importance and application of structural lightweight aggregate concrete, this study proposed a model to predict the compressive strength of the concrete by indirect ultrasonic testing method which is a non-destructive test where elevation of direct ultrasonic pulse velocity is not possible. The important results of this study are as follows: The model is simple to use, easy to use and relatively low cost to gain strength lightweight concrete which is not studied yet.

2. Experimental Program

To study the above-mentioned issues one type of lightweight aggregate concrete were made and tested. The lightweight aggregates used were in compliance with the standard ASTM C330 [23] and determination of the lightweight aggregate concrete mixing ratio was based on standard ACI 211.2 [24]. Measuring, mixing, transporting, and placing
operations for lightweight concretes are similar to the procedures for normal weight concrete. However, there are certain differences, especially in proportioning and batching procedures that should be considered to produce a finished product of the highest quality [25].

In this study more than 33 concrete samples have been tested. The batches were made in Shiraz University of Technology laboratory using the specific gravity method. The weight method procedure is applicable to sand-lightweight concrete comprised of lightweight coarse aggregate and normal weight fine aggregate. Estimating the required batch weights for the lightweight concrete involves determining the specific gravity factor of lightweight coarse aggregate and from this the first estimate of the weight of fresh lightweight concrete can be made. Also the absorption of lightweight coarse aggregate may be measured by the method described in ASTM C 127 [26]. When concrete was made with lightweight aggregates that have low initial moisture and relatively high rates of absorption they were mixed with one-half to two-thirds of the mixing water for a short period prior to the addition of cement to minimize slump loss. In addition the specific gravity of the lightweight aggregates were determined at the moisture content anticipated prior to use. For convenience each type of concrete mix was named. The concrete made of expanded clay was named lightweight aggregate LWA01. In all concrete mixes sand was used as the fine aggregate and expanded clay, or as the coarse aggregate.

2.1. Material proportion and mix design

Before any amendment the fineness modulus of the fine sand used was 3.46 which reached to 3 after sifting and amendment. For the experiments undertaken the sand moisture content was 1%. In the mix design the saturated surface dry (SSD) condition for the sand was 6% water. The specific gravity of the fine sand was 1717.65Kg/m3, which was determined in accordance with ASTM C 29 [27].

In the standard ASTM C 330 there are some implications and requirements for gradation of lightweight aggregates. The gradation of aggregates was based on these requirements [23]. The lightweight aggregates used in this study were coarse aggregates with the sizes of 9.6mm for expanded clay. Specific gravity for the expanded clay was 365.72Kg/m3 and the humidity of the aggregates in the natural environment was zero in order to be used outside and exposed to the sun. Also for the experiments undertaken for the aggregates of expanded clays and mineral pumices to reach to the SSD condition 13% and 15% of water was needed respectively. Pycnometric specific density factor for expanded clays was 1.1.

The proportioning follows a sequence of straightforward steps that, in effect, fit the characteristics of the available materials. The first step in the study was to choose a slump based on the type of construction being considered, which in this case, comprised beams, reinforced walls and buildings columns. The next step was to choose one of the maximum size of lightweight aggregates mentioned in previous paragraphs. Based in
these two initial steps the appropriate amount of mixing water was determined. The fourth step involved selection of the approximate water-cement ratio based on the expected compressive strength of the concrete.

Table 1. Summary of the concrete mix design and density of the lightweight concretes (1)

<table>
<thead>
<tr>
<th>Name of samples</th>
<th>Type of LWA</th>
<th>Water Cement Ratio</th>
<th>Water $W_W$ (Kg)</th>
<th>Cement $W_C$ (Kg)</th>
<th>Sand $W_S$ (Kg)</th>
<th>LWA $W_L$ (Kg)</th>
<th>Density (Kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LWA01</td>
<td>Expanded Clay</td>
<td>0.4</td>
<td>210</td>
<td>525</td>
<td>710</td>
<td>155</td>
<td>1659</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.6</td>
<td>230</td>
<td>380</td>
<td>835</td>
<td>155</td>
<td>1529</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.8</td>
<td>210</td>
<td>260</td>
<td>975</td>
<td>155</td>
<td>1631</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.6</td>
<td>212</td>
<td>353</td>
<td>795</td>
<td>539</td>
<td>1840</td>
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<tr>
<td></td>
<td></td>
<td>0.8</td>
<td>212</td>
<td>265</td>
<td>884</td>
<td>539</td>
<td>1781</td>
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</table>

Assessment of the cement content was the next step, with the amount of cement per unit volume of concrete being fixed. The next step was estimating the lightweight coarse aggregate content based on the maximum size of aggregates and volume of oven-dry loose coarse aggregates per unit volume of concrete for different fineness moduli of sand. With the quantities of water, cement, and coarse aggregate established, the remaining material, fine aggregate content, was estimated in the last step. The concrete mix designs and density of the hardened concretes are presented in the Table 1. All the weights are for making 1m$^3$ of fresh concrete and all weights are based on saturated surface dry (SSD) aggregates.

2.2. Ultrasonic pulse velocity test

Standard method of Ultrasonic testing is conducted according to ASTM C 597 [28]. The frequency was 54 KHz and the voltage was 500 V in conducting the test. Figure (1) shows the device. The device includes a processor unit that carries out sending and receiving ultrasonic pulses and measuring the time between the two operations (sending and receiving). It also includes two probes and two cables, which do the transmission of ultrasonic pulses. To calibrate the device a cylinder is used. The sound pulse passing time has been inserted on the cylinder [7]. The device is lightweight, portable and easy to use and can also be used with and without electricity in and out of the lab. The device has two probes that actually transfer the sound energy. One probe sends the sound energy to the concrete and the recipient probe receives this energy and the pulse flow rate has been achieved according to the time difference between these two acts [7]. As figure 2 shows, in this study, the indirect transfer (opposite surfaces) is used to measure the pulse flow rate.
Fig. 1. Ultrasonic testing device (Pundit)

Fig. 2. Velocity test for indirect transmission of pulses

3. Experimental results

The compressive strength of concrete is the most common performance measure used by engineers in designing buildings and other structures [29]. The compressive strength is measured by breaking cylindrical concrete specimens in a compression-testing machine [30]. For convenience a few cylindrical samples were made of WLA01 in order to find coefficients which would convert the compressive strength results of cubic samples to that of cylindrical samples. The measurement method of concrete compressive strength was assessed according to the standard ASTM C 39-83b [31]. Loading rate was considered constant ranging from 0.15 to 0.34MPa/sec. The laboratory velocity of 0.3MPa/sec was used to break the samples. The type of lightweight aggregate and its volume fraction in a mix determined the density of lightweight concrete [32]. Customarily the theoretical density is a laboratory determination; for which the value is assumed to remain constant for all batches made using identical component ingredients and proportions. It is calculated from ASTM C 138/C 138 M [31].

3.1. Factors effecting the compressive strength and indirect ultrasonic pulse velocity in lightweight concrete

According to the Table 2 it can be seen that by increasing the compressive strength of samples, the density and ultrasonic pulse velocity increase. Also this table shows that the direct ultrasonic pulse velocity has a direct relationship with indirect ultrasonic pulse velocity, which means that indirect pulse velocity can be used to estimate the compressive strength on concrete as well. Figure 3 shows diagrams for Compressive strength of concrete according to the indirect transfer velocity of sound pulses for Lightweight Aggregate Concrete for LWA01 (in SSD condition). This diagram indicates a clear relationship in relation to different mixes of lightweight concretes and their corresponding compressive strengths. By increasing the weight ratio of lightweight aggregate to all aggregates concrete compressive strengths decrease with an inverse relationship between them.
Table 2. Compressive strength and density of samples [7]

<table>
<thead>
<tr>
<th>No. of sample</th>
<th>$F_C$ (MPa)</th>
<th>Density (Kg/m³)</th>
<th>$V$ (Km/s)</th>
<th>$U$ (Km/s)</th>
<th>No. of sample</th>
<th>$F_C$ (MPa)</th>
<th>Density (Kg/m³)</th>
<th>$V$ (Km/s)</th>
<th>$U$ (Km/s)</th>
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<td>12.84</td>
<td>1693.3</td>
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<tr>
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<td>12.16</td>
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<td>9</td>
<td>9.52</td>
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<td>1638.8</td>
<td>2.8</td>
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<td>2.5</td>
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<td>16.47</td>
<td>1700.7</td>
<td>3.45</td>
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<td>13</td>
<td>11.82</td>
<td>1642.4</td>
<td>2.99</td>
<td>2.54</td>
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<td>15.99</td>
<td>1689.2</td>
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<td>14</td>
<td>9.76</td>
<td>1664.6</td>
<td>2.83</td>
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<td>13.15</td>
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<td>1743.4</td>
<td>3.19</td>
<td>2.78</td>
<td>32</td>
<td>9.29</td>
<td>1645.0</td>
<td>2.78</td>
<td>2.32</td>
</tr>
</tbody>
</table>
4. Comparing the mix design used in this study to other research results

In experimental work by Bastos et al. to introduce a method for the design of lightweight concrete with expanded clay aggregates 28 lightweight concrete sample mixes using four different quantities of cement (126, 155, 185 and 214 kg/m³) with densities between 853 and 1,418 kg/m³ were made by expanded clay (LECA Portugal) and tested. In comparison with the study referred in this paper the size of the lightweight aggregates were smaller and in some samples they were used as fine aggregates. In Figure 5, the results for both works are shown. Samples in the study referred to in this paper have higher densities and compressive strengths than those of the study by Bastos et al. [31].

5. Prediction of lightweight concrete compressive strength using indirect ultrasonic pulse velocity through GEP

Gene expression programming is a method for learning the most fit computer programs by means of artificial evaluation [34, 35]. GP as a subset of genetic algorithms, is a modern regression method that has a great ability to automatically evolve computer programs. Koza introduced GP in the late 1980s after experiments on symbolic regression which is also called tree-based GP [36, 37]. The main difference between the GA and GP approaches is in their evolving programs where in GP they are parse trees rather than fixed-length binary strings in GA [1, 36, and 38].

Gene Expression Programming (GEP) as an evolutionary algorithm that incorporates both the idea of a simple, linear chromosome of fixed length used in Genetic Algorithms (GAs) and the tree structure of different sizes and shapes used in Genetic Programming (GP) was first invented by Ferreira [14, 39]. In GEP, the genetic operators act on the chromosome level that leads to an extreme simplification in the creation of genetic diversity. Due to multi-genic nature of GEP, more complex programs with several subprograms can be generated during the
evolutionary process. The symbols are elements from function or terminal sets and a gene in GEP is composed of a list of symbols. Each of the functions takes any value of data type which can be returned by a function or assumed by a terminal [38, 40]. A typical GEP gene is as given below:

\[ + \times \sqrt{a \cdot 3 \cdot b} \]  \hspace{1cm} (1)

Where a, b and 3 are elements of the function set; +, √ and × are the terminal nodes, and “.” is the element separator for easy reading. This expression is called Karva notation or K-expression [14]. Figure 6 illustrates the expression tree of the above sample gene as an example. A K-expression can be represented by a diagram as an ET. The first position in the K-expression denotes the root of the ET. The transformation process starts from the root and reads through the string one by one [38, 39].

Fig. 6. Example of expression trees (ETs).

The gene expression programming method has been used to estimate the compressive strength of lightweight aggregate concrete. The fit function (name Mean Squared Error) and the correlation coefficient were used to illustrate the accuracy of the relationship. The MSE assesses the quality of an estimator or a predictor [40]. In statistics, the mean squared error (MSE) or mean squared deviation (MSD) of an estimator measures the average of the squares of the errors or deviations, that is the difference between the estimator and what is estimated [1, 41].

The relationship between elasticity modulus and compressive strength and the velocity of ultrasonic pulses can lead to introduce a reason for measuring and testing the velocity of ultrasonic pulses to evaluate the Compressive strength of concrete [7, 9].

To estimate the compressive strength according to the pulse rate several relationships has been suggested. One of these relationships is an exponential function according to Equation 2. These relationships are empirical and their constant coefficients are empirically derived from multiple experiments [7, 12, 16 and 42]. In order to have better equations, these was an odd date in experimental results which has been deleted.

\[ F_c = Ae^{BV} \]  \hspace{1cm} (2)

This relationship has been proposed for estimating the compressive strength of ordinary concrete the method of minimum set of squares is used to fit the diagram and to determine these constant coefficients. MATLAB software is used to perform the fit. The achieved relation for the LWA01 is as follows [7]:

In these relations the velocity of direct ultrasonic pulses is according to Km / s and the compressive strength of 28 days concrete is estimated according to MPa.

\[ F_c = 0.466 \times e^V \]  \hspace{1cm} (3)

The correlation coefficient in this case is 0.76 [7]. The following equation is obtained for LWA01 concrete. In this equation, the velocity of ultrasonic pulses is according to
Km / S. and the compressive strength of the 28-day concrete sample is estimated according to MPa. Also, the accuracy and the correlation coefficient are mentioned [7].

\[ F_c = 0.101V - 0.220 \]  \hspace{1cm} (4)

Fitness Function: MSE, Training Fitness: 996.1, Training R-square: 0.96, Testing Fitness: 993.8, Testing R-square: 0.96

For LWA01 concrete the following equation is obtained. Where \( V \) is the indirect ultrasonic pulse velocity according to Km/s and \( F_c \) is the compressive strength on the 28-day concrete samples according to Mpa. The correlation coefficient in this case is 0.81. Figure 7 and 8 show relationships between the actual compressive strength and the predicted compressive strength using equations derived from GEP. In table 3 the correlation coefficients of three obtained equations are presented.

\[ F_c = 29.89 - \frac{51.7}{U} \]  \hspace{1cm} (5)

Fig. 7. Actual compressive strength vs. the estimated compressive strength

<table>
<thead>
<tr>
<th>Samples</th>
<th>Equation 3</th>
<th>Equation 4</th>
<th>Equation 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>LWA01</td>
<td>0.76</td>
<td>0.96</td>
<td>0.81</td>
</tr>
</tbody>
</table>

**6. Conclusion**

By examining the results of tests and the presented graphs and tables it can be concluded that using ultrasonic pulse velocity test can predict compressive strength of the lightweight concrete as a non-destructive test. Results indicate that GEP can generate equations which have more accurate than former presented equations. By fitting the diagram with the method of the least sum of squares it was found that the provided exponential equation is less accurate in a comparison with equations obtained through GEP. The correlation coefficient of equation based on direct ultrasonic pulse velocity shows that the direct ultrasonic pulse velocity has more reliable outcomes than indirect test for estimation of compressive strength of lightweight concrete. However, the accuracy
of the obtained equation using indirect ultrasonic pulse velocity through GEP is acceptable. Therefore, in some conditions, where evaluating of direct ultrasonic pulse velocity is not possible or difficult to obtain, the indirect ultrasonic pulse velocity can lead to predict compressive strength of lightweight concrete through the presented equation with acceptable results.

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