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## Innovations in Sustainable Earthquake Resisting Rocking Wall-Frames

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### ABSTRACT

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This paper introduces a novel design concept for the development of efficient, sustainable Rocking-Wall Moment Frames (RWMFs) under seismic conditions. The proposed concepts lead to a novel structural configuration with provisions for Collapse Prevention (CP), Self-Centering (SC), reparability, performance control (PC), damage reduction, and energy based seismic analysis. It introduces the merits of design led analysis (DLA) over the traditional methods of approach, followed by the development of a lateral resisting system that is more efficient than its conventional counterparts. The fundamental idea behind the proposed methodology is that seismic structural response is mainly a function of design and construction, rather than numerical analysis. In design led analysis the rules of mechanics and structural design are induced rather than followed. The new system is a combination of grade beam restrained moment frames and articulated shear walls, tied to each other by means of post tensioned (PT) stabilizers and Gap Opening Link Beams (GOLBs).

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## 1. Introduction

“Sustainability has been enshrined as a goal of society to ensure that the satisfaction of present needs does not compromise the ability of future generations to meet their own needs. It is thus a social objective, achievable only where all areas of society cooperate in fulfilling the associated demands.”

[1] The historical background and

development of dual rocking wall-frame systems under seismic conditions have been amply discussed in recent publications, [2-21]. While the present paper is concerned with somewhat futuristic ideas, its basis are simple and seek to develop a structural system that inherently complies with the same design objectives as those investigated for through standard methods of analysis. Here, an effort is made to develop a

structural system that is more suitable for the purpose and is more likely to perform as expected than its traditional counterparts. The fundamental idea behind the proposed methodology is that seismic structural response is mainly a function of design and construction, rather than numerical analysis [24, 25]. Both strength and stiffness are induced rather than investigated. Stability conditions and failure mechanisms are enforced rather than tested. The proposed concept is materialized by first developing a purpose specific structural system followed by target oriented, design led analysis. In design led analysis the rules of mechanics and structural design are induced rather than followed. The first step in developing the proposed system is to identify the flaws and deficiencies experienced with traditional methods of dual system design as practiced all over the world.

## **2. Traditional systems and conventional methods of approach**

### **2.1. The general scheme**

Consider the basic scheme of a conventional fixed base wall-frame (dual) system, Figs. (1b and 1c), subjected to an arbitrary distribution of lateral forces, Fig. (1a). The forthcoming arguments are equally valid for reinforced concrete as well as steel moment frames in combination with concrete/ steel plate shear walls and/or steel braced frames. A careful study of the kinematics of the subject structure, together with lessons learned from the traditional methods of approach reveal some of the physical flaws and shortcomings associated with this type of construction, e.g.;

The large stiffness of the combined structure due to fixed boundary conditions (walls and columns) results in shorter natural periods

and larger dynamic forces during seismic events.

The cantilever bending of the wall reduces the dominance of the first mode of natural vibrations.

Unless the wall is extremely rigid, drift concentration may cause undesirable conditions.

Cantilever bending does not conform to a straight line displacement profile. This is in conflict with the commonly utilized assumption of triangular distribution of seismic forces.

Cantilever bending may encourage soft story failure in upper level sub frames

The sequences and patterns of formations of plastic hinges are unpredictable and uncontrollable.

Damage to column feet, base plates and footings is unavoidable. Reparability is questionable.

Premature formation of plastic hinges at column feet is tantamount to early development of large displacements and undesirable P-delta effects.

Premature formation of plastic hinges at wall base is also tantamount to early development of large displacements and may trigger progressive collapse.

The system does not lend itself well to reparability, self-centering and collapse prevention. Fixed supports tend to collect residual deformations due to manufacturing processes and seismic events.

The system does not lend itself well to performance control and damage reduction.

The sharp changes in inter-story stiffnesses and differing demand capacity ratios hinder practical weight optimization efforts.

Lack of gap openings limits the natural damping of the dual system.

In fixed base shear-wall /MFs, residual drifts result in cost prohibitive repairs after major earthquakes.

Most of these problems can be alleviated by replacing the wall fixity by an articulated joint, the column boundary conditions by a system of grade beam restrained pinned supports and the introduction of post tensioned stabilizers and rigid links. Such strategies eventually lead to the development of new sets of closed form design formulae for replacing fixed base wall frames with idealized, fully supplemented, mode shaping

systems. While the engineering community has witnessed remarkable advances in both the technological as well as computational aspects of seismic structural design, the same cannot be claimed for system development and design methodologies as a whole. The current presentation does not aim at discussing the limitations of traditional methods of design, but rather to introduce the basis of a newer philosophy that might stimulate and advance the customary thinking on the subject.

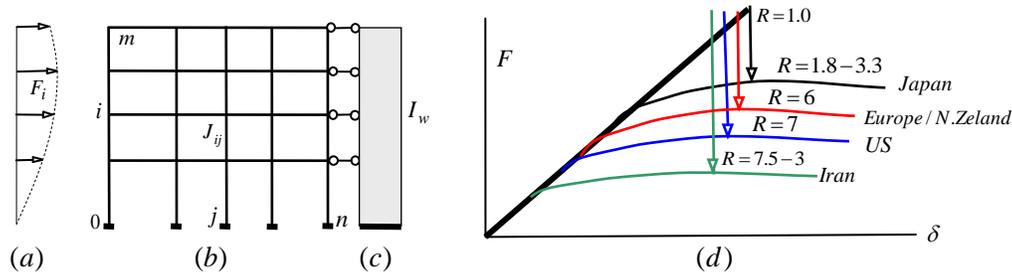


Figure 1 (a) Loading, (b) Moment frame, (c) Shear wall, (d) Comparative  $R$  factors

### 2.2. Earthquake loading

Problems associated with initial-stiffness structural characterization in traditional force-based seismic design, and use of code-specified force reduction factors  $R$  have been fully discussed in several previous publications [16] and will not be reiterated here, except for the depiction of the expected response of regular dual system.

To this end an attempt has been made to utilize Housner's [26, 27] equal energy

concept that defines the demand-capacity relationship of SDOF systems in terms of their base shear and the corresponding energy absorption capacities at plastic failure. In the proposed structural combination the pin supported rocking wall retains a straight line deflection profile during all phases of the elastoplastic loading. The first mode dominates and suppresses all higher modes of natural vibrations [54, 55].

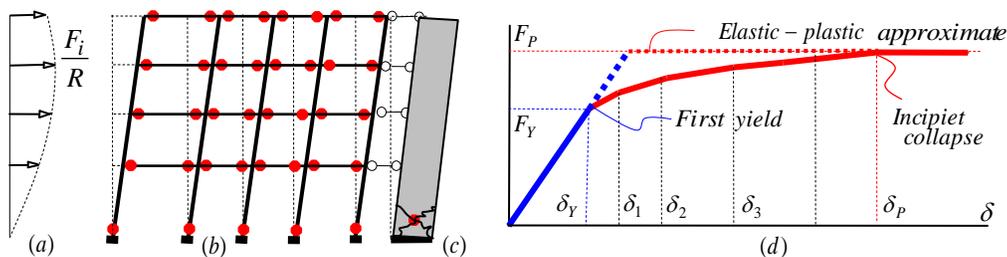


Figure 2 (a) reduced loading, (b) Preferred failure mechanism, (c) Wall failure, (d) Push over curve

### 2.3. Design strategies

Despite their technical limitations, Allowable stress (ASD), Limit state design (LSD), Load resistance factor design (LRFD) and to a lesser extent, Plastic design (PD) are still the most widespread methods of seismic structural design worldwide. These methodologies have served the engineering communities well in the past, at the expense of the consumer. Functional and societal demands require higher levels of performance including safety, economy and life-cycle costs. The main weakness of the ASD is in its limitation to the elastic range of response and at best to the onset of plasticity. LSD and LRFD are essentially rationalized elastic methodologies and may at best be regarded as lower bound plastic designs. ASD, LSD and LRFD result in almost identical products. PD [28-31] tends to produce more economic results, but is also limited in nature, depends on the correct prediction of the failure mechanism and does not address service level behavior. Push over (PO), as depicted in Fig. 2, is more of an instrument of investigation rather than design and as such is better suited for the seismic evaluation of existing buildings [32].

However, PO can be used to investigate pre-selected ultimate target drift and failure mechanism as performance criteria, and as such offers major improvements over traditional thinking. The fallacy that PD cannot be associated with ultimate displacements, together with the dominating influences of electronic computations has hindered the progress of the more rational performance-based plastic designs, such as the now well recognized Performance based plastic analyses, [33,34] and similar methods of approach. However, the recently introduced purpose specific PC [35, 36]

which is a more comprehensive design procedure, embraces the merits of both the elastic as well as plastic methods of approach. Design led PC utilizes the complete elastoplastic characteristics of ductile structures, including plastic deformations at incipient collapse, as part of the inclusive design strategy A symbolic and self-explanatory comparison of the prevailing methods of design; ASD, LRFD, PD and the proposed design led PC is presented in Fig.(3), where it may be observed that the most significant difference between PC and the traditional methods of design is that in PC the design of the structure is based on the performance of the entire structure rather than the capacity of the weakest member of the system.

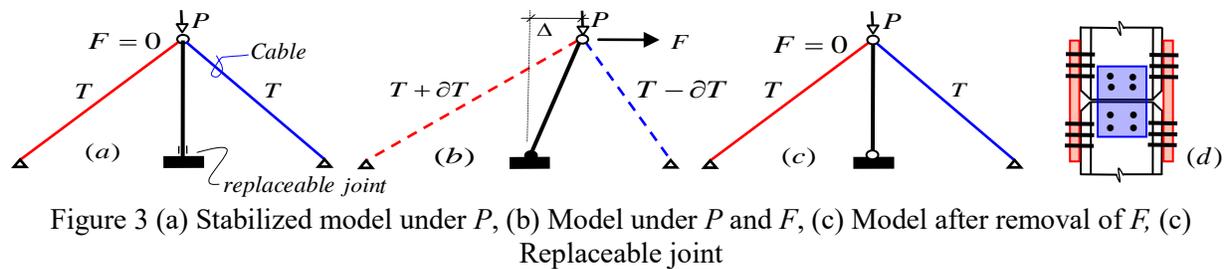
The primary purpose of this section is to first develop a structural configuration that complies with scientifically observed needs rather than resorting to intuitive guesswork and the investigation of a given system for compliance with the same requirements. And, second to introduce a multi-stage, simple PC formula with a view to self-centering and collapse prevention.

### 3. Technical definitions

The following definitions have been provided to facilitate the presentation of the forthcoming discussions.

Design led analysis aims chiefly at knowledge based application of structural concepts, details and materials, in order to achieve predetermined objectives, rather than investigating an assumed system for compliance with the same criteria. Theories of structures are applied rather than followed. Seismic load distribution is controlled rather than accepted.

- Performance control is the means by which design led analysis is implemented.
- Collapse prevention is the means and methods that are used to prevent a structural mechanism from falling down. See Fig.( 3)
- Damage reduction/avoidance is the strategy that aims at reducing and/or eliminating known types of damage due to adverse loading conditions.
- Self-centering or self-alignment is the ability that tends to realign a structural mechanism back to its original undisturbed form. A pictorial presentation of self-centering and collapse prevention is provided in Fig.(3), where the lateral force  $F$  causes the formation of a plastic hinge at the base, Fig.(3b).The stressed tendons return the system to its original position after  $F$  is removed.



However, it should be born in mind that residual deformations under seismic loading can significantly affect the re-centering capabilities of the structure [37-40].

- *Repair ability* is the provision that allows repairs/replacements (full functional restoration) to be achieved faster with less cost and effort than those associated with conventional structures. See Fig. 3 (d).
- *Cost effectiveness* = realistic design assumptions, detailing simplicity + minimum material consumption + use of repetitive members and connections.
- *Modular modeling* is a simplified, somewhat approximate, structural modeling technique for multi-bay, multi-story buildings. Each horizontal sub frame is represented by an equivalent closed loop module.
- *Moment Frames of Uniform Response (MFUR)* are special weight optimized lateral resistant systems in which story level drift ratios are constant and members of similar groups such as beams, columns and braces share the same demand-

capacity ratios regardless of their location within the group. In other words,  $\phi_i = \phi = V_i / K_i$ , where  $V_i$  and  $K_i$  stand for racking shear and module stiffness at level  $i$ . For a pictorial representation of a typical MFUR see Figs. (4a-4d). MFUR are used to compare drift/weight efficiencies of geometrically similar moment frames.

- *Moment Frames of Uniform Shear or sections (MFUS)* are also special, weight optimized lateral resistant frames in which story level drift ratios are constant and members of similar groups such as beams, columns and braces share the same demand-capacity ratios regardless of their location within the group. In MFUS story level shears and subframe stiffness are proportional but constant in magnitude, i.e.,  $V_i = V, K_i h_i = Kh$ . For a pictorial representation of a typical MFUS see Figs.(4e-4g). Note that both MFUR and MFUS are structures of uniform response. However, in order to overcome the issues discussed in subsections 2.1, 2.2 and 2.3

resort may be made to some or most of the following design strategies;

- By providing energy dissipating, replaceable, moment connections at beam ends

- By actually reducing dead load, and height of building, e.g., flat slabs and steel plate shear walls.
- By increasing energy dissipation by means of controlled gap openings and stressed tendon arrangements, etc.
- By providing wall mounted and other types of structural dampers and fuses, e.g., Taylor devices
- By reducing global stiffness in order to increase the natural period of vibrations.

- By allowing the first natural mode of vibration to suppress all higher modes, thereby the seismic loading.
- By reducing drift concentration, thereby improving overall structural performance [41].
- By selecting purpose specific structural options such as grade beam restrained pinned column boundary conditions [42-43]
- By introducing a unique rocking wall moment frame combination incorporating stressed tendon stabilizers such as those presented in Figs. (6a and 9) below.

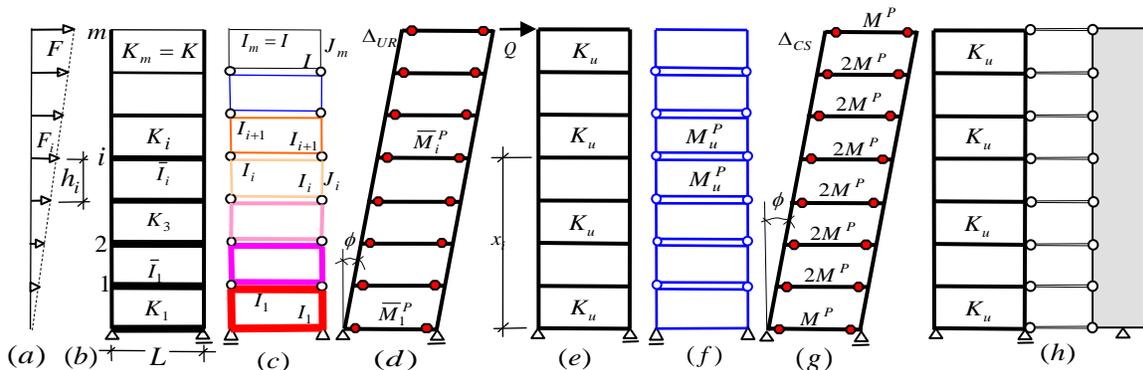


Figure 4 (a) loading, (b) MFUR, (c) MFUR modules, (d) MFUR at collapse, (e) MFUS, (f) MFUS modules, (g) MFUS at collapse, (h) Rigid wall-MFUS combination with link beam

In this scheme  $i$  and  $j$  represent the joint coordinates of an  $m \times n$ , grade beam supported, regular moment frame.  $\bar{I}_{ij} = I_{ij} + I_{i+1,j}$  And  $\bar{J}_{ij} = J_{ij}$  stand for moments of inertias of beams and columns related to joint  $ij$  respectively. The moment frame is connected to the rocking wall by means of pin ended rigid links. The rotational stiffness of the post-tensioned wall at the base is symbolized as  $K_C$ . The equivalent rotational stiffnesses of the supplementary devices connecting the link beams to the wall and the frame are given as  $K_{Di}$ . The use of the proposed supplementary device in the

form of a newly developed rigid link beam is illustrated in Fig (6). The theoretical development presented here in is based on the assumption that any multi-story, multi-bay, regular moment frame, under lateral loading can be construed as being composed of imaginary horizontal sub frames as depicted in Fig which may in turn be modeled as an equivalent symmetrical single bay moment frame [44,45]. The use of the single bay modular model not only simplifies the task of otherwise complicated frame analysis, but also facilitated the rational presentation of the more important conceptual arguments.

### 3.1. Practical design advantages

The practical advantages of grade beam supported moment frame-rocking wall combinations with supplementary devices over fixed base moment frame-shear walls may be summarized as follows;

- Rocking wall-grade beam supported moment frames, with or without supplementary devices, tend to deform with *zero* to negligible *drift concentration* along the height of the structure.
  - Rocking wall-grade beam supported moment frames tend to prevent *soft story failure* and the formation of *column base plastic hinges*.
  - Rocking wall-grade beam supported moment frames lend themselves well to *self-centering*, *collapse prevention* and *damage avoidance* treatments.
  - Rocking wall-grade beam supported moment frames attract substantially less residual stresses and deformations due to extraordinary loading conditions.
  - Overturning moments are transmitted to the footings *only* through axial reactions.
  - The rocking wall tends to bend as an upright *simply supported* beam rather than a vertical cantilever.
  - Gap openings at the ends of the link beams and at the bottom of the rocking wall *add* natural damping and provide opportunities for self-centering, damage reduction and collapse prevention. The use of gap opening in conjunction with pre-stressing reduces frame moments and drift ratios.
  - Rocking walls can be used as elements of structural *control* for pre and post-earthquake conditions. The uniform drift is *not* sensitive to minor changes in wall stiffness.
  - In grade beam supported frames *no* moments are *transmitted* to the footings.
- No anchor bolt, base plate and footing *damage* can occur due to seismic moments.
- The grade beams *prevent* the formation of plastic hinges at column supports. The grade beam provides means of controlling column base rotation and the overall drift.
  - For *equal* wall mass, RWMF have *longer* natural periods of vibration and attract significantly smaller seismic forces.
  - Rigid rocking walls *suppress* contributions of higher modes of vibrations. In other words the dominant mode shape remains *unchanged* during *all phases* of loading.
  - Rocking walls tend to rotate as rigid bodies *without* significant in-plane deformations.
  - The normalized displacement function is a straight line and remains *unchanged* throughout the loading history of the structure. Loss of stiffness changes only the value of the *drift angle* but not the drift profile.
  - The displacement profile remains a function of the *single variable* for all loading conditions.
  - The structure is a *SDOF* system, and as such lends itself well to equivalent energy studies.
  - The lateral displacements of well-proportioned grade beam supported moment frames could be *smaller* than those of identical frames with fixed and pinned boundary support conditions [43].
  - The magnitude and the distribution of the P-delta moments are more *favorable* in rocking-wall MFUS combinations than in geometrically similar MFUR [46].
  - Removal of the flange plates allows the joints to rotate freely until re-centering is achieved.
  - Replacement of the flange plates removes sources of residual deformations due to earthquakes.

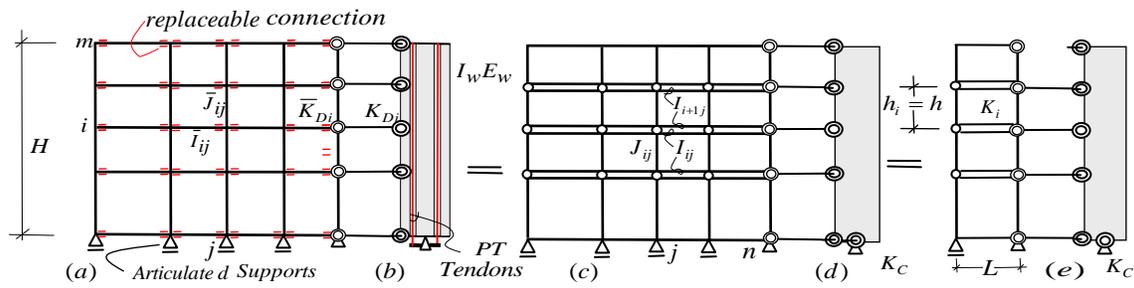


Figure 5 (a) Proposed structural system, (b) Rocking wall with stressed tendons, (c) Subframe model, (d) Equivalent base level rotational device, (e) Modular model

### 3.2. Main design features

The proposed earthquake resisting system consists of five essential components;

The grade beam supported moment frame, the rocking wall, the link beam, the replaceable moment connections and the stabilizing tendons.

The gravity system and the earthquake resisting moment frames are designed in accordance with the rules and regulations of the prevailing codes of practice with provisions to accommodate the unbonded stressed tendons and special attention to the weak beam–strong column requirement. The additional moments induced by the link beam should be taken into account when designing the column affected by gap opening.

The rigid rocking wall, whether constructed out of concrete or steel plates, should be free to pivot about its base and rotate freely at all slab wall junctions. Figs. (6a and 6b) show one such detail that allows horizontal shear transfer from the slab to the wall without inhibiting the vertical component of the movement of the wall at the same junction. The detail also provides out of plane stability at all floor levels. The high rigidity of the wall causes all wall attached link beams, to absorb proportional amounts of energy, symbolized by  $M_{D,i} \approx K_{D,i} \bar{\phi}$ ,  $M_C \approx K_C \phi$  etc.,

where  $M_D$  and  $M_C$  represent the linear moments of resistance of the restraining devices at levels  $i$  and the base respectively,  $\bar{\phi} = \alpha \phi$ .

The most commonly utilized post-tensioned gap opening beam or link beam system with butting flat ends against column sides, Fig (7), tends to expand the frame beyond its original span length. As the gap widens at the beam–column interface, the beam rotates upwards, and bends the column inwards. This in turn exerts a compressive force against the beam and opposes the post-tensioning force. Note that in this particular case the angle of rotation of the column,  $\psi$  is larger than  $\phi$ . In order to alleviate or reduce these effects, the author is proposing the use of a truncated version of the same link beam as shown in Fig.(7). The proposed link beams comprise full length co-axial very rigid compression elements surrounded by bulk material, such as concrete, that houses the PT tendons and provides buckling stability for the compression core. It follows that in order to prevent contact between the column and the truncated end of the link beam the width of the initial gap  $\gamma$  should be larger than  $\bar{\phi}d/2$ . in Fig. 7 instead of using axial springs at the ends of the link beams and the wall base, equivalent rotational schemes have been

utilized to capture the restraining effects of the post tensioned tendons.

- While post-tensioning cables and their attachments are commercially available, their continuous disposition along the length of the frame, the link beams and the walls should be in strict conformance with the pertinent engineering principles. Their length, layout, cross sectional areas and the pre-stressing forces should be assessed in terms of the required drift angle, self-centering and collapse prevention requirements. The special cable layouts presented in Fig. (7) have been devised to eliminate loss of stretching due to simultaneous gap opening and closing at the ends of the same link beam.
- The slab acts as a rigid horizontal diaphragm. Seismic shear is transferred to the RWMF system through stressed tendons, compression of the link beam as well as direct shear connectors between the slab and the wall, Fig. (6). The physical separation between the slab and the wall and the link beam, prevents the slab from

being damaged during strong ground motion.

- Collapse prevention and self-centering become even more attractive if practical repairs following a severe earthquake can also be implemented. The replaceable joints symbolized in Fig.6 (a) are meant to dissipate seismic energy, control locations of the plastic hinges within the beams and prevent major damage to the frame. Similar pairs of repairable joints at the ends of the grade beams prevent damage to the footings and the formation of plastic hinges at column supports. The proposed detail consists of two parts, a welded beam-column moment connection and a repairable splice joint. The stub joint and the rest of the beam are designed to remain elastic while the splice joint develops the full plastic moment of the section. The versatility of the of proposed moment connection helps reduce initial out of plumbness and eliminate post-earthquake residual deformations.

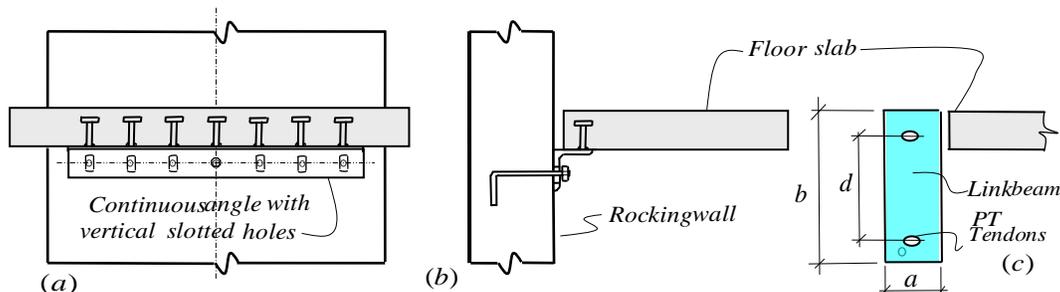


Figure 6 (a) Symbolic wall-slab shear connection, (b) Section showing same, (c) Link beam section

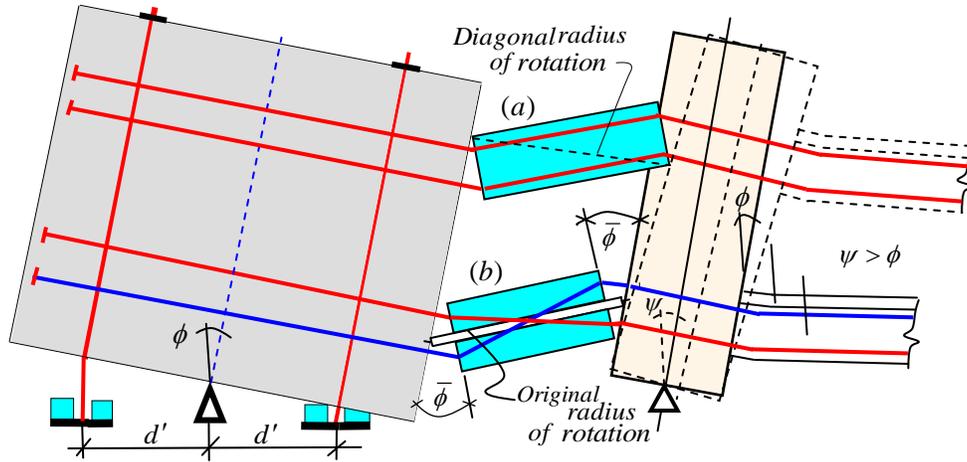


Figure 7 Rotations of link beams and adjoining walls (a), Full bearing both sides, (b) Both ends hinged

The mathematical treatment of the proposed structural system is presented as follows.

#### 4. Formulation of the mathematical equations

In Fig (8)  $S_i$  are the unknown interactive forces and  $Q_m$  is the roof level wall reaction.  $K_D$  And  $\bar{K}_D$  are the actual and the pseudo Symbolized by  $M_{D,i} \approx K_{D,i}\bar{\phi}$  and  $M_C \approx K_C\phi$ , etc., where  $K_C = d'^2 A_{cw} E_c / H$ .  $A_{cw}$  and  $E_c$

stiffnesses of the stressed link beams. The challenge here is to determine  $\phi_i, S_i$  and  $Q_m$ . Here, a rational assumption has been made that the high rigidity of the wall causes all wall mounted devices to absorb proportional amounts of energy, stand for the cross sectional area and the modulus of elasticity of the wall cables respectively.

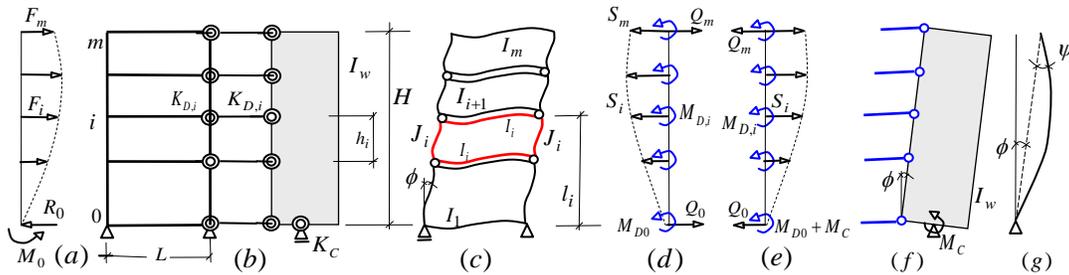


Figure 8 (a) Loading, (b) Proposed dual system, (c) Modularized free body diagram, (d) Forces acting on the frame, (e) Forces acting on the wall, (f) Rocking wall and added devices, (g) Displacement profile of the wall.

##### 4.1. Linear behavior

Next, consider the elastic response of the modularized model  $\phi_i = V_i / K_i h_i$  [47], where,  $K_i = 12EJ_i / h_i^3 (1 + \rho_i)$  is the elastic stiffness of module  $i$ . If  $V_i h_i$  is the story level racking

moment, then it may be shown that  $\sum_{i=1}^m V_i h_i = M_0$ , where  $M_0$  is the global overturning moment caused by the external forces  $F_i$ . Since,  $\phi_i = \bar{K}_{D,i} \phi$ , and that for constant drift  $\phi_i = \phi$  then the net shear force

acting on the roof module can be computed as  $V_m = Q_m + F_m - S_m$ . The net drift and the interacting force  $S_i$  can be expressed as;

$$S_i = F_i - [K_i h_i (1 + \bar{K}_{D_i}) - K_{i+1} h_{i+1} (1 + \bar{K}_{D_{i+1}})] \phi \quad (1)$$

$$\phi = \frac{M_0}{K_C + \sum_{i=1}^m [K_i h_i^2 (1 + \bar{K}_{D,i}) + \alpha K_{D,i}]} = \frac{M_0}{K^*} \quad (2)$$

#### 4.2 Plastic failure analysis

Assuming that the wall is strong enough to prevent soft story failure [46], the strong column-weak beam condition has been observed and that the plastic moments of resistance of the beams of the modules and supplementary devices are given by  $M_i^P$ ,  $M_{D,i}^P$  and  $M_C^P$  respectively then the following plastic failure scenarios can be envisaged.

1.  $M_C^P = M_{D,i}^P = 0$ . In which case the free standing frame will fail in a purely sway mode, as depicted in Figs.(4d, 4g). The corresponding collapse load can be related to;

$$M_{0,1} = 4 \sum_{i=1}^m M_i^P \quad (2.1)$$

2-  $M_{D,i}^P = 0$  and  $M_C^P \neq 0$ . In this case the frame will still fail in a purely sway mode, with the wall tendons yielding in tension. The corresponding collapse load can be shown to be equal to;

$$M_{0,2} = M_C^P + 4 \sum_{i=1}^m M_i^P \quad (2.2)$$

The corresponding drifts,  $\phi_2$ , can now be estimated by inserting  $K_i = K_{D,i} = \bar{K}_{D,i} = 0$  in Eq. (2).

3-  $M_C^P = 0$  and  $M_{D,i}^P \neq 0$ . In this particular case the frame will also fail in a purely sway mode, with the link beam tendons yielding in tension. The corresponding collapse load can be expressed as;

$$M_{0,3} = \sum_{i=1}^m (4M_i^P + 2M_{D,i}^P) \quad (2.3)$$

The corresponding drift,  $\phi_3$  can now be computed by replacing  $K_i h_i^2 (1 + \bar{K}_{D,i})$  with  $\alpha K_D$  and inserting  $K_C = 0$  in Eq. (2). And lastly;

4-  $M_D^P \neq 0$  and  $M_C^P \neq 0$ . In this scenario all moment resisting elements reach their ultimate carrying capacities. The corresponding failure load of the entire system can now be estimate as;

$$M_{0,4} = M_C^P + \sum_{i=1}^m (4M_i^P + 2M_{D,i}^P) \quad (2.4)$$

These failure scenarios can be used to study collapse prevention and self-centering strategies, in which case it would be appropriate to assume;  $M_C^P > M_{D,i}^P > M_i^P$ . If the wall tendons are the last moment resisting element at incipient collapse then the corresponding drift  $\phi_4$  may be estimated by inserting  $K_i = K_{D,i} = \bar{K}_{D,i} = 0$  in Eq. (2).

#### 5. Development of the moment frames

The novelty of Eq. (2) is in that it contains the additional term  $\bar{K}_{D,i}$ . It is interesting to note that Eq. (2) coincides with its original form for  $\bar{K}_{D,i} = 0$ , [21,46], and that the effect of the link beam is to increase the stiffness of the module from  $K_i$  to  $K_i (1 + \bar{K}_{D,i})$ . Two important cases arise,  $S_i = 0$  and  $S_i = F_i$ . The

first case implies no wall-frame interaction. i.e., either  $E_w I_w = 0$  or the moment frame is a structure of uniform response, as depicted in Fig. (5b), in which case Eq. (1) reduces to;

$$F_i = [K_i h_i (1 + \bar{K}_{D,i}) - K_{i+1} h_{i+1} (1 + \bar{K}_{D,i+1})] \phi = [V_i - V_{i+1}]$$

Or 
$$\phi = \frac{V_i}{K_i (1 + \bar{K}_{D,i}) h_i} \quad (3)$$

The obvious conclusion here is that it would be counterproductive to use rocking walls in conjunction with MFUR. The second case indicates that the wall is infinitely rigid,  $E_w I_w = \infty$ , and absorbs the entire external load. Eq. (1) yields;

$$[K_i h_i (1 + \bar{K}_{D,i}) - K_{i+1} h_{i+1} (1 + \bar{K}_{D,i+1})] \phi = 0$$

Or 
$$\phi = \frac{V}{K(1 + \bar{K}_D)h} \quad (4)$$

Where,  $V_i = V = Q = \sum_{i=1}^m F_i(x_i/H)$ ,  $K_i = K$  and  $\bar{K}_{D,i} = \bar{K}_{D,i+1}$ . It is tentatively concluded that

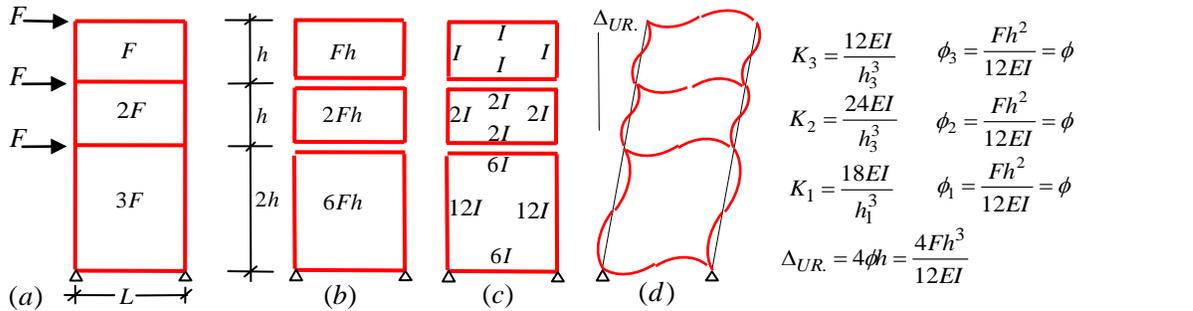


Figure 9 (a) 3-story MFUR and story shears, (b) Module racking moments, (c) Assigned sections, (d) Deformed shape

A summary of the results of Eqs. (5 and 6) is provided in Fig.(9c). Fig. (9d) depicts the deformed shape and maximum lateral displacement of the free standing MFUR. Substitution in Eq. (2) gives;

$$\phi = \frac{M_0}{\sum_{i=1}^m K_i h_i^2} = \frac{(4Fh + 3Fh + 2Fh)h}{12EI + 24EI + 18EI \times 4} = \frac{Fh^2}{12EI} \quad (7)$$

under certain circumstances, MFUS could be more economical than the corresponding MFUR.

**5.1. Case 1, Free standing MFUR-**  $S_i = 0$ ,  $m=3$  and  $K_{D,i} = \bar{K}_{D,i} = 0$ . Let  $L=h$  and  $I_3 = J_3 = I$ .

**Solution:** The story shears and racking moments can be computed as indicated in Figs (9a, 9b).

The drift formula for a doubly symmetric module has been established [46] as;

$$\phi_i = \frac{M_i h_i}{12E} \left[ \frac{h_i}{J_i} + \frac{L}{I_i} \right] = \frac{V_i}{K_i} = \frac{M_i}{K_i h_i} \quad (5)$$

The proportionality rules for uniform drift,  $\phi_i = \phi$ , require that;

$$\frac{J_i}{J_m} = \left( \frac{M_i}{M_m} \right) \left( \frac{h_i}{h_m} \right)^2 \quad \text{and} \quad \frac{I_i}{I_m} = \left( \frac{M_i}{M_m} \right) \left( \frac{h_i}{h_m} \right) \quad (6)$$

Similar substitutions in Eq. (1) verify the statement of the problem, thus;

$$S_3 - Q_3 = F - K_3 h_3 \phi = F - F = 0$$

$$S_2 = F - (K_2 h_2 - K_3 h_3) \phi = F - (2F - F) = 0$$

$$S_1 = F - (K_1 h_1 - K_2 h_2) \phi = F - (3F - 2F) = 0 \quad (8)$$

Assuming that unit weight is somehow proportional to its moment of inertia, then it may be shown;

$$G_{UR} = 4\eta lh + 8\eta IL + 12\eta IL + 24\eta l \times 2L = 72\eta IL \quad (9)$$

G stands for total weight. The maximum flexural stress of the lowermost module can be computed as;

$$\sigma_{UR} = \frac{6Fh(d_s/2)}{4 \times 6I} = \frac{Fhd_s}{8I} \quad (10)$$

$d_s$  is the effective depth of the section.

**5.2. Case 2, Free standing MFUS- $S_i = 0$ ,**  $m=3$  and  $K_{D,i} = \bar{K}_{D,i} = 0$ . Let  $L=h$  and  $\bar{I}_3 = \bar{J}_3 = \bar{I}$ .

**Solution:** The story shears and rocking moments can be computed as indicated in Figs. (10 a, 10b).

Since the story shear  $V_i$  is not uniform, drift ratio  $\phi_i$  can not be uniform either. However, the rules of proportionality for uniform drift,  $\phi_i = \phi$ , require that;

$$K_m h_m = \dots K_i h_i = \dots K_1 h_1 \quad \text{or} \quad K_3 h_3 = K_2 h_2 = K_1 h_1 \quad (11)$$

A summary of the results of Eqs.(11) and  $V_i = K_1 h_1 \phi$  is provided in Fig. (10c). Fig. (10d) depicts the deformed shape and maximum lateral displacement of the subject MFUS. It is apparent that  $\Delta_{CS} > \Delta_{UR}$ . In order to compare the two systems,  $\bar{I}$  should be selected in such a way that  $\Delta_{CS} = \Delta_{UR}$ . This gives

$$\bar{I} = \frac{9I}{4}, \quad \phi_1 = \frac{3Fh^2}{27EI}, \quad \phi_2 = \frac{2Fh^2}{27EI}, \quad \phi_3 = \frac{Fh^2}{27EI} \quad (12)$$

The total weight and the corresponding maximum flexural stresses can be computed as;

$$G_{CS} = 2 \times 4\eta \bar{I} h + 4\eta \bar{I} L + 8\eta \bar{I} \times 2L = 28\eta \bar{I} L = \left(\frac{28\eta 9I}{4}\right) L = 63\eta IL < 72\eta IL \quad (13)$$

$$\sigma_{CS} = \frac{6Fhd_s \times 4}{2 \times 4 \times 2 \times 9I} = \frac{Fhd_s}{6I} > \frac{Fhd_s}{8I} \quad (14)$$

It can be seen that while MFUS is lighter in weight,  $G_{CS} < G_{UR}$ , drift concentration, Eq. (12) and the maximum elastic stresses,  $\sigma_{CS} > \sigma_{UR}$  exceed those recorded for the corresponding MFUR. This indicates that the combination of an MFUS with a rigid rocking wall could alleviate these problems and lead to the development of an even more efficient structural configuration.

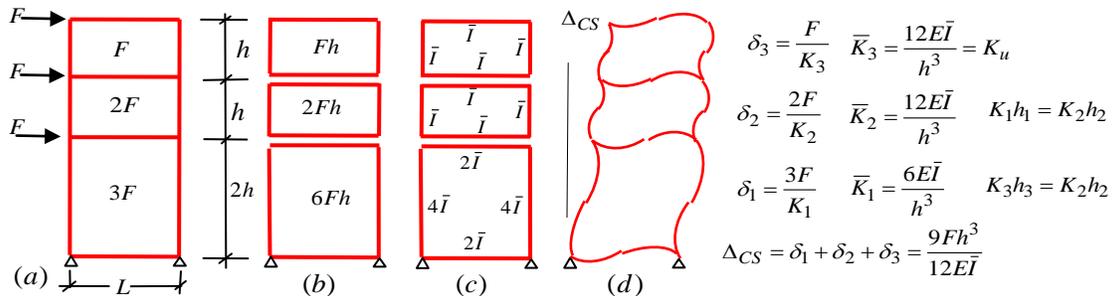


Figure10 (a) 3-story MFUS and story shears, (b) Module racking moments, (c) Assigned sections,(d) Deformed shape

**5.3. Case 3, RW-MFUS combination, m=3,**  
 $K_{D,i} = \bar{K}_{D,i} = 0, \bar{I}_3 = \bar{J}_3 = \bar{I}$  and  $E_W I_W = \infty_i$ .  
 Let  $L=h$ .

**Solution:** Since the wall is infinitely rigid then  $S_i = F_i$ . Note that as opposed to case 2, the story level shears are uniform, Fig.11 (a). A summary of story level racking moments and module moments of inertias has been provided in Figs.11 (c and d). A computation of the overturning moment  $M_0$  and maximum flexural stress is presented at the right hand end of Fig. 11. In the simplified form of Eq.(16);  $\phi = M_0 / \sum_{i=1}^m K_i h_i^2$ ,

$M_0$  is constant, and for fixed  $\phi$ ,  $\sum_{i=1}^m K_i h =$  Constant. Next, consider the combined response of an MFUS and rigid wall as follows;

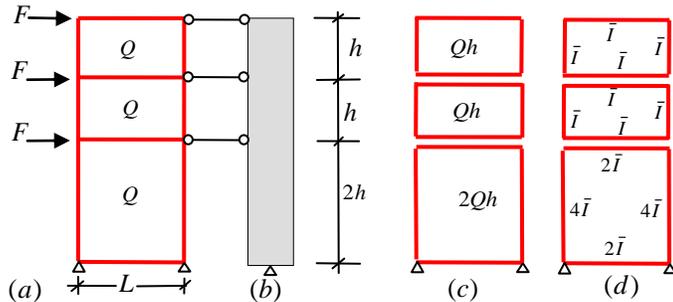


Figure 11 (a) 3-story MFUS and story shears, (b) Rocking wall(c) Module racking moments, (d) Assigned sections

**6. Wall stiffness analysis**

The configuration of the rocking wall and the nature of the self- equilibrating forces  $S_i$  and  $Q_m$  suggest that the wall tends to bend as an upright simply supported beam with a rigid body tilt  $\phi$ , rather than a fixed base cantilever. In other words the wall is a mechanism and its reference line of displacements passes through both the pin and the free end. Hence, its bending actions may be looked upon as that of an equivalent

$$\sum_{i=1}^m K_i h_i^2 = \sum \bar{K} h_1^2 + \bar{K} h_2^2 + \bar{K} h_3^2 = \left[ \frac{12E\bar{I}h^2}{h^3} + \frac{12E\bar{I}h^2}{h^3} + \frac{6E\bar{I}(2h)^2}{h^3} \right] = \frac{48E\bar{I}}{h} \tag{15}$$

Eqs.(1, 3 and 4) can be used to verify the validity of the original assumption that  $S_i = F_i$ . i.e.,

$$S_1 = F - \left( \frac{6E\bar{I} \times 2h}{h^3} - \frac{12E\bar{I} \times h}{h^3} \right) \phi = F - 0 = F,$$

$$S_2 = F - \left( \frac{12E\bar{I} \times h}{h^3} - \frac{12E\bar{I} \times h}{h^3} \right) \phi = F - 0 = F$$

$$\text{and } (S_3 - Q_3) = F - \bar{K}_3 h_3 \phi = F - \frac{9F}{4} = \frac{-5F}{4} .$$

Equilibrium of wall requires that;  
 $(F_3 - Q_3)4h + S_2 3h + S_1 2h = (F - Q_3)4h + 5Fh = 0$   
 or  $Q_3 = \frac{9F}{4}$ .

$$M_0 = 4Fh + 3Fh + 2Fh = 9Fh$$

$$\therefore 9Fh = 4Qh \quad Q = \frac{9F}{4}$$

$$\sigma = \frac{9F \times 2h \times 4(d_s / 2)}{4 \times 4 \times 9I \times 2} = \frac{Fhd}{8I}$$

$$\sigma = \sigma_{UR}$$

simply supported beam. For practical design purposes the stiffness of the wall can be related to a fraction of the prescribed uniform drift  $\phi$  of the system, say  $5\% \phi$  or  $\psi_{max} = \epsilon \phi$ . the following design data may be found useful for the preliminary estimation of high-rise wall stiffnesses under uniform and triangular distribution of lateral forces.

$$\text{Uniform load } \psi_{max} = \frac{Fh^2 m(m-1)(m^2 + m - 2)}{24E_w I_w}$$

$$\& I_{w,\min} = \frac{Fh^2(m-1)(m^2+m-2)}{24E_w \varepsilon \phi} \quad (16)$$

Triangular load

$$\psi_{\max} = \frac{Fh^2(m-1)(2m-1)(2m^2+3m-4)}{180E_w I_w m} \quad \&$$

$$I_{w,\min} = \frac{Fh^2(m-1)(2m-1)(2m^2+3m-4)}{180E_w \varepsilon \phi m} \quad (17)$$

## 7. Use of energy equivalency for base shear analyses

Eq. (15) can be used to plot accurate load deformation curves for the RWMF types discussed in this article. Three distinct displacement limits can be associated with RWMF of Fig. (18a),  $\phi_Y$  at first yield,  $\phi_P$  at incipient collapse and  $\phi_\mu$  at the ductility limit. If the wall remains elastic and that because of any reasons, such as RBS treatment of all other beams, the last set of plastic hinges form at the ends of the grade beam, then the corresponding drift ratios at first yield and at incipient collapse can be expressed as;

$$\phi_Y = 9Fh^2/48E\bar{I} \quad \text{and} \quad \phi_P = 9Fh^2/96E\bar{I} \quad (18)$$

respectively. While  $\phi_P$  is twice as high as its linear counterpart,  $\phi_\mu = \mu\phi_Y$  and is a function of the period related ductility factor  $\mu$ . The total energy absorption capacity of the structure in terms elastic energy  $U_Y$  and plastic energy  $U\mu = U_P + \partial U_\mu$  as displayed in Fig. (18f) can be computed as;

$$U = U_Y + U_P \quad (19)$$

The utility of Eq.(19) becomes apparent when used in conjunction with Housner's [26,27] equal energy concept;

$$U = U_S = \frac{\gamma M}{2} \left( \frac{T}{2\pi} S_a g \right)^2 \quad (20)$$

Where,  $S_a$  and  $\gamma = (2\mu_s - 1)/R_\mu^2$  are the Spectral acceleration and the Energy equivalency factors respectively.  $M$  and  $g$  are the total mass of the system and the gravitational acceleration.  $R_\mu$  is the period dependent ductility reduction factor respectively. a thorough discussion of the classical of  $T - \mu_s - R_\mu$  relationship can be found in [51].

## 8. Collapse prevention

The proposed structural system contains two independent drift restraining mechanisms, the post-tensioned rocking wall and the link beams. These devices can be utilized either on their own or in combination with each other. However, it would be safe to assume that for all practical intents and purposes the wall alone is capable of performing as expected. Seismic collapse is usually triggered by structural instability or the  $P$ -delta phenomenon, preceded by the formation of ductile failure mechanisms. Plastic failure mechanisms often undergo large lateral displacements that in turn lead to catastrophic collapse. While gravity forces, as active component of the  $P$ -delta effect, are constant quantities, lateral displacements can be controlled, even reversed or re-centered by means of RWMF technologies related with Eq. (6), provided that the wall remains elastic and suppresses soft story failure. The accumulative effect of the local  $P$ -delta moments on the rotational capacity of the combined structure can be expressed as Eq.(21):

$$\phi = \frac{M_0}{f_{CR}K^*} \quad (21)$$

Where,  $f_{CR} = [1 - P^*H / 2K^*]$  may be interpreted as the global load reduction function due to destabilizing effects of the gravity loading.  $P^*$  is the equivalent total gravity load defined as ;

$$\frac{P^*\phi H}{2} = \frac{P_w H \phi}{2} + \sum_{i=1}^m P_i \times l_i \phi \quad (22)$$

$P_w$  is the total weight if the wall. At incipient collapse or formation of the anticipated plastic failure mechanisms all  $K_i$  become zero, and  $K_{D,i}$ ,  $\bar{K}_{D,i}$  may conservatively assumed to be zero. Subsequently, the global stiffness of the combined system, Eq. (2), reduces to  $K^* = K_C$ . In other words if complete collapse is to be prevented after formation of the preferred plastic mechanism, then the surviving vertical cable system should be strong enough to withstand the entire seismic demand and stiff enough to develop the drift ratio associated with collapse prevention. There are several commercially available devices and technologies that can be utilized as base level stabilizers for pin supported rocking walls. However the generic stressed tendon arrangement adopted in this study serves well to introduce the basic issues involved in the preliminary design of such items. The post tensioned tendons not only act as lateral stabilizers, but also add strength and stiffness to the wall. The moment capacity and rotational stiffness of the base level stressed cable system have been defined as  $M_C = T_w d'$  and  $K_C = d'^2 A_{cw} E_c / H$ .  $T_w$  is the tensile force of the cables due to lateral forces.  $A_{cw}$  and  $E_c$  stand for the cross

sectional area and the modulus of elasticity of the wall cables respectively. With  $P^*$  and  $\phi_{collapse}$  known, the basic design parameters for collapse prevention can be computed as;

$$T_w > \Omega(M_0 + \bar{M}_0) / d' = \Omega(2M_0 + P^* \phi_{collapse} H) / 2d' \\ \text{and } A_{cw} E_c = T_w H / \phi_{collapse} d' \quad (23)$$

Where  $\Omega$  is the over strength factor defined by the pertinent code of practice.

## 9. Concluding remarks

Construction is one of the largest end users of environmental resources and one of the largest polluters of manmade and natural environments. The improvement in the performance of buildings with regard to the environment will indeed encourage greater environmental responsibility and place greater value on the welfare of future generations.

A relatively new seismic structural system incorporating post tensioned rocking walls and moment frames has been presented. Both vertical as well as horizontal stressed tendons and co-linear gap opening devices have been provided to ensure collapse prevention and active re-centering. The post-tensioning produces a resisting moment to service loading along the frame and provides restoring forces at the ends of the link beams that tend to return the frame and the wall to their pre-earthquake position. The proposed mathematical model lends itself well to SDOF treatment. A theoretically exact formula for the preliminary design of such systems has also been presented. The proposed configuration satisfies the theoretical conditions of minimum weight. A new gap opening link mechanism that does not induce unwanted moments in the

columns has also been introduced. The proposed structural scheme is neither perfect nor complete. It is still under development and needs the test of time and scrutiny before it becomes a viable earthquake resisting system.

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