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Estimating Inter-Story Drift in High Rise Buildings with the Flexural and Shear Cantilever Beam and Mode-Acceleration Method

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ABSTRACT

In this study, the seismic inter-story drift of structures is estimated by a combination of mode-acceleration equations with the modelling of high-rise buildings with flexural and shear cantilever beams. In the equation presented for calculating the inter-story drift, having less knowledge of the building is adequate and this issue is of significance in estimating the nonstructural component forces, especially in high-rise buildings and also in the initial design of structures. Also, a comparison of inter-story drift estimated by the approximation method with an exact method indicates that the application of the mode-acceleration method compared to mode-displacement with a fewer number of modes comes close to the exact calculation, which facilitates and expedites the analysis. In order to carry out an exact evaluation of the presented equation, inter-story drift is calculated and compared in 10, 15 and 50 story buildings during three seismic records using approximate relations. Exact analysis of those structures is done in finite element Opensees software. The results of comparisons show that the presented equation provides an adequate estimation without the need for modelling and lengthy software analysis.

1. Introduction

During the past few years, with regard to the retrofit of buildings and optimization of new

buildings, the seismic behavior of structures has been improved. On this basis, damages from earthquakes occur inside buildings and in addition to fatalities, damages to

nonstructural components result in great financial loss [1]. In most buildings, the expenditures for nonstructural components and building content are many times that of constructing the building itself. Also, damages to nonstructural components of buildings result in the performance of many specific or important buildings to come to a halt. For example, damage to nonstructural components and content of a hospital during a relatively severe earthquake results in great hazards [2,3]. Thus, the issue of preventing fatalities and damages to nonstructural components and building content is very serious and the significance of paying attention to this issue even with approximate calculations also results in less fatalities and financial loss.

With regard to Figure 1, a comparison of the financial value of various building components with regard to their use is shown

As it is observed in Figure 1, the financial value of the structural components of buildings compared to the nonstructural component and building content is less than preventing 20%. Thus. damage to nonstructural components in buildings is of great significance.

The inter-story drift in buildings during earthquakes is often among issues that result in great loss to nonstructural components. Nonstructural components sensitive to interstory drift in buildings are observed in Table 1.





Table 1. Nor	structural components sensitive to
	inter-story drift [5].
Samaitivity	Commonant

Schlivity	Component
	Building walls
	Windows
	Inner doors
	Partitions
Sensitive to drift	Ceiling plaster
	Electronic systems
	Partitions
	Doors
	Elevator cabin

Therefore, with regard to the sensitivity of nonstructural components to inter-story drift (Table 1), obtaining an approximate seismic response of inter-story drift in buildings is very important to prevents irreparable financial damages and fatalities.

In this study, an equation is proposed for estimating the inter-story drift using the mode-acceleration equation based on the structural modelling method with shear beams.

1.1 Literature Review

1.1.1 Inter-Story Drift

The ratio of inter-story drift (IDR) in buildings is defined as the difference in displacement of upper and lower floors of a story divided by the height of that floor [12,16].studies by Miranda calculate the ratio of inter-story drift for the j^{th} story using the following equation:

$$IDR(j,t) = \frac{1}{-\sum_{i=1}^{n} \Gamma} \prod_{i=1}^{n} \left[\phi_{i}(j+1) - \phi_{i}(j) \right]_{i} D_{i}(t)$$
(1)

In this equation, $\phi i(j)$ and $\phi i(j+1)$ are the ith vibration mode in *j* and *j*+*1* stories, respectively. *h* is the story height, *n* the number of stories, and Γ_i Modal participation factor of mode i^{th} [14].

1.1.1. 1.1.2 Modelling High Rise Buildings with the Flexural and Shear Cantilever Beam Method

The simple building model includes one continuous elastic model. In previous studies, continuous models were also suggested for estimating the structural response to wind or earthquakes [6, 7]. The model used in this study includes a combination of flexural and shear cantilever beam, which transforms under flexure and shear, respectively. Flexural and shear cantilever beams are connected by means of a rigid member and transfer the horizontal forces. Therefore, throughout the model height, both beams transformations experience equal [8]. Numerous research studies have been carried out regarding the cantilever beam methods for modelling and obtaining an approximate structural response in various loading. Nevertheless, Miranda [9] has had the main role in completing this method. Research carried out by Miranda and his colleagues [9, 10] have increased the precision of this method and proposed numerous revisions, hence improving it. The continuous model used includes a flexural cantilever beam and a shear cantilever beam, which transform in flexure and shear, respectively (Figure 2).



Fig. 2. Flexural and shear transformation in flexural and shear cantilever beam [11].

In the model proposed in Figure 2, the flexural and shear cantilever beam are connected together with an unlimited number of rigid members, which transfer the horizontal forces. The flexural and shear cantilevers in the hybrid system along the entire height are situated under equal lateral displacement.

The equation below is the response of the modeled structure using the flexural and shear cantilever continuum beam under base acceleration, which is not dissimilar to the equation of motion:

$$\frac{\partial^{2}u(x,t)}{\partial t^{2}} + \frac{c}{\rho}\frac{\partial u(x,t)}{\partial t} + \frac{EI}{\rho H^{4}}\frac{\partial^{4}u(x,t)}{\partial x^{4}}$$
(2)
$$-\frac{EI\alpha^{2}}{\rho H^{4}}\frac{\partial^{2}u(x,t)}{\partial x^{2}} = -\frac{\partial ug(t)}{\partial t^{2}}$$

In Equation 2, ρ is the mass per unit length in the cantilever beam model, H is the overall building height, u(x,t) is the normalized relative lateral displacement in height (which is variable between 0 at the structure foundation and 1 at the roof surface) and at time *t*, *c* is the damping coefficient per unit length, EI is the flexural rigidity of the flexural beam and α is the ratio of lateral stiffness which is defined as Equation 3:

$$\alpha = H_{\sqrt{\frac{GA}{EI}}}$$
(3)

With regard to the equation, GA is shear cantilever beam rigidity and EI is the flexural cantilever beam rigidity. The amounts of α equal to zero indicate a pure flexural cantilever beam (Euler-Bernoulli beam) and unlimited amounts show a pure shear cantilever beam. Average amounts of α show multistory buildings in which the lateral displacements are a combination of flexural shear lateral displacements. and The simplified model of the shear and flexural cantilever beam is obtained based on Equation 4:

$$\phi_{i}(x) = \sin(\gamma_{i}x) - \frac{\gamma_{i}}{\beta_{i}} \sinh(\beta_{i}x) - \eta_{i}\cos(\gamma_{i}x) \qquad (4)$$
$$+\eta_{i}\cosh(\beta_{i}x)$$

in which β_i and η_i are dimensionless parameters for the *i*th vibration mode which is obtained based on the following equations:

$$\beta_i = \sqrt{\alpha^2 + \gamma_i^2} \tag{5}$$

$$\eta_{i} = \frac{\gamma_{i}^{2} \sin(\gamma_{i}) + \gamma_{i} \beta_{i} \sinh(\beta_{i})}{\gamma_{i}^{2} \cos(\gamma_{i}) + \beta_{i}^{2} \cosh(\beta_{i})}$$
(6)

and γ_i is the eigenvector of the *i*th vibration mode which is dependent on the *i*th root of the equation character below:

$$2 + \left[2 + \frac{\alpha_0^4}{\gamma_i^2 \beta_i^2}\right] \cos(\gamma_i) \cosh(\beta_i) + \left[\frac{\alpha_0^2}{\gamma_i \beta_i}\right] \sin(\gamma_i) \sinh(\beta_i) = 0$$
(7)

the vibration period of higher modes (T_i) can be relatively obtained using the main vibration building mode (T1), as follows:

$$\frac{T_i}{T_1} = \frac{\beta_1 \gamma_1}{\beta_i \gamma_i}$$
(8)

With regard to the fact that masses are considered constant at the cantilever beam height, the modal participation factor Γ_i can be calculated by the following equation:

$$\Gamma_{i} = \frac{\int_{0}^{1} \phi_{i}(x) dx}{\int_{0}^{1} \phi_{i}^{2}(x) dx}$$
(9)

1.1.3 Mode-Acceleration

In 1983, Cornwell, Craig and Johnson presented a new mode-acceleration equation to obtain the structural response to dynamic loading. The mode-acceleration equation is of higher accuracy in convergence to the correct response compared to the modedisplacement equation [4]. The modeacceleration equation is as follows:

$$u(x,t) = \sum_{i=1}^{M} \phi_i(x) \left[\left(\frac{2\xi_i}{\omega_i} \right) \dot{D}_i(t) + \left(\frac{1}{\omega_i^2} \right) \ddot{D}_i(t) \right]$$
(10)

The parameters of the equation are defined as follows:

 $\begin{array}{ll} \begin{array}{c} \mbox{relative system speed for one} \\ \dot{D}_{i}(t) & \mbox{degree of freedom of the } i^{th} \mbox{ mode} \\ (m/s). \\ \omega_{i} & i^{th} \mbox{ mode frequency.} \\ \xi_{i} & i^{th} \mbox{ mode damping.} \\ \end{array} \\ \begin{array}{c} \mbox{Displace mode figure for the } i^{th} \end{array}$

The mode-acceleration equation compared to the mode-displacement equation requires less participation of vibration modes in order to reach the appropriate dynamic response. Also, the mode-acceleration equation in high-rise buildings shows better performance [14].

2. Methodology

 $\phi_i(x)$

mode.

In this study, by combining the modeacceleration and flexural and shear cantilever beam response equations, a new equation is presented for calculating inter-story drift in structures. The derivative of the modeacceleration equation with respect to the dimensionless parameter (x=z/H) provides drift response based on the modeacceleration equation:

$$\theta(x,t) = \frac{\partial u(x,t)}{\partial x}$$
$$= \frac{1}{H} \sum_{i=1}^{\infty} \Gamma_{i} \phi_{i}'(x) \left[\left(\frac{2\xi_{i}}{\omega_{i}} \right) \dot{D}_{i}(t) + \left(\frac{1}{\omega_{i}^{2}} \right) \ddot{D}_{i}(t) \right]$$
(11)

in which $\phi'_i(x)$ is the first derivative of the i^{th} mode figure compared to dimensionless altitudinal parameter x. A comparison of inter-story drift for the j^{th} story is calculated by the following equation:

$$IDR(j,t) = \frac{1}{H}$$

$$\times \sum_{i=1}^{n} \Gamma_{i} \left[\phi_{i}(j+1) - \phi_{i}(j) \right] \left[\left(\frac{2\xi_{i}}{\omega_{i}} \right) \dot{D}_{i}(t) + \left(\frac{1}{\omega_{i}^{2}} \right) \ddot{D}_{i}(t) \right]$$
(12)

in which $\phi_i(j+1)$ and $\phi_i(j)$ are the *i*th vibratory modes in floors *j* and *j*+1, respectively. *h* is the story height and *n* are the number of stories. Here, the ratio of inter-story drift in the *j*th story, based on mode-acceleration equation, is e equal to the rotation of the midheight of story j in the model as shown in the following equation:

$$IDR(j,t) \approx \theta(x,t) = \frac{1}{H} \sum_{i=1}^{\infty} \Gamma_i \phi_i'(x) \left[\left(\frac{2\xi_i}{\omega_i} \right) \dot{D}_i(t) + \left(\frac{1}{\omega_i^2} \right) \ddot{D}_i(t) \right]$$
(13)

in which x is the level at mid-height of floors j and j+l. By using the mode-acceleration equation for presenting the overall inter-story drift equation and inter-story drift, we can benefit from the association of much fewer modes for the calculation of drift. In the mode-acceleration method, using the

acceleration and velocity response of the one degree of freedom system has higher accuracy and convergence speed in estimating structural response compared to the use of displacement response, which was used in the mode-displacement equation [14,15]. Nevertheless, the ratio of inter-story drift in dimensionless height x, is estimated by the following equation:

$$IDR(\mathbf{x},t) \approx \frac{1}{H} \times \sum_{i=1}^{M} \Gamma_{i} \phi_{i}'(\mathbf{x}) \left[\left(\frac{2\xi_{i}}{\omega_{i}} \right) \dot{D}_{i}(t) + \left(\frac{1}{\omega_{i}^{2}} \right) \ddot{D}_{i}(t) \right]$$
(14)

in which m is the number of modes which take part in obtaining the drift response. By using the seismic response equation, the inter-story drift in the simple flexural-shear beam model is obtained by using the modeacceleration equation.

3. Introducing the Models and Results of Analyses

For evaluating the presented equation (Equation 14), 10, 15 and 50 story frames are considered. The evaluated columns are three flexural steel frames designed based on UBC94 regulations. It is supposed that the structures are placed in seismic zone 4 and soil type 2 region . The height of 10, 15 and 50 story buildings are 40.21, 51.8, and 150.2 m, respectively with a basic period of 1.9, 2.11, and 3.66 s. The damping percentage is considered as 5% and the overall weight of each story is 1107, 900 and 855 kN in three frames, respectively. The beam and column

arrangement in the frames is shown in Tables 2, 3 and 4.

Table 2. Beam and Column Layout in 10-storyframe with 3 openings.

Material Properties					
Inner Beams	Side Beams	Inner Columns	Side Columns	Stories	
W24*55	W24*55	W14*176	W14*109	1,2	
W21*50	W21*50	W14*132	W14*82	3,4	
W21*50	W21*50	W14*109	W14*74	5,6	
W21*44	W21*44	W14*99	W14*68	7,8	
W16*40	W16*40	W14*68	W14*48	9,10	

Table 3. Beam and Column Layout in 15-storyframe with 3 openings.

Material Properties					
Inner Beams	Side Beams	Inner Columns	Side Columns	Stories	
IPE30	IPE27	2IPE33	2IPE27	1-3	
IPE30	IPE27	2IPE30	2IPE24	4-6	
IPE30	IPE27	2IPE27	2IPE22	7-9	
IPE30	IPE27	2IPE24	2IPE20	10-12	
IPE30	IPE27	2IPE22	2IPE18	13-15	

Table 4. Beam and Column Layout in 50-storyframe with 4 openings.

Material Properties				
Inner Beams	Side Beams	Inner Columns	Side Columns	Stories
W14*176	W14*176	Box 50*50	Box 50*50	1-10
W14*132	W14*132	Box 45*45	Box 45*45	11-20
W14*109	W14*109	Box 40*40	Box 40*40	21-30
W14*99	W14*99	Box 35*35	Box 35*35	31-40
W14*68	W14*68	Box 30*30	Box 30*30	41-50

By using Equation 3, the amount of lateral stiffness ratio for two frames of 10 and 15 stories is equal to 0.30 and for the 50-story frame it is obtained as 3.3. Also, the damping coefficient is considered 5% for all vibration

modes. For the lateral stiffness ratio equal to α =0, 3, 7 and 30, mode figures are obtained using equation 8 as follows. The mode participation obtained from Equation 9 are implemented in obtaining the mode figures.



----- ALPHA=7 ALPHA=30







Third Mode



Fig. 3. The effect of dimensionless parameter of lateral stiffness ratio on mode figures.

In order to evaluate the accuracy of the presented equation, the mentioned structures have been modeled and analyzed in opensees software under three earthquake records considering linear material behavior. The records consists of 1) Imperial Valley Earthquake El Centro Station, 2) Kern County earthquake Taft Station, 3) Loma Prieta earthquake Coralitus station. Drift results in the structures stories are obtained from the software. Figures 4 (a-i) compare the maximum inter-story drift of the stories resulting from the time history analyses in OpenSEES software with the results obtained from flexural and shear cantilever beam model and using Equation 14.



a) 10-story Frame-Imperial Valley Earthquake



b) 10-story frame- Kern County Earthquake



c) 10-story frame- Loma Prieta Earthquake



d). 15-story frame- Imperial Valley Earthquake



e) 15-story frame- Loma Prieta Earthquake



f) 15-story frame- Kern County Earthquake



g) 50-story frame- Imperial Valley Earthquake



h) 50-story frame- Kern County Earthquake



i) 50-story frame- Loma Prieta Earthquake

Fig. 4. Comparison of inter-story drift response obtained from time history analysis in the software with acceleration obtained from the presented equation.

As it is observed in Figures 4(a-i), using structural modelling with the cantilever beam method and using the mode-acceleration equation is an appropriate method for estimating the inter-story drift in high-rise buildings. In other words, the accuracy of the drift response equation using the modeacceleration converges to an accurate response with fewer modes.

In Figures 5 (a-g), the effective mode numbers are evaluated in obtaining an accurate response in the equations provided for acceleration in 10, 15 and 50-story structures.



■ Mode-Displacement L'Mode-Acceleration



a) 10-story building- 5th story- Imperial Valley Earthquake

b) 10-story building- 5th story- Kern County Earthquake





c) 10-story building- 4th story- Loma Prieta Earthquake

d) 15-story building- 9th story- Imperial Valley Earthquake



■ Mode-Displacement ■ Mode-Acceleration

e) 15-story building- 11th story- LomaPrieta Earthquake



■ Mode-Displacement ■ Mode-Acceleration



f) 50-story building- 25th story- Kern County Earthquake

Mode-Displacement Node-Acceleration



In fact, in Figures 5, two mode-acceleration mode-displacement methods and are evaluated. With regard to Figures 5 (a-g), in the 10-story structure, the seismic response of the structure with the cooperation of 6 vibration modes with the mode-displacement method converges to the response of the mode-acceleration method with the cooperation of 3 vibration modes. Also, in the 15-story structure, structural seismic response with the cooperation of 8 vibration

modes with the mode-displacement method converges to the mode-acceleration response with the cooperation of 5 vibration modes. In the 50-story structure, the seismic response with cooperation of 6 vibration modes with the mode-acceleration method converges to the mode-displacement response with the cooperation of 18 vibration modes. In fact, the figures for inter-story drift indicate suitable results of using the modeacceleration response for high-rise buildings, which come closer to computer analyses with the cooperation of fewer modes.

High-rise structure modeling using the flexural-shear cantilever beam method is an appropriate method for structural analysis. In fact, by using structural modelling techniques in the form of flexural and shear beam utilizing less information of the structure, an adequate estimation of structural seismic behavior is obtained.

4. Conclusion

As the results indicate, the proposed equation appropriate provides an estimate bv considering the number of modes for obtaining inter-story drift in various earthquakes. In the equations presented for drift, the mode-acceleration inter-story method is used. Contrary to the inter-story drift of Miranda, which was based on the mode-displacement technique, instead of using six mode vibration, three modes have been used. Therefore, it is clear that using the mode-acceleration method instead of the mode-displacement method requires the cooperation of fewer modes to estimate the response. Reduction in the number of cooperative modes in the mode-acceleration method does not reduce the precision in its seismic response estimation method. With regard to the capability of the proposed equation, in normal buildings with governing first mode, the first mode is used, and in high-rise buildings, the effect of higher modes can be considered. With an increase in the number of modes, the accuracy of the proposed equation increases and the response obtained from the equation comes closer to the accurate solution. In the equation proposed for analyzing and obtaining interstory drift, much less information is

necessary compared to the dynamic structural parameters being evaluated. This fact will result in an increase of approximate analysis speed and estimation of structural inter-story drift. In the introductory design of high-rise buildings, an appropriate estimate of story drifts can be obtained by using the proposed equation. This can provide a better outlook on nonstructural element design which is sensitive to inter-story drift. For existing buildings, inter-story drift obtained from the proposed equation can result in implementing provisions nonstructural for securing elements of buildings.

5. Nomenclature

 β : Mass per unit length in the model (N/m) H : Overall structural height (m) u(x,t): Seismic response of displacement in a specific place and time (m). Evaluated x : height in the model (dimensionless). t: Time (s) c : Damping per unit length EI: Flexural rigidity of flexural beam (Kg.m²) α : Lateral stiffness ratio (dimensionless) GA: Shear rigidity of shear beam (Kg.m²) h_i : Height of ith floor (m) L: Aperture of floor beams i (m) E: reactionary coefficient (Kg/m^2) I_{c} : moment of inertia of columns (m⁴) I_{g} : moment of inertia of girders (m⁴) A: area column (m^2) d: Distance of column from neutral warp (m) β_i : Dimensionless parameter for the i^{th} vibration mode(m) γ_i : Eigen vector of the ith vibration mode T_i : Vibration period of the ith mode T₁: Basic period (s)

- $$\begin{split} & \Gamma_{i}: \text{Cooperation coefficient of the } i^{\text{th}} \text{ mode} \\ & \frac{\partial^{2} u(x,t)}{\partial t^{2}}: \text{Relative acceleration } (\text{m/s}^{2}) \end{split}$$
- $\frac{\partial^2 u_g(t)}{\partial t^2}$: Earth acceleration (m/s²)

 $D_i(x)$: One degree of freedom system displacement response for the ith mode (m)

 $\phi_i(x)$: Displace mode figure for the ith mode

 $u_i(x,t)$: Displacement response of ith mode (m)

 $\ddot{u}'(x,t)$: Absolute acceleration at dimensionless height *x* in time *t* (*m*/s²).

 $\xi_i \colon i^{th} \text{ damping mode}$

 $\boldsymbol{\omega}_{\mathbf{i}}: \mathbf{i}^{\text{th}} \text{ mode frequency}$

 $\dot{D}_{i}(t)$: relative system speed for one degree of freedom of the ith mode (m/s).

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