

Adaptive Neural Fuzzy Inference System Models for Predicting the Shear Strength of Reinforced Concrete Deep Beams

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ARTICLE INFO

Article history:

Received: 27 June 2015

Accepted: 25 August 2015

Keywords:

Shear strength,

RC deep beams,

Adaptive Neural Fuzzy Inference System (ANFIS).

ABSTRACT

A reinforced concrete member in which the total span or shear span is especially small in relation to its depth is called a deep beam. In this study, a new approach based on the Adaptive Neural Fuzzy Inference System (ANFIS) is used to predict the shear strength of reinforced concrete (RC) deep beams. A constitutive relationship was obtained correlating the ultimate load with seven mechanical and geometrical parameters. These parameters contain Web width, Effective depth, Shear span to depth ratio, Concrete compressive strength, Main reinforcement ratio, Horizontal shear reinforcement ratio and Vertical shear reinforcement ratio. The ANFIS model is developed based on 214 experimental database obtained from the literature. The data used in the present study, out of the total data, 80% was used for training the model and 20% for checking to validate the model. The results indicated that ANFIS is an effective method for predicting the shear strength of reinforced concrete (RC) deep beams and has better accuracy and simplicity compared to the empirical methods.

1. Introduction

Deep beam has usually been a subject of interest in structural engineering practice. With the strong growth of construction work in many developing countries, deep beam design and its behavior prediction is a issue

of considerable relevance. Some examples of deep beams include bridge bent caps, transfer girders, and pile caps [1]. The prediction of the ultimate shear strength of RC deep beams is critical particularly when this value is used in designing. An un-conservative value of shear capacity leads to failure, a classic

example is the partial collapse of the wilkins Air Force Depot warehouse in ohio [2]. The main obstacle to the shear problem is the large number of parameters involved, some of which many not be known [3]. The most important parameters affect the behavior of the load capacity of deep beams were extracted from literature review [4-7].

Several empirical formulas are proposed to determine the shear strength of reinforced concrete (RC) deep beams, such as The American Concrete Institute (ACI) code [8-11], Canadian Standard Association (CSA) [12], strut-and-tie model (STM) [13,14].

ANN model suggested to the prediction of RC beams shear strength. At the first time Gohutilized ANN to predict the ultimate shear strength of RC deep beams [15]. Yeh used augment-neuron networks to model concrete strength [16]. Atici predicted the strength of mineral admixture concrete by ANN [17]. Oreta and Kawashima applied the neural network in the modeling of the confined compressive strength and strain of circular concrete columns [18].Sanad and Saka used ANN for prediction of ultimate shear strength of RC deep beams [19].Lee used ANN for concrete strength prediction [20]. Kim et al. applied ANN for estimation of concrete strength [21].Mansour et al. used ANN to predict the shear strength of RC beams [22]. Cladera and Mari used ANN inbeams with stirrups for the shear design procedure of normal and high strength reinforced concrete beams [23]. Tang CWutilized radial basis function neural networks to estimate the torsional strength of reinforced concrete beams [24].Abdalla et al. simulated the shear of RC beams with ANN[25]. Caglar et al. applied the neural network in dynamic analysis of reinforced concrete buildings [26]. Arslan predicted the

tension strength of RC beams by ANN [27].GEP is one of the other methods suggested for predicting the shear strength of RC beams [28-30].

Addition to the above-mentioned methods, adaptive neuro-fuzzy inference system (ANFIS) proved to be simple and powerful tool for predicting the shear strength of reinforced concrete is (RC) deep beams. Adaptive neuro-fuzzy inference system (ANFIS) is a fuzzy inference system (FIS) implemented in the framework of adaptive neural networks which includes both the fuzzy logic qualitative approximation and the adaptive neural network capability. It has been successfully utilized for modeling in many engineering field [31-35].

The main purpose of this paper is to utilize the ANFIS technique for predicting the shear strength of RC deep beams. The proposed model is developed based on 214 experimental database obtained from the literature [36-43]. The performance of the ANFIS is compared with results obtained by empirical CSA and ACI codes and ANN and GPE models.

2. Materials and methods

2.1. Data description

Experimental data used in this study consist of 214 RC deep beams collected from the literature. The dataset includes 52, 25, 37, 53, 4, 12, 19 and 12 RC deep beams taken from [36] to [43], respectively. 80% of total data are randomly chosen as training data for developing the model and the remaining 20% of dataset which were not exposed in the training data chosen as test set. the range and average of the training and test datasets are presented in Tables1 and 2.Input parameters used to estimate the shear strength of RC

deep beams consist of the web width (b), effective depth (d), shear span to depth (a/d), concrete compressive strength (f'_c), main reinforcement ratio (ρ), horizontal shear reinforcement ratio (ρ_h) and vertical shear

reinforcement ratio (ρ_v). shear strength (V) is used as the output variable. The geometrical parameters of RC deep beam are shown in Fig.1.

Table1. The Range and average of the training dataset

| Parameter | Training data (171 data set) | | |
|---|------------------------------|--------|------|
| | Max | Avg. | Min |
| b Web width (mm) | 310 | 130 | 80 |
| d Effective depth (mm) | 800 | 420 | 220 |
| a/d Shear span to depth | 2.7 | 1.21 | 0.27 |
| f'_c Concrete compressive strength (MPa) | 73.6 | 33.32 | 13.8 |
| ρ Main reinforcement ratio (%) | 4.08 | 1.82 | 0.52 |
| ρ_h Horizontal shear reinforcement ratio (%) | 2.45 | 0.31 | 0 |
| ρ_v Vertical shear reinforcement ratio (%) | 2.65 | 0.56 | 0 |
| V Ultimate load strength of RC beam (kN) | 1214.7 | 306.45 | 43 |

Table2. The Range and average of the testing dataset

| parameter | Testing data (43 data set) | | |
|---|----------------------------|--------|------|
| | Max | Avg. | Min |
| b Web width (mm) | 310 | 130 | 80 |
| d Effective depth (mm) | 800 | 430 | 310 |
| a/d Shear span to depth | 2.34 | 1.24 | 0.27 |
| f'_c Concrete compressive strength (MPa) | 73.6 | 33.51 | 16.2 |
| ρ Main reinforcement ratio (%) | 4.08 | 1.78 | 0.52 |
| ρ_h Horizontal shear reinforcement ratio (%) | 0.94 | 0.32 | 0 |
| ρ_v Vertical shear reinforcement ratio (%) | 2.65 | 0.47 | 0 |
| V Ultimate load strength of RC beam (kN) | 1286 | 296.66 | 110 |

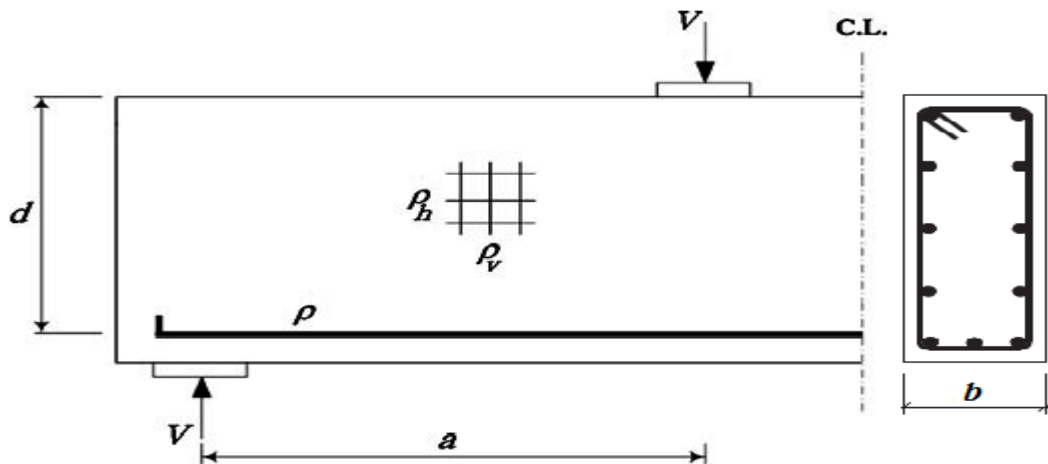


Fig. 1. The geometrical parameters of RC deep beam.

2.2. Adaptive neuro-fuzzy inference system (ANFIS)

As an excellent function approximation tool, ANFIS is one of the best tradeoffs between neural and fuzzy systems providing smoothness due to the fuzzy control interpolation and adaptability, due to the neural network back-propagation. It was introduced by Jang in 1993 [31] based on Takagi-Sugeno-Kang (TSK) [44] inference model. In ANFIS, neural network

automatically extracts the fuzzy rules and establishes the optimal distribution of membership functions through the learning process. ANFIS contains five layers in its architecture including, the fuzzify layer, product layer, normalized layer, defuzzify layer, and total output layer. Figure 2 shows an example of simple five layers architecture of a typical.

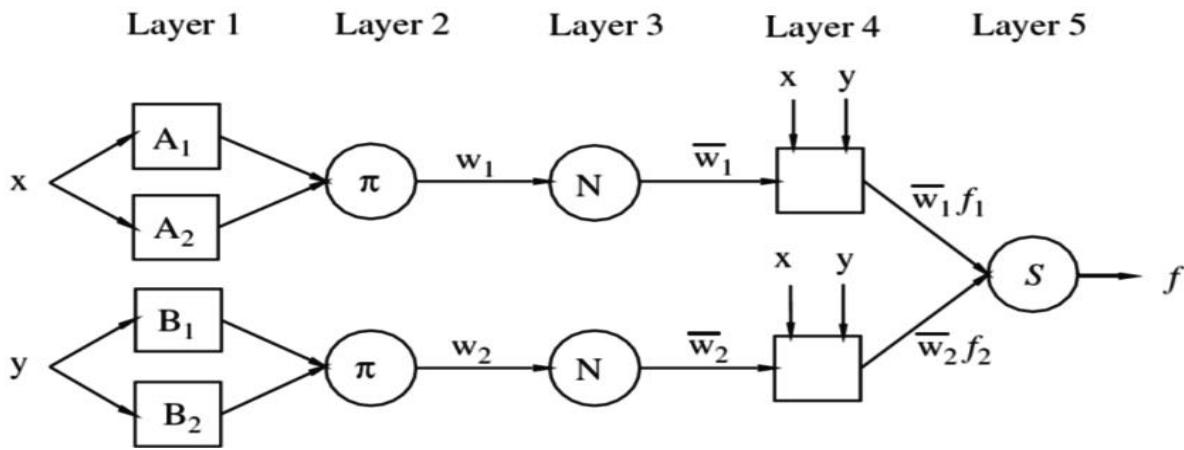


Fig.2.An ANFIS network structure for a simple FIS.

It is highlighted here that by assuming just two membership functions for each of the input data x and y , the general form of a first-order TSK type of fuzzy if-then rule has been given by Eq. 1. Here, we re-write the rule i of the ANFIS as:

Rule i : If x is A_i and y is B_i THEN

$$f_i = p_i x + q_i y + r_i \quad i = 1, 2, \dots, n \quad (1)$$

Where, n is the number of rules and p_i , q_i and r_i are the parameters determined during the training process. At first stage of the learning process, the membership function (μ) of each of the linguistic labels A_i and B_i are calculated as follow:

$$O_{1,i} = \mu_{A_i(x)}, \quad i = 1, 2 \quad (2)$$

$$O_{1,i} = \mu_{B_{i-2}(y)}, \quad i = 3, 4 \quad (3)$$

Where, $\mu_{A_i(x)}$, $\mu_{B_{i-2}(y)}$ can adopt any fuzzy membership function. For example, if the Gaussian membership function is employed, $\mu_{A_i(x)}$ is given by:

$$\mu_{A_i}(x) = \frac{1}{1 + \left[\left(\frac{x - c_i}{a_i} \right)^2 \right]^{b_i}} \quad (4)$$

Where, a_i , b_i and c_i are parameters of the membership function, governing the Gaussian functions accordingly.

The second layer consisting of fixed nodes represent the t -norm operators that combine the possible input membership grades in

order to compute the firing strength of the rule. The outputs of this layer are given by Eq. 5

$$O_{2,i} = w_i = \mu_{A_i(x)}\mu_{B_i(y)}, \quad i = 1,2 \quad (5)$$

which are the so-called firing strengths of the rules.

The third layer implements a normalization function and the outputs of this layer can be represented as follows:

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1,2 \quad (6)$$

The fourth layer is the de-fuzzification layer with adaptive nodes and every node i has the following function:

$$O_{4,i} = \bar{w}_i f_i = w_i(p_i x + q_i y + r_i), \quad i = 1,2 \quad (7)$$

Where, \bar{w}_i is the output of layer 3, and $\{p_i, q_i, r_i\}$ is the parameter set.

Finally in the fifth layer, the aggregation of the outputs performed by weighted summation. The output of the system is the final result

$$O_{5,i} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{w_1 + w_2} \quad (8)$$

By using a hybrid learning procedure [31], ANFIS can determine fuzzy inference parameters and construct an input-output mapping based on some collection of input-output data. ANFIS uses a hybrid learning procedure consisting of the combination of the least-squares method, which is used for identifying the consequent parameters, and gradient descent method for assigning the error signals propagate backward and the premise parameters. Once network have been trained to extracting the fuzzy rules and establishment the optimal distribution of membership functions, it can be used to predict an output value corresponding to a new group of input values.

3. Results and discussions

To evaluate the performance of the ANFIS model presented in this study, the data were randomly divided into two subsets, 80% was used for training the model and 20% for checking to validate the model. The test set data were not used during the model development.

Fig. 3 shows the model structure of the ANFIS that is to be built for modeling of shear strength of RC deep beams in this study. The model structure is implemented using the fuzzy logic toolbox of MATLAB 2010 software package.

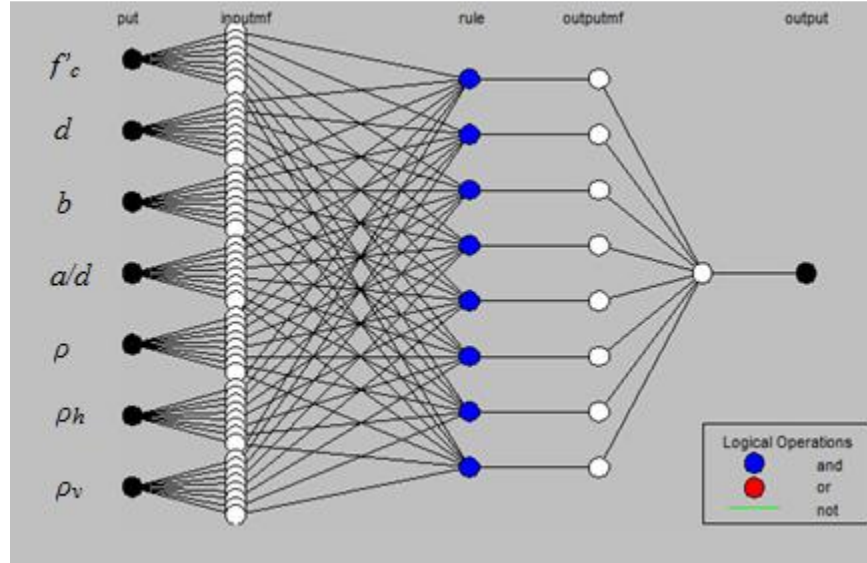


Fig. 3. Architecture of ANFIS model.

In this section, an experimental database is used to investigate the accuracy of ACI and CSA codes, ANN, GEP and ANFIS models in the prediction of the shear strength of RC deep beams.

The present study used squared correlation coefficient (R^2), Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) to compare ANFIS performance against other models.

These three statistical parameters are used to compare the performance of various methods as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i^{\text{exp}} - y_i^{\text{calc}})^2}{\sum_{i=1}^n (y_i^{\text{exp}} - \bar{y}^{\text{calc}})^2} \quad (9)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i^{\text{exp}} - y_i^{\text{calc}})^2}{n}} \quad (10)$$

$$MAE = \frac{\sum_{i=1}^n |y_i^{\text{exp}} - y_i^{\text{calc}}|}{n} \quad (11)$$

where y_i^{exp} , y_i^{calc} , and \bar{y} are the experimental, calculated and average values of shear strength of RC deep beams and n is

the number of compounds in dataset. Moreover the criteria recommended by Golbraikh and Tropsha [45] and Roy [46] were also used for the external validation of the test set. These criteria are presented as follows:

$$0.85 \leq k \leq 1.15 \quad (12)$$

$$0.85 \leq k' \leq 1.15 \quad (13)$$

$$m = \frac{(R^2 - R_0^2)}{R^2} \leq 0.1 \quad (14)$$

$$n = \frac{(R^2 - R_0'^2)}{R^2} \leq 0.1 \quad (15)$$

$$R_m^2 = R^2 \times \left(1 - \sqrt{|R^2 - R_0^2|}\right) \geq 0.5 \quad (16)$$

where

$$k = \frac{\sum y_i^{\text{exp}} y_i^{\text{calc}}}{\sum (y_i^{\text{calc}})^2} \quad (17)$$

$$k' = \frac{\sum y_i^{\text{exp}} y_i^{\text{calc}}}{\sum (y_i^{\text{exp}})^2} \quad (18)$$

$$R_0 = 1 - \frac{\sum_{i=1}^n (y_i^{cal} - ky_i^{calc})^2}{\sum_{i=1}^n (y_i^{cal} - \bar{y}^{cal})^2} \quad (19)$$

$$R'_0 = 1 - \frac{\sum_{i=1}^n (y_i^{exp} - k'y_i^{exp})^2}{\sum_{i=1}^n (y_i^{exp} - \bar{y}^{exp})^2} \quad (20)$$

These parameters (m, n, k and k') are presented in table3 for the test data and this shows that the proposed model are substantially valid and can be used to predict shear strength of RC deep beams. The values

of R^2 , *RMSE* and *MAE* parameters for the proposed model, ANN and GEP models and CSA and ACI codes are presented in Table 4. Comparison of the results indicates that the performance of the proposed model with low values of the *RMSE*, *MAE* and the values close to unity for R^2 is more accurate and better than the previous models. In addition, the performance of the ANN model is better than the GEP model and the two models perform a much better prediction, compared to ACI and CSA empirical codes.

Table 3. Statistical parameters for the test set in ANFIS models.

| | k | k' | m | n |
|-------|--------|--------|---------|---------|
| ANFIS | 1.0083 | 0.9828 | -0.0276 | -0.0276 |

Table 4. Comparison results obtain using different models and codes.

| Model | Reference | RMSE | MAE | R^2 |
|--|---------------|--------|--------|-------|
| ACI | [31] | 140.62 | 113.54 | 0.87 |
| CSA | [31] | 114.70 | 91.34 | 0.82 |
| ANN | [31] | 42.27 | 30.28 | 0.95 |
| GEP | [31] | 51.57 | 40.99 | 0.93 |
| ANFIS(test) | Present study | 34.76 | 25.24 | 0.97 |
| Multiple linear regression(MLP) method | Present study | 94.68 | 67.63 | 0.8 |

Moreover, a simple linear model was developed for prediction the shear strength of reinforced concrete (RC) deep beams by using multiple linear regression (MLP) method as follows:

$$V(\text{KN}) = 3.88\hat{f}_c + 451.25d + 2756.2b - 203.1\frac{a}{d} + 43.18\rho + 20.88\rho_h + 18.93\rho_v - 231.88 \quad (21)$$

The standardization of the regression coefficients in Eq. (21) enables to assign more importance to the shear strength of

(RC) deep beams with larger absolute standardized coefficients.

It is seen that the web width, shear span to depth, Concrete compressive strength with the absolute standardized coefficients of 0.689, 0.551 and 0.318, respectively, have the highest influence on the shear strength.

Figure 4 shows the comparison between the predicted shear strength and observation shear strength. This figure showed that the predicted shear strength using the ANFIS has a good agreement with the experimental result.

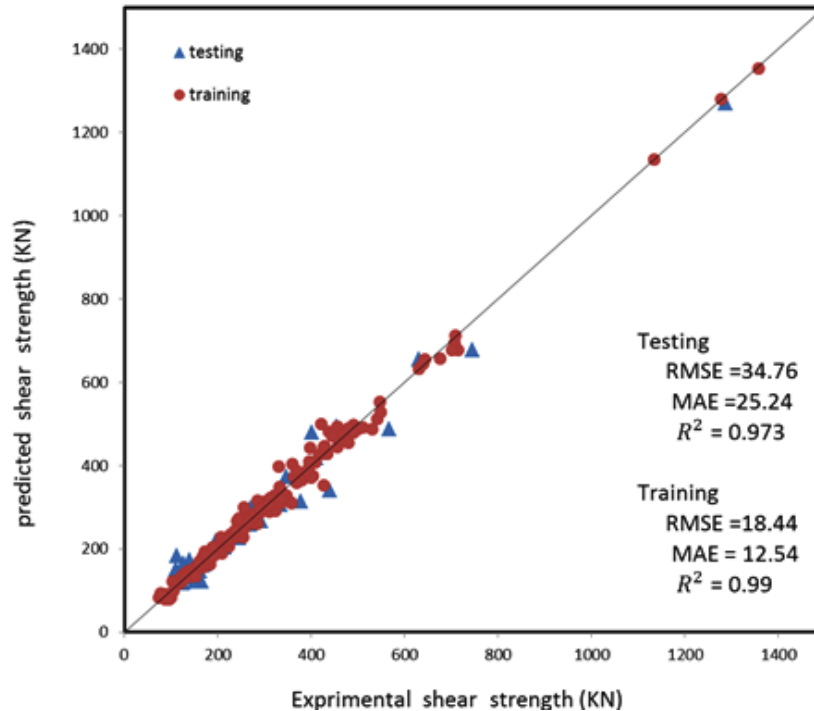


Fig.4. Experimental versus predicted strength values using the ANFIS model.

4. Conclusion

In this study the Adaptive Neural Fuzzy Inference System (ANFIS) was utilized to predict the shear strength of reinforced concrete (RC) deep beams. ANFIS model were constructed trained and test using the experiments of 214 dataset obtained from the literature. A constitutive relationship was obtained correlating the ultimate load with seven mechanical and geometrical parameters. The performances of the model were evaluated and the results were compared with empirical CSA and ACI

concrete codes, ANN and GPE models. The results show the goodness, robustness, and the predictive capacity of proposed model. Also, ANFIS, ANN and GEP models are more accurate than empirical ACI and CSA codes. Based on the results of the sensitivity analysis, web width, shear span to depth, Concrete compressive strength have the highest influence on the shear strength.

The results indicated that ANFIS is effective method for predicting the shear strength of reinforced concrete (RC) deep beams and has better accuracy and simplicity in compression with the empirical methods.

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