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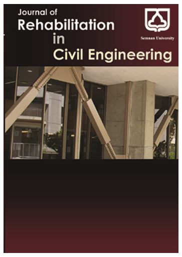

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Nondestructive Evaluation of Damage in Beams Using Displacement Curvature

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ABSTRACT

In this paper, the capabilities of displacement curvature derived from static response data for finding the location and severity of damage in Euler-Bernoulli beams are assessed. Static response of a beam is obtained using the finite element modeling. In order to reduce the number of measured nodal displacements, the beam deflection is fitted through a polynomial function using a limited number of nodal displacements. An indicator based on displacement curvature obtained for healthy and damaged structure is utilized to identify the damage. The influence of many parameters may affect the efficiency of the method such as the number of elements, the value and location of applied load as well as noise effect is investigated. Two test examples including a simply supported and a cantilever beam are considered. Numerical results show that using the method, the locations of single and multiple damage cases having different characteristics can be well determined.

1. Introduction

Structural damage detection is of great importance, because early detection and repair of damage in a structure can increase its life and prevent from an overall failure. During the last years, a considerable attention has been dedicated to identify damage in structures and therefore many damage detection methods have been introduced. The main aim of a damage detection method is to identify the occurrence, location and severity of damage. For this, the responses of a structure perform a vital role. Structural responses are divided into two main group including static and dynamic responses. Several studies related to using dynamic responses such as the natural frequencies and mode shapes of a structure can be found in the literature [1-10]. Also, damage detection methods based on employing static data has attracted much attention. Since static methods only depend on the stiffness matrix, therefore relations are easier with less complexity. In addition, static techniques have more accurate data, more inexpensive tools of measurement and also the speed of access to the right data in comparison with dynamic ones. Meanwhile, the results of applying the methods are more reliable on some civil structures. Data from static measurements has been frequently used for damage detection by many researchers. A static method of parameter identification using a finite element model for orthotropic labs was proposed by Sanayei and Scampoli [11]. Banan *et al*. [12, 13] proposed an algorithm for estimating member constitutive properties of the finite element model from measured displacements under a known static loading. The algorithm was based on the concept of minimizing an index of discrepancy between the model data and measurement data using the constrained least-square minimization. A two-stage damage detection method based on a grey system theory for damage localization and an optimization technique for damage quantification using the measured static displacement of a cantilever beam was proposed by Chen *et al*. [14]. They showed that the grey relation analysis based method can localize the slight to moderate damage and the optimization can identify the damage magnitude with a high accuracy. Unfortunately, they did not mention the sensitivity of damage to number of load cases, intensity of loading, limited measured static data, and the application of the approach on large-scale and complex structures. Abdo [15] has made a parametric study using static response based displacement curvature for structural damage detection. The results showed that changes in displacement curvature can be used as a good damage indicator even for a small amount of damage. Seyedpoor and Yazdanpanah [16] have proposed an efficient indicator for structural damage localization using the change of strain energy based on static noisy data. The acquired results clearly showed that the proposed indicator can precisely locate the damaged elements.

The main purpose of this study is to assess the efficiency of displacement curvature based on a limited static data for determining the location and severity of damage in beams. The influence of many parameters affecting the efficiency of the method such as the number of finite elements, the value and location of applied load as well as noise effect is investigated here. Numerical results demonstrate that the method can well determine the locations of single and multiple damage cases having different characteristics.

2. Theory

In solid mechanics [17], the curvature = $\left(1/\rho\right)$ (ρ is the radius of curvature) and deflection y can be related by Eq. (1):

$$\frac{1}{\rho}=\frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}=\frac{M}{EI} \tag{1}$$

where M is the bending moment, E is the modulus of elasticity and I is the moment of inertia of the cross section. Neglecting the second order of slope, the curvature can be approximated by Eq. (2):

$$\frac{1}{\rho}\approx y''=\frac{d^2y}{dx^2} \tag{2}$$

Then, the relationship between curvature, bending moment and stiffness can be as follows:

$$\frac{d^2y}{dx^2}=\frac{M}{EI} \tag{3}$$

Eq. (3) shows that the curvature is a function of stiffness. Any change in stiffness due to any damage at a section should be evidenced by a change in curvature at that location.

3. Damage Identification Method

In this paper, damage detection of a prismatic beam with a specified length is studied. First, the beam is divided into a number of finite elements. Then, nodal displacements of the beam in measurement points are evaluated using the finite element method. Deformation equation (y) can be obtained by fitting a polynomial curve via specified nodal displacements. By having the deformation equation, the equation of the slope ($\theta = dy/dx$) can be achieved. The displacement curvature equation of healthy beam can now be determined with differentiating the slope equation. By having the mathematical relationship of displacement curvature, the curvature of the beam can be evaluated at any arbitrary point. This process can also be repeated for damaged beam. It should be noted in this paper, it is assumed that the damage decreases the stiffness and therefore can be simulated by a reduction in the modulus of elasticity (E) at location of damage (element). Finally, using the displacement curvature obtained for two states the *index* introduced in [18] can be utilized to identify the damage. The step by step of damage detection method can be described as follows:

1) Divided the beam subjected to an arbitrary concentrated load into n elements as shown in Fig. 1.

2) Analyze the beam using the finite element method for determining the displacements of measurement points shown in Fig. 2.

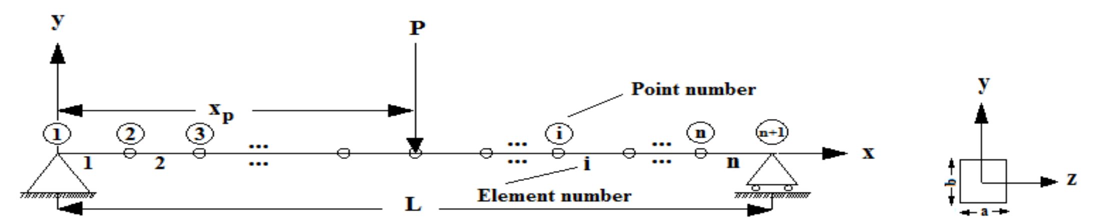


Fig. 1. (a) The geometry of the simply supported intact beam (b) Cross-section of the beam

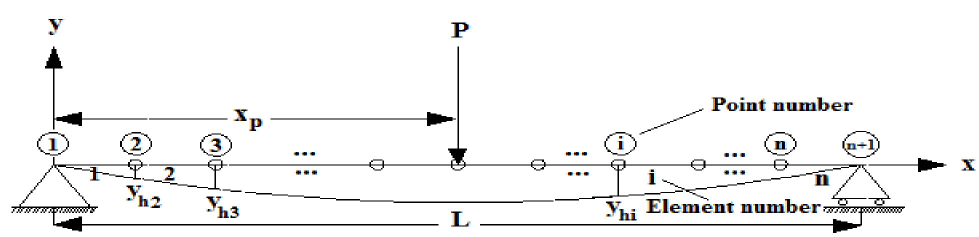


Fig. 2. Displacements of the simply supported intact beam under a concentrated load

3) Consider the nodal coordinates and displacements obtained as follows:

$$[x,y_h]=\left[(x_1,y_{h1}),(x_2,y_{h2}),(x_3,y_{h3}),\,\ldots\,,(x_i,y_{hi}),\,\ldots\,,(x_{n+1},y_{h(n+1)})\right]$$

Now the goal is to obtain the best curve which passes through the determined points. A polynomial curve that passes through above points can be defined as follows:

$$y_h=\sum_{j=0}^{m} a_{j+1}x^j = a_1+a_2x^1+a_3x^2+a_4x^3+\ldots+a_{m+1}x^m \tag{4}$$

where $(a_1, a_2, a_3, ..., a_{m+1})$ are the polynomial coefficients and *m* is polynomial degree.

4) Determine the slope (θ) equation with differentiating from Eq. (4).

$$\theta_h = \frac{dy_h}{dx} = \sum_{j=1}^{m} (j)(a_{j+1} x^{j-1}) = a_2 + 2a_3 x + 3a_4 x^2 + ... + m a_{m+1} x^{m-1} \quad (5)$$

5) Determine the displacement curvature of healthy beam with differentiating from Eq. (5).

$$\kappa_h = \frac{1}{\rho_h} = y_h'' = \frac{d\theta_h}{dx} = \sum_{j=2}^{m} (j-1)(j)(a_{j+1} x^{j-2}) = 2a_3 + 6a_4 x + ... + (m-1) m a_{m+1} x^{m-2} \quad (6)$$

The curvature of the beam can now be evaluated at any arbitrary point.

6) Induced a hypothetical damage in an arbitrary element as shown in Fig. 3, and analyze the beam for determining the nodal displacements in measurement points.

7) Consider the nodal coordinates and displacements of damaged beam as follows:

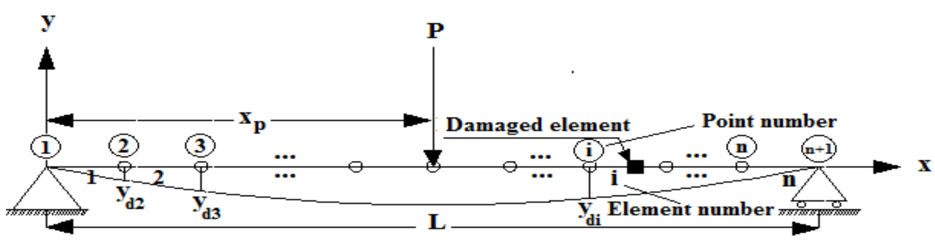


Fig. 3. Displacement of the simply supported damaged beam under a concentrated load

$$[x, y_d] = \left[(x_1, y_{d1}), (x_2, y_{d2}), (x_3, y_{d3}), \ ... \ , (x_i, y_{di}), \ ... \ , (x_{n+1}, y_{d(n+1)})\right]$$

The displacement curve of damaged beam can be fitted as

$$y_d = \sum_{j=0}^{m} a_{j+1} x^j = a_1 + a_2 x^1 + a_3 x^2 + a_4 x^3 + ... + a_{m+1} x^m \quad (7)$$

8) Determine the slope (θ) equation withdifferentiating from Eq. (7).

$$\theta_d = \frac{dy_d}{dx} = \sum_{j=1}^{m} (j)(a_{j+1} x^{j-1}) = a_2 + 2a_3 x + 3a_4 x^2 + ... + m a_{m+1} x^{m-1} \quad (8)$$

9) Determine the displacement curvature ofdamaged beam with differentiating from Eq. (8).

$$\kappa_d = \frac{1}{\rho_d} = y_d'' = \frac{d\theta_d}{dx} = \sum_{j=2}^{m} (j-1)(j)(a_{j+1} x^{j-2}) = 2a_3 + 6a_4 x + ... + (m-1) m a_{m+1} x^{m-2} \quad (9)$$

The curvature of the damaged beam can now be evaluated at any arbitrary point.

10) Use the index bellow [18] for damage localization.

$$Index_2 = \max\left[0, \left(Index_1 - \text{mean}\left(Index_1\right)\right)/\text{std}\left(Index_1\right)\right] \tag{10}$$

where $Index_1$ is defined by Eq.(11). Also, mean ($Index_1$) and std ($Index_1$) represent the mean and standard deviation of $Index_1$, respectively.

$$Index_1 = \kappa_d - \kappa_h \tag{11}$$

4. Numerical Examples

In order to assess the efficiency of the proposed method, two test examples including a simply supported beam and a cantilever beam are considered. Various parameters that may affect the performance of the method are studied.

4.1. Example 1: a simply supported beam

A simply supported beam with span L=1 (m) shown in Fig. 4 is selected as the first example. The beam has a square cross-section with dimensions of 0.05×0.05 m. Modulus of elasticity is $E = 2.1\times10^7\ \text{t/m}^2$. As shown in Fig. 5, for assessment of the method, ten different damage scenarios are considered. The first six scenarios (case 1-6), consist of a single damage. Seventh and eighth scenarios (case 7-8), include multiple damage cases with different intensity and finally the ninth and tenth scenarios (case 9-10) are introduced for considering the measurement noise effect.

One of the important parameter for accurately identifying damage is the number of measurement points for displacement curvature. In order to consider this effect, two different finite element meshes are used for the beam in scenarios 1 to 6. The first mesh is consists of 10 elements for the beam (damage scenarios 1-4, the length of each element is equal 0.1 L). The second mesh models the beam with 20 elements (damage scenarios 5-6, the length of each element is equal 0.05 L). The influence of position and value of load is also considered here. In scenario 1, the concentrated load is applied at midpoint while the scenario 3 is considered to study the effect of load position. The forth case (scenario 4) is similar to the first case, but, the load value is two times. In fact, this case is considered for investigating the effect of load value on damage detection. Measurement noise can not be avoided. Hence, the effect of noise is considered to perturb the responses of damaged structure. In this example, 3% noise is assumed.

For evaluating the index given by Eq. (10), the deflection equation of the beam before and after damage is needed to be determined. The curve fitting toolbox of MATLAB [19] is employed here for this purpose. For example, the deformed shape and corresponding equations obtained for the intact beam and damaged beam of case 2 are shown in Figs. 6-7, respectively.

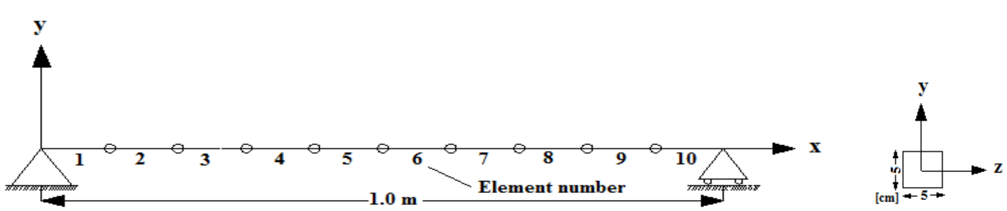


Fig. 4. (a) Geometry of the simply supported beam (b) Cross-section of the beam

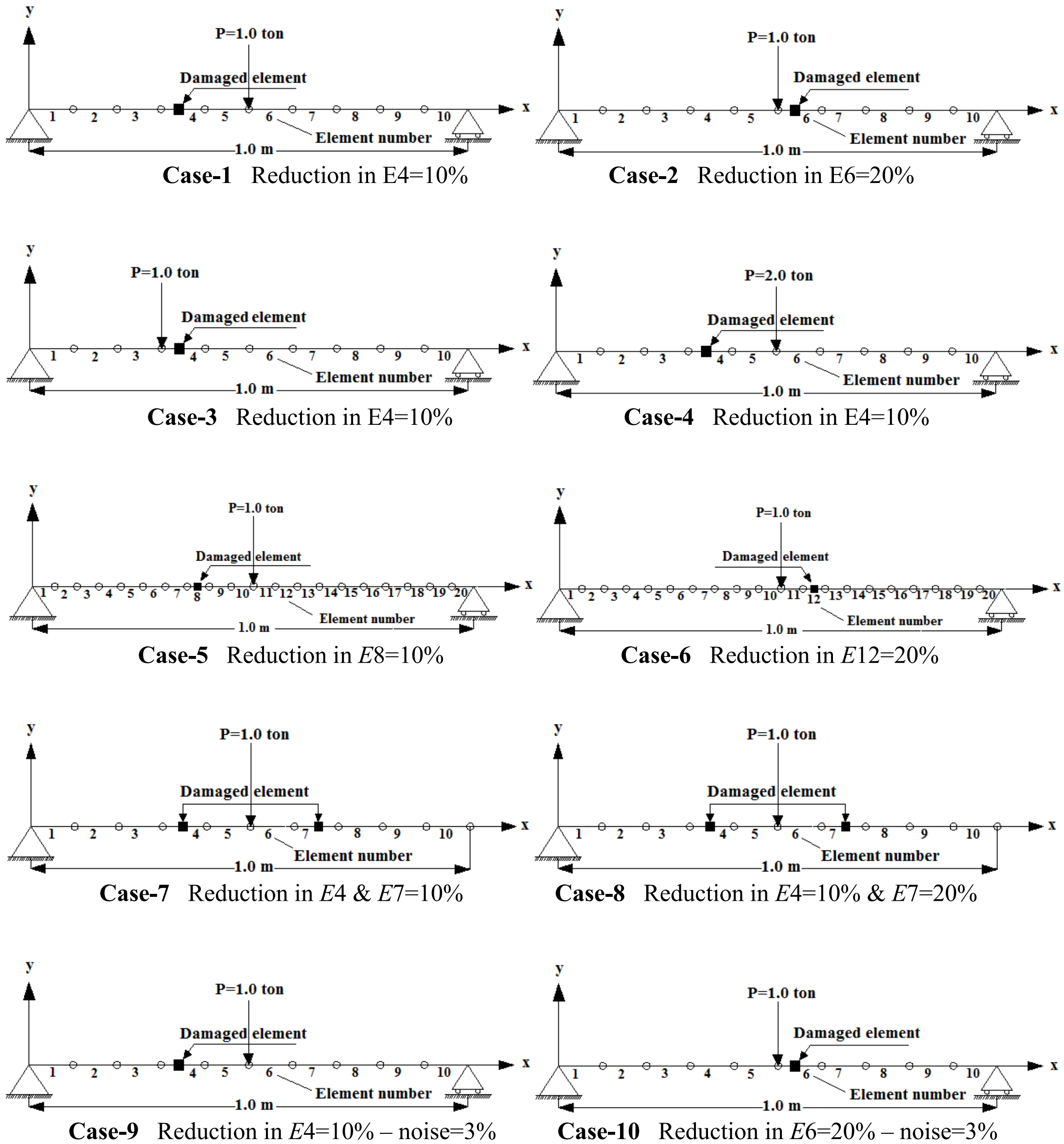


Fig. 5. Ten different damage scenarios for the simply supported beam

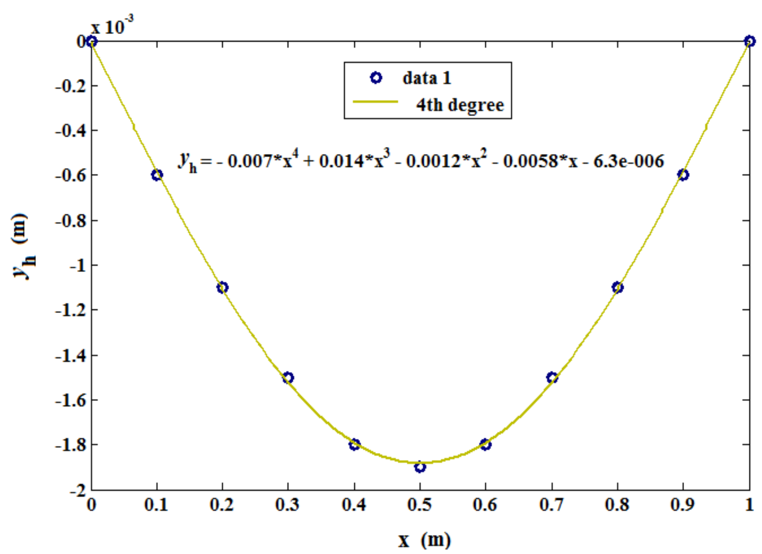


Fig. 6. Deformed shape and deflection equation of the simply supported intact beam

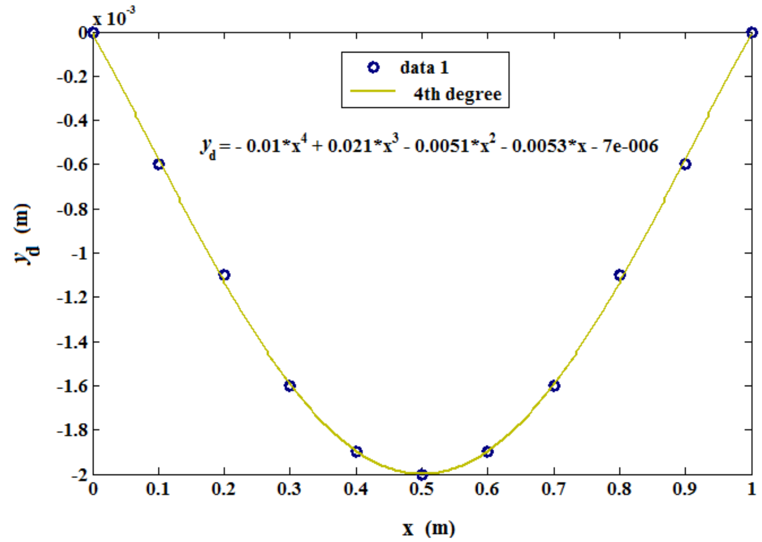


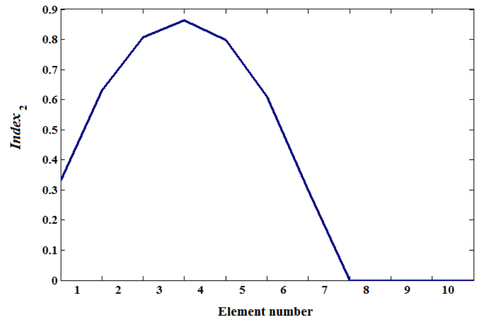
Fig. 7. Deformed shape and deflection equation of the simply supported damaged beam

Damage identification charts of simply supported beam for cases 1-10 are shown in Fig. 8. As shown in the figure, the value of $Index_2$ is further in vicinity of some elements that this indicates there is damage in these elements. The $Index_2$ is shown for damage scenarios 1 and 2 (with 10 elements) and 5 and 6 (with 20 elements) in Figs. 8 (a)-(b) and 8 (e)-(f), respectively. The results show that the values of $Index_2$ almost are identical in both cases. This means that the number of measurements is not very important, however, the most important factor in determining the damage location is data and the accuracy of measurement.

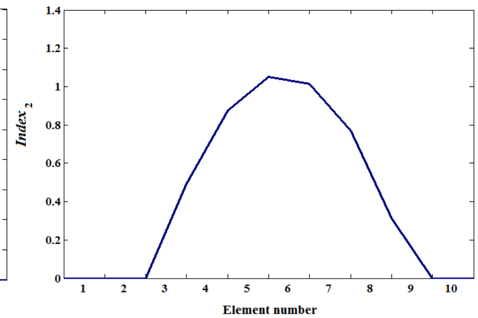
For investigating the effect of changing the position of the load on the method, the results of scenario1 (concentrated load at node 6) and scenario 3 (concentrated load at node 4) are compared. The results are plotted in Figs 8 (a) and (c), respectively. The results show that the damage index in the beam does not depend on the position of load between two supports. The values of $Index_2$ for damage scenarios 7 and 8 (multiple damage) are shown in Figs. 8 (g) and (h), respectively. It is reveal that the method can also locate the multiple damage cases properly.

For examining the effect of load value on damage detection method, the results of scenario1 (concentrated load of 1 ton) and scenario 4 (concentrated load of 2 ton), are compared. The identification charts are shown in Figs. 8 (a) and (d), respectively. As can be observed, the values of $Index_2$ are identical in both cases. In fact, the difference in displacement curvature for all measurements (nodes) in scenario 4 is twice values that are obtained from scenario1. This leads to doubling of the values of numerator and denominator of $Index_2$. It can be concluded that the use of $Index_2$ as a method for determining the damage sites does not depend on load value.

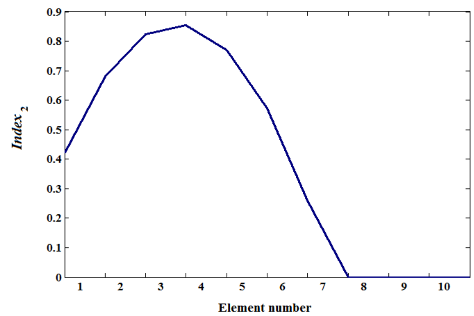
Figs. 8 (i) and (j) show damage charts for the damage scenarios 9 and 10 considering 3% noise, respectively. When comparing them with those shown in Figs 8 (a) and (b) for scenarios 1 and 2 (states without noise), it can be indicated that there is a good compatibility between these values. In other words, the measured noise has a negligible effect on $Index_2$. All of the results prove that the use of the static response (displacement curvature) can be useful for identifying the damage of the beam. It seems to be better than a vibration based method that needs more expensive sensors and in case of ambient vibration, will have a lot of noise [15].



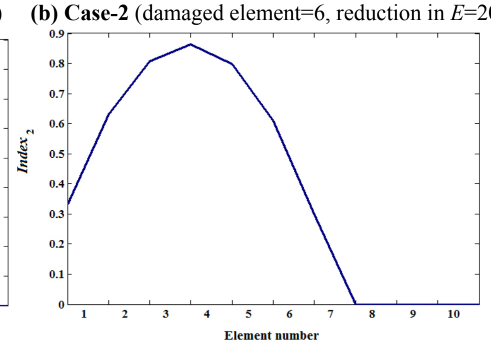
(a) Case-1 (damaged element=4, reduction in *E*=10%)



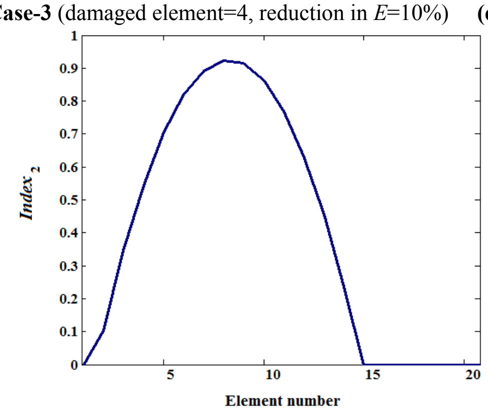
(b) Case-2 (damaged element=6, reduction in *E*=20%)



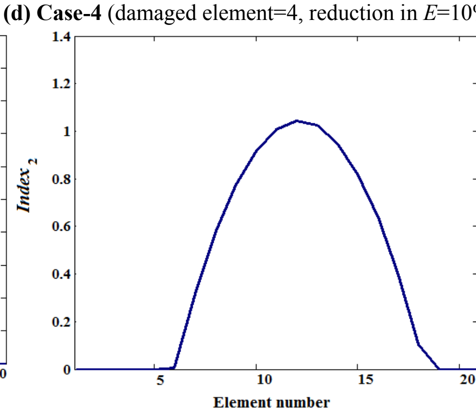
(c) Case-3 (damaged element=4, reduction in *E*=10%)



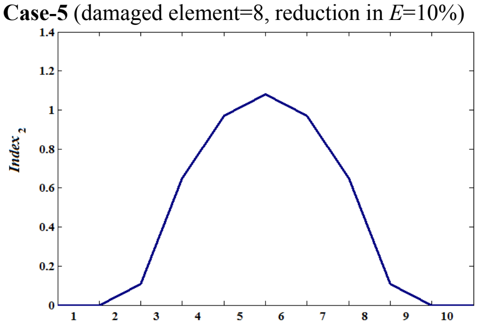
(d) Case-4 (damaged element=4, reduction in *E*=10%, *P*=2.0 ton)



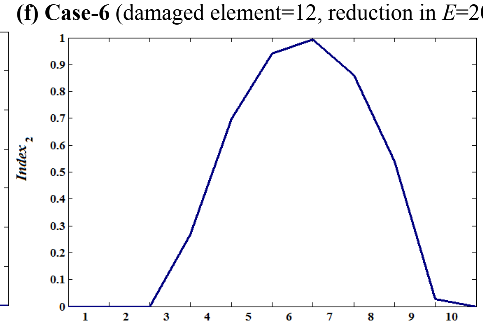
(e) Case-5 (damaged element=8, reduction in *E*=10%)



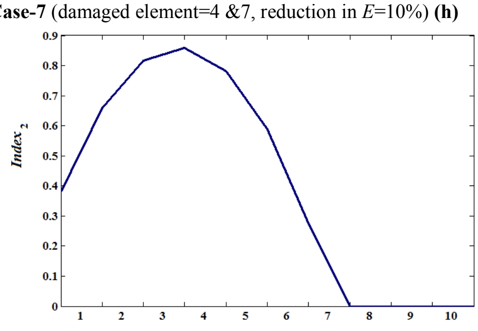
(f) Case-6 (damaged element=12, reduction in *E*=20%)



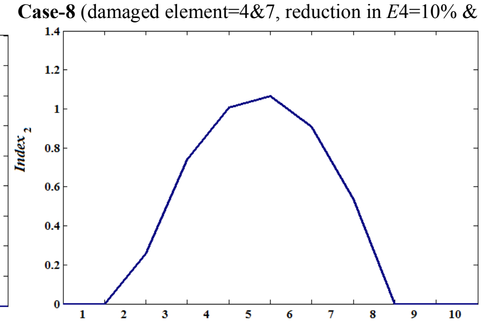
(g) Case-7 (damaged element=4 &7, reduction in *E*=10%)



(h) Case-8 (damaged element=4&7, reduction in *E*4=10% & *E*7= =20%)



(i) Case-9 (damaged element=4, reduction in *E*=10%, noise=3%)



(j) Case-10 (damaged element=6, reduction in *E*=20%, noise=3%)

Fig. 8. Damage identification of simply supported beam for cases 1-10

4.2. Example 2: A cantilever beam

A cantilever beam with span *L*=1 m shown in Fig. 9 is selected as second example. The beam has a square cross-section with dimensions of 0.05×0.05 m. Modulus of elasticity is $E = 2.1\times10^{7}\ \mathrm{t/m^{2}}$. As shown in the first example, damage index used could locate damage correctly. However, in the cantilever beam for accurately locating damage, the index is modified as:

$$Index_3 = \max\left[0, \left(\text{mean}\left(Index_1\right) - Index_1\right) / \text{std}\left(Index_1\right)\right] \tag{12}$$

As shown in Fig. 10, ten different damage scenarios are considered for the beam. The first six scenarios (case 1-6), include single damage case. Seventh and eighth scenarios (case 7-8), represent multiple damage cases and finally in ninth and tenth scenarios (case 9-10) noise effect is also considered. For studying the number of measurement point effect on the efficiency of damage detection method, two different finite element meshes are taken in scenarios 1-6. The first mesh consists of 10 elements (damage scenarios 1-4, the length of each element is equal 0.1*L*) and the second one includes 20 elements (damage scenarios 5-6, the length of each element is equal 0.05 *L*). The influence of position and amount of load is also considered. In scenario 1, the beam is subjected to a concentrated load applying at the beam end while in scenario 3, the load is applied to another point. The scenario 4 is similar to scenario 1 excluding the value of the load has been two times. In this example, the 3% and 5% noise are considered in ninth and tenth scenarios, respectively.

The values of $Index_3$ for damage scenarios 1 and 2 (with 10 elements) and 5 and 6 (with 20 elements) are shown in Figs. 11 (a)-(b) and 11 (e)-(f), respectively. The results show that the $Index_3$ are approximately identical in both cases. This means that the number of measurement points cannot affect the results considerably, while the most important factor for determining the damage location is the accuracy of the measurement data. As shown in Fig. 11, the $Index_3$ in the vicinity of some elements is more than other ones and it indicates that there is damage in the elements.

For investigating the effect of load position on the performance of the method, the results of scenario1 (concentrated load at node11) and scenario 3 (concentrated load at node 9) are compared. The results are shown in Figs.11 (a) and 11 (c), respectively. The results indicated that the method does not depend on the position of load. The identification charts of damage scenarios 7 and 8 (multiple damage cases) are shown in Figs. 11 (g) and11 (h), respectively. It is revealed the efficiency of the method for locating the multiple damages.

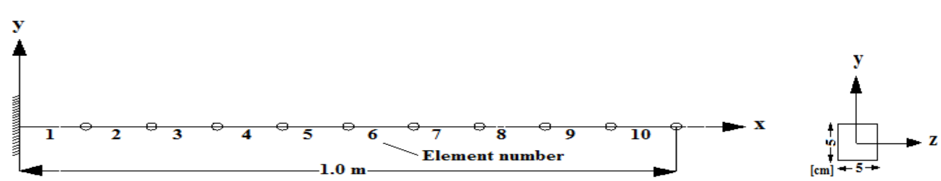


Fig. 9. (a) Geometry of the cantilever beam (b) Cross-section of the beam

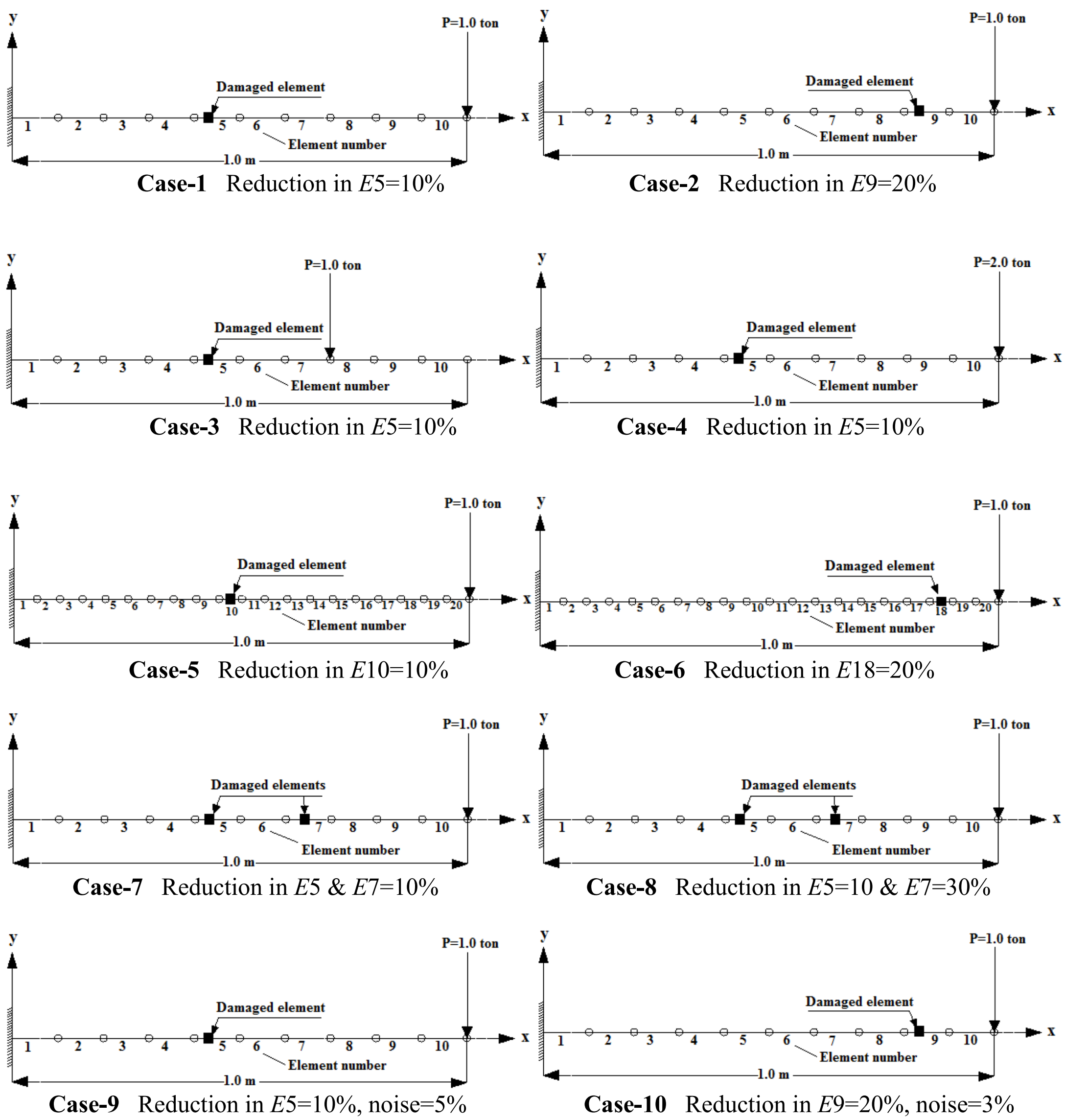
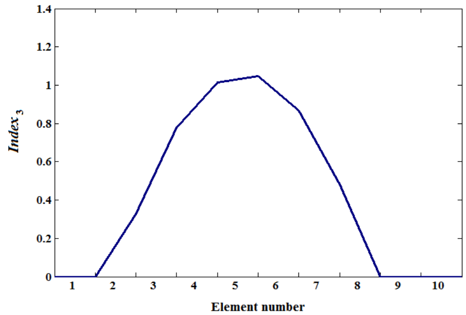


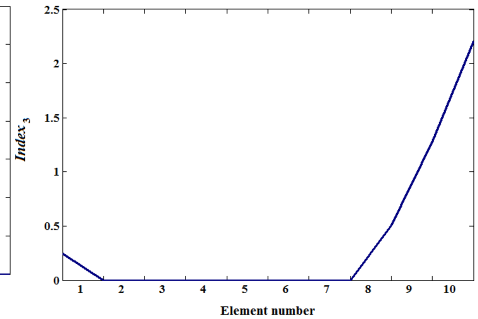
Fig. 10. Ten different damage scenarios for the cantilever beam

In order to examine the effect of the amount of load on damage detection method, the results of scenario1 (concentrated load with amount 1 ton) and scenario 4 (concentrated load with amount 2 ton) are compared in Figs. 11 (a) and (d), respectively. As can be observed, the $Index_3$ is the same for both the cases. It can be concluded that the use of $Index_3$ not depends on the value of load.

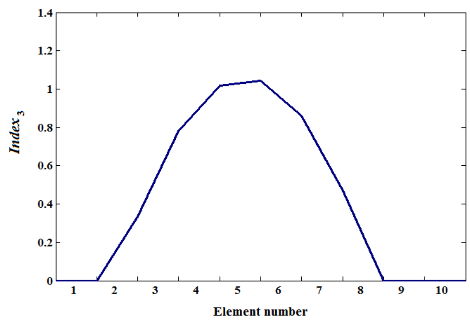
Figs. 11 (i) and (j) show $Index_3$ for the damage scenarios 9 and 10 considering 3% and 5% noise, respectively. Comparing these results with those shown in Figs 11 (a) and (b) for scenarios 1 and 2, it can be concluded that there is a good compatibility between them. In other words, considering the measurement noise has a negligible effect on damage detection method.



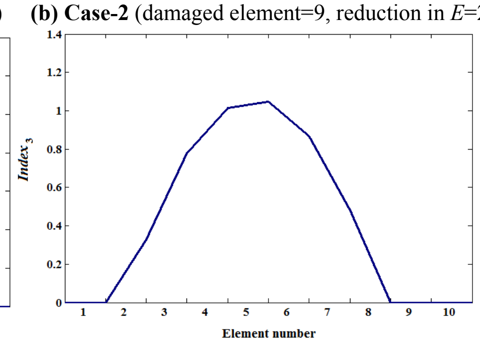
(a) Case-1 (damaged element=5, reduction in *E*=10%)



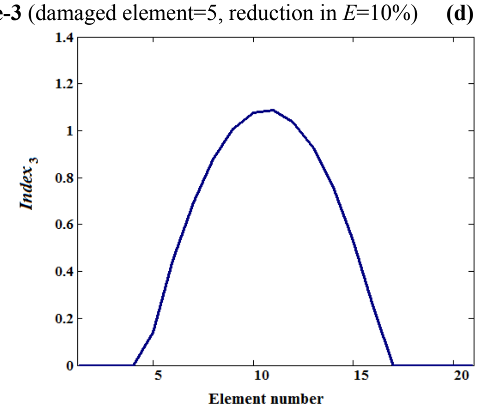
(b) Case-2 (damaged element=9, reduction in *E*=20%)



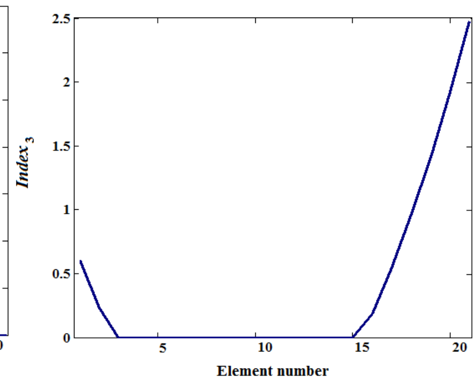
(c) Case-3 (damaged element=5, reduction in *E*=10%)



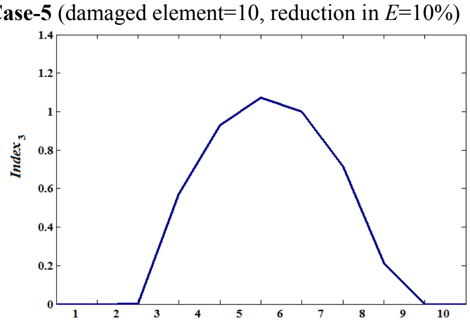
(d) Case-4 (damaged element=5, reduction in *E*=10%, P=2.0 ton)



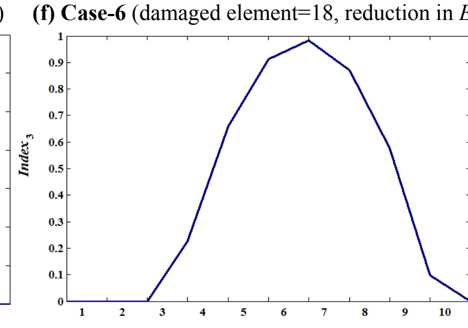
(e) Case-5 (damaged element=10, reduction in *E*=10%)



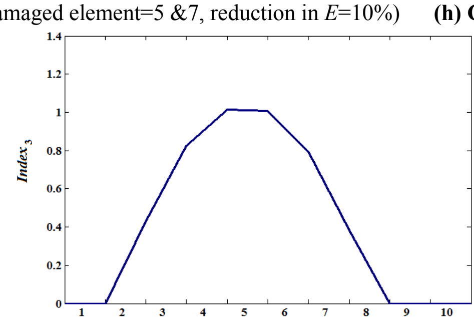
(f) Case-6 (damaged element=18, reduction in *E*=20%)



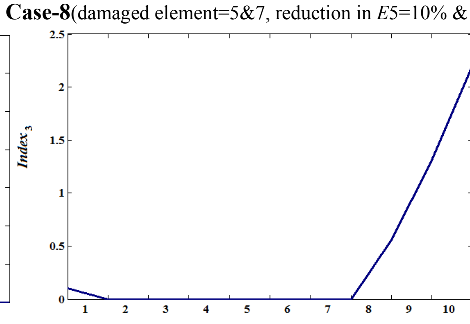
(g) Case-7 (damaged element=5 &7, reduction in *E*=10%)



(h) Case-8(damaged element=5&7, reduction in *E*5=10% & *E*7 =30%)



(i) Case-9 (damaged element=5, reduction in *E*=10%, noise=5%)



(j) Case-10 (damaged element=9, reduction in *E*=20%, noise=3%)

Fig. 11. Damage localization in cantilevered beam for damage cases1-10

5. Conclusion

In this paper, damage identification in beams using displacement curvature extracted from a static analysis has been investigated. The effects of many parameters may affect the efficiency of the method with considering a simply supported and a cantilever beam as test examples have been assessed. Based on the numerical studies, the following results can be concluded:

1) Displacement curvature obtained from the static response is sensitive to the stiffness reduction (reduction of Young's modulus). In other words it has characteristics from damaged area and can be used as a good indicator for damage detection. It may fairly be better than a dynamical method that needs more expensive instruments.
2) As achieved from the example results, the number of measurement points is not very important, however, the most important factor is the accuracy of measurement.
3) The proposed method does not depend on the position and amount of the load and it can be effectively used for locating the single and multiple damage cases.
4) Measurement noise has a negligible effect on the efficiency of the method for damage detection.

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