Stepwise Regression for Shear Capacity Assessment of Steel Fiber Reinforced Concrete Beams

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ABSTRACT

Inclusion of steel fibers to concrete progresses the flexural and tensile capacities of concrete. Consequently the shear capacity of concrete flexural members improve. Predicting the shear capacity of concrete beams containing steel fiber is an important issue not only in structural design but also to retrofitting of existing structures. Since there are several variables to assess the shear capacity of steel fiber reinforced concrete (SFRC) beams, presenting a suitable equation is a complicated task. The aim of the present paper is to evaluate an empirical formulae based stepwise regression (SR) method for shear capacity of SFRC beams. A series of reliable experimental data has been provided from literatures for model development. The obtained results based SR model were compared with experimental data in training and testing state. A practical formulae based SR method has been developed for shear capacity assessment of SFRC beams. Besides, several equations based models also presented to compare with the equation based SR model. The comparison showed the SR formulae gives the most exact accuracy than others in terms of shear capacity assessment of SFRC beams.

1. Introduction

In practical applications concrete has been consider as a brittle material which is not able to resist tensile and shear forces. Reinforced concrete beams prone to suffer shear stresses as subjected to external forces. The tensile and shear failures of such members are the most essential concerns of practical engineers. Incorporating of steel fibers to concrete mixtures increase the tensile capacity and accordingly the shear strength of reinforced concrete beams. Flexural reinforced concrete members contained web reinforcement to prevent the sudden shear failure. Incorporating steel fibers in concrete mixtures act as crack prevention, and consequently improve the
shear capacity. A large numbers of researches which done in current and past years acknowledged the effectiveness of steel fibers as shear reinforcement [1–30]. However, it is very difficult to predict the shear capacity of flexural members contained steel fibers accurately. Presenting an exact model to assess the shear capacity of SFRC flexural members is a challenging action in reason of existing several influences parameters. In order to predict the shear capacity of SFRC flexural members a series of empirical formulations were introduced [1-34], but it needs an accurate and simple formulae for employing the practical engineers. Intelligent methods give a very strong a reliable procedure to develop formulation for comprehensive and explicit functions consisting of several variables. In the current research, a method based SR approach has been developed to present an empirical formulation for predicting the shear capacity of SFRC beams. A series of 239 experimental data was gathered from the literatures in order to develop the SR model. It was considered six main effective variables that affect the shear capacity of SFRC beams. Compressive strength of concrete ($f'_c$), effective depth ($d$), shear span to depth ratio ($a/d$), longitudinal reinforcement ratio ($\rho_l$), length of fiber to diameter of fiber ($l_f/d_f$) and fibre content by volume ($V_f$) have been candidate for input variables. It should be mentioned that the only output variable is shear capacity of abovementioned beams ($\nu_{frc}$). In order to show the accuracy of the obtained formulae based SR method, a comparison between the existing recommendation formula and the presented equation has been implemented. Besides a parametric investigations also were conducted to predict the capability of presented formulation as a function of several input variables. The existing formulations based methods have been summarized in Table 1.

Table 1. Existing formulations based methods for FRC beams.

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Predictive equation for ultimate shear strength(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al–Ta’an and Al-Fee [2]</td>
<td>$\nu_{frc} = e \left(0.17\sqrt{f'_c} + 106\rho_l \frac{d}{a}\right) + 1.128F$</td>
</tr>
<tr>
<td></td>
<td>$e = 1$ if $a/d &gt; 2.5$; $e = 2.5d/a$ if $a/d \leq 2.5$</td>
</tr>
<tr>
<td>Mansur et al. [7]</td>
<td>$\nu_{frc} = 0.41\left(\tau V_f \frac{l_f}{d_f}\right) + \left(0.16\sqrt{f'_c} + 17.2 \frac{\rho V_f d}{M}\right)$</td>
</tr>
<tr>
<td>Swamy et al. [30]</td>
<td>$\nu_{frc} = 0.37\tau V_f \frac{l_f}{d_f} + \nu_c$</td>
</tr>
<tr>
<td></td>
<td>$\nu_c = 0.167\sqrt{f'_c}$</td>
</tr>
<tr>
<td></td>
<td>Factor of $\tau d/a$ to multiply $\nu_c$ for $a/d &lt; 2$</td>
</tr>
<tr>
<td>Ashour et al. [10]</td>
<td>$\nu_{frc} = \left(0.7\sqrt{f'_c} + 7F\right) \frac{d}{a} + 17.2 \rho_l \frac{d}{a}$</td>
</tr>
<tr>
<td>Khuntia et al. [15]</td>
<td>$\nu_{frc} = \left(0.167\alpha + 0.25F_1\right) \sqrt{f'_c}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha$ is the arch action factor</td>
</tr>
</tbody>
</table>
\[ \frac{\alpha}{d} \geq 2.5; \quad 2.5d/\alpha \leq 3 \text{ for } \frac{a}{d} < 2.5 \]

\( v_{fc} = k f_t \left( \frac{d}{a} \right)^{0.25} \)

\( k = 1 \) if \( f_t \) is obtained by direct tension test

\( k = 2/3 \) if \( f_t \) is obtained by indirect tension test

\( k = 4/9 \) if \( f_t \) is obtained using modulus of rupture

\( f_t \) are the tensile strength of fiber reinforced concrete; \( F \) is the fiber factor equal to bond factor \( \frac{v_f l_f}{d_f} \); \( M \) and \( V \) are the moment and shear force; \( \tau \) is the fiber-matrix interfacial bond stress, \( \alpha \) is the actor for arch action; \( v_c \) is the concrete contribution toward shear strength.

\( v_{fc} = \frac{2d}{a} (\rho f_c + 0.41\tau F) + \frac{d}{2a} \left( \frac{\rho}{288(\rho-1)} \right)^{4} + 2 \)

\( v_{fc} = \left( \frac{\rho}{3.022} \left( \frac{a}{d} \right) \right)^{3} + \frac{F d^{1/4}}{2.289} \left( \frac{9.436 f_c^2}{d} \right) \)

2. Database

A series of 239 samples from reliable literatures was gathered to develop the SR model [1, 3–14, 16–21, 23–27, 31]. The statistics characteristics of parameters for developing the model tabulated in Table 2. Besides, to show the distributions of applied parameters frequency histograms have been also presented in Fig. 1.

### Table 2. Statistics characteristics of parameters for developing the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( v_f )</th>
<th>( l_f/d_f )</th>
<th>( \rho_t )</th>
<th>( d )</th>
<th>( a/d )</th>
<th>( f_t^2 )</th>
<th>( v_{fc}(\text{Mpa}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0074</td>
<td>74.3682</td>
<td>0.0268</td>
<td>228.5586</td>
<td>3.0031</td>
<td>44.8594</td>
<td>3.9494</td>
</tr>
<tr>
<td>Std. Error of Mean</td>
<td>0.0003</td>
<td>1.6171</td>
<td>0.0007</td>
<td>9.37072</td>
<td>0.0676</td>
<td>1.1265</td>
<td>0.1867</td>
</tr>
<tr>
<td>Median</td>
<td>0.0075</td>
<td>63</td>
<td>0.0282</td>
<td>175</td>
<td>3</td>
<td>40.2</td>
<td>3.0299</td>
</tr>
<tr>
<td>Mode</td>
<td>0.01</td>
<td>60.00</td>
<td>0.031</td>
<td>127</td>
<td>2.50</td>
<td>33.20</td>
<td>3.30</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>0.0041</td>
<td>24.9991</td>
<td>0.107</td>
<td>144.8679</td>
<td>1.0456</td>
<td>17.4151</td>
<td>2.8865</td>
</tr>
<tr>
<td>Variance</td>
<td>0</td>
<td>624.954</td>
<td>0</td>
<td>20986.690</td>
<td>1.093</td>
<td>303.285</td>
<td>8.332</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.810</td>
<td>0.780</td>
<td>0.775</td>
<td>1.646</td>
<td>-0.168</td>
<td>1.657</td>
<td>2.455</td>
</tr>
<tr>
<td>Std. Error of Skewness</td>
<td>0.157</td>
<td>0.157</td>
<td>0.157</td>
<td>0.157</td>
<td>0.157</td>
<td>0.157</td>
<td>0.157</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.525</td>
<td>-0.251</td>
<td>0.855</td>
<td>1.973</td>
<td>-0.575</td>
<td>2.593</td>
<td>7.520</td>
</tr>
<tr>
<td>Std. Error of Kurtosis</td>
<td>0.314</td>
<td>0.314</td>
<td>0.314</td>
<td>0.314</td>
<td>0.314</td>
<td>0.314</td>
<td>0.314</td>
</tr>
<tr>
<td>Range</td>
<td>0.018</td>
<td>114.00</td>
<td>0.0472</td>
<td>690.50</td>
<td>4.54</td>
<td>90.90</td>
<td>18.42</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.002</td>
<td>19.00</td>
<td>0.01</td>
<td>76.00</td>
<td>0.46</td>
<td>20.60</td>
<td>0.77</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.02</td>
<td>133.00</td>
<td>0.0572</td>
<td>766.50</td>
<td>5.00</td>
<td>111.50</td>
<td>19.19</td>
</tr>
</tbody>
</table>
Fig. 1. Histograms of used parameters in the model development.

3. Stepwise Regression (SR) Model

SR is a powerful statistical method that obtains the relationship between independent variables and dependent variable. SR method examines the predictor variables which considered in a regression model. In other words, SR examines the best compositions of input parameters for assessment of dependent parameter (output) [35]. In order to find the best input variables combinations, it needs to find the best fit between input and output variables. This can be found by add or delete variables which
affect the remaining sum of square [36]. The stepwise procedure, as consider either add and delete variables at a time, has been introduced as a strong method [37-39].

Forward SR selection of parameters selects the subset models using including one variable at a time to the earlier selected subset. This will be the variable that include the most value of correlation with dependent variable. On the other hand, backward eliminating of parameters selects the subset models using omitting one parameter whose elimination will cause the remaining sum of squares to increase the least. It should be kept in mind that neither forward selection nor backward elimination treats the influence which the including or elimination of a parameter is able to have on the contributions of other parameters. First, SR selecting variable managed using forward selection which rechecks at each state to examine the importance of introduced variables. When the convergence gives no enough minimum power, the selection method changes to backward elimination and parameters dropped one at a time till all existing parameters conclude the minimum power of convergence.

As one knows, SR selection of parameters needs more computing procedure than forward or backward selection. It is rational to expect SR selection to have a greater chance of selecting the best subsets in the sample data, but selection of the best subset for each subset size is not guaranteed.

The stopping criterion for SR selection of parameters employs both forward and backward elimination criteria. The parameter selection procedure finished when all parameters in the model conclude the rule to stay and no parameters out of the model conclude the rule to join. The rule for a parameter to join the model need not be the same as the rule for the parameter to keep.

3.1. Accuracy Measurements

In this study four measurements are applied to assess the strength of the offered ANN model, including: absolute percentage error (Err) for the i\textsuperscript{th} output, mean absolute error (MAE), mean squared error (MSE) and correlation coefficient (R). Err, MAE, MSE and R have been presented as below:

\[ Err_i = \left( \frac{y_i - t_i}{t_i} \right) \times 100 \]  \[ R = \frac{\sum_{i=1}^{N} (y_i - \bar{y})(t_i - \bar{t})}{\sqrt{\sum_{i=1}^{N} (y_i - \bar{y})^2 \sum_{i=1}^{N} (t_i - \bar{t})^2}} \]  \[ MSE = \frac{1}{N} \sum_{i=1}^{N} (t_i - y_i)^2 \]  \[ MAE = \frac{1}{N} \sum_{i=1}^{N} |y_i - t_i| \]

where N show the number of samples, t\textsubscript{i} and \bar{\text{t}} show the exact and average of the exact outputs, respectively, and y\textsubscript{i} and \bar{\text{y}} show the examined and average of the examined outputs, respectively (for the i\textsuperscript{th} output).

3.2. Model Developing

As mentioned above, SR method employed to develop an empirical formulae to shear capacity assessment of SFRC beams. Therefore, it needs to select variables which may affect the shear capacity of SFRC beams. In order to present them, the variables that used in existing equations along with some others which may influence have been employed in a sensitivity analysis to investigate the powers of variables on the
target (shear capacity). Thereafter, input and output vectors parameters of the developed model based on SR method consisted of six and one variables as follow:

\[
\text{Input} = \left\{ V_f, \frac{l_f}{d_f}, \rho_l, d, \frac{a}{d}, f'_c \right\}
\]

\[
\text{Output} = \left\{ v_{frc} \right\}
\]

where \( v_{frc} \) is the shear capacity of SFRC beams. \( V_f, \frac{l_f}{d_f}, \rho_l, d, \frac{a}{d} \) and \( f'_c \) are the most important parameters which affect the shear capacity of SFRC beams and have been used as input components. As mentioned previously, it is possible to use different functions to evaluate equations based SR method. Here, three different models based different functions were developed to progress practical formulations for shear capacity assessment of SFRC beams. Here, 80% and 20% of the specimens were used for training and testing sets, previously. The available forms for used independent variables in SR procedure has been given as below:

\[
X_i, \quad \frac{X_i}{X_j}
\]

where \( X_i \) considered as the independent variables that shown in \{Input\}.

The applied models for developing of SR procedure are shown in Table 3. As it can be seen from Table 3, the applied functions for SR development are adjusted for two independent \((X_i, X_j)\) and one dependent \((y)\) variables. Consequently, all of developed equations based applied functions are shown in Table 4. In order to operate the SR process, all available data divided into two sets for training and testing subsets. 239 specimens divided into 191 and 48 specimens for training and testing analysis states. SPSS software application has been implemented for analysis process. From Table 4, it is evident that the developed equation consisting more function parameters gives more exact than others. For practical applications it is desirable to use simple yet accurate formula. Here, as mentioned above, three different equations based different input functions were investigated. As it is obvious the equation 3 gives more exact accuracy than others and beside this it can be calculated simple enough for practical engineers. Therefore, in order to assess the shear capacity of SFRC beams the equation 3 can be presented to engineers who deals with practical considerations.

| Table 3. Models considered in SR process (inputs vs equations). |
|---|---|---|
| Model | Input | Equation |
| 1 | \(x_1, x_2\) | \(y = b_0 + b_1 \times X_1 + b_2 \times X_2\) |
| 2 | \(x_1, x_2, x_1^2, x_2^2, x_1 \times x_2\) | \(y = b_0 + b_1 \times X_1 + b_2 \times X_2 + b_3 \times X_1^2 + b_4 \times X_2^2 + b_5 \times X_1 \times X_2\) |
| 3 | \(x_1, x_2, \frac{1}{X_1}, \frac{1}{X_2}, x_1 \times x_2\) | \(y = b_0 + b_1 \times X_1 + b_2 \times X_2 + b_3 \times \frac{1}{X_1} + b_4 \times \frac{1}{X_2} + b_5 \times X_1 \times X_2\) |
Table 4. Statistical details and equations of best subsets for each stepwise regression model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[ v_{frc} = 4.847 +118.379 \times V_f -0.02 \times \frac{l_f}{d_f} +118.379 \times \rho_f -0.002 \times 340 -1.369 \times \frac{a}{d} +0.026 \times f'_c ]</td>
<td>0.7268</td>
</tr>
<tr>
<td>2</td>
<td>[ v_{frc} = -9.331 -210.61 \times V_f \times \frac{a}{d} +1336.324 \times V_f + 371.091 \times \rho_f -19313.74 \times V_f^2 -2.899 \times \frac{l_f}{d_f} \times \rho_f +0.092 \times \frac{l_f}{d_f} -3.431 \times V_f \times \frac{l_f}{d_f} ]</td>
<td>0.8441</td>
</tr>
<tr>
<td>3</td>
<td>[ v_{frc} = -9.385 +5.365 \times \frac{d}{a} -4376.967 \times V_f \times \rho_f +224.624 \times \frac{1}{d} -3.031 \times \frac{l_f}{d_f} \times \rho_f +0.491 \times \rho_f \times f'_c +0.022 \times \frac{l_f}{d_f} \times \frac{a}{d} +582.927 \times V_f -125.526 \times V_f \times \frac{a}{d} +570.546 \times \rho_f -68.762 \times \rho_f \times \frac{a}{d} +0.003 \times d \times \frac{a}{d} -0.008 \times d ]</td>
<td>0.8893</td>
</tr>
</tbody>
</table>

The proposed SR-based formulation of the shear capacity of SFRC beams is as given below:

\[ v_{frc} = -9.385 +5.365 \times \frac{d}{a} -4376.967 \times V_f \times \rho_f +224.624 \times \frac{1}{d} -3.031 \times \frac{l_f}{d_f} \times \rho_f +0.491 \times \rho_f \times f'_c +0.022 \times \frac{l_f}{d_f} \times \frac{a}{d} +582.927 \times V_f -125.526 \times V_f \times \frac{a}{d} +570.546 \times \rho_f -68.762 \times \rho_f \times \frac{a}{d} +0.003 \times d \times \frac{a}{d} -0.008 \times d \]  \hspace{1cm} \text{(7)}

3.3. Results Performance

The obtained results for \( v_{frc} \) using SR model development versus experimental have been tabulated in Figs. 2 (a) and (b) in training and testing states, respectively. As it can be found from the examination the R values for training and testing states are 0.9011 and 0.8512, respectively.
As mentioned previously, six input variables were considered as the affected parameters on the shear capacity assessment of SFRC beams. Thereafter, the SR formulae was introduced to apply with suitable accuracy. The statistics characteristics of presented formula and existing equations Al–Ta’an and Al Fee, Mansur et al, Swamy et al, Ashour et al, Khuntia et al, Sharma, Gandomi et al and Ilker Fatih Kara were tabulated in Table 5. The parameters of $R$, $MSE$ and $MAE$ are chosen to assess the performance of the presented formulation.

For model validity, an accepted criteria introduced by Gandomi et al. [40]. This criteria expressed as follows:

1- If $|R| > 0.8$, a robust correlation obtained between the measured and predicted values.
2- If $0.2 < |R| < 0.8$, a correlation obtained between the measured and predicted values.
3- If $|R| < 0.2$, a weak correlation obtained between the measured and predicted values.

Moreover, it is clear that for a strong model, the error values have to be at minimum [42]. The evaluation measures presented in Table 5 confirm that the proposed formulation (Eq. 7) is capable for prediction of $v_{frc}$. From Table 5, it is obvious the proposed formulae gives most exact accuracy than others.
4. Parametric Analyses

Moreover, in order to picture confirmation of the SR development model, a parametric analysis was accomplished based on the procedure that proposed in [41]. The mentioned procedure examines the response of the developed formulae to a set of assumed data. Based on this method, one input is changed while the other inputs are remained constant at their average. If this analysis yields conformed results to the underlying of problem, the strength of developed formulae is proved. For this study, the results of the mentioned parametric analysis show in Fig. 3. In fact, Fig. 3 illustrates the tendency of the shear capacity of SFRC beams to the variations of $V_f$, $l_f/d_f$, $\rho_l$, $d$, $a/d$ and $f'_c$. Therefore, these results confirm that the proposed formulae is enough correct to be used by practical engineers.

![Fig. 3. Shear capacity analysis based SR model.](image)
5. Conclusions

In the current study, the ANN was applied to assess the shear strength of SFRC beams. The experimental dataset including 239 specimens was obtained using reliable technical literatures for SR model development. Consequently, the SR model with six input parameters were considered including: compressive strength of concrete ($f'_c$), effective depth ($d$), shear span to depth ratio ($a/d$), longitudinal reinforcement ratio ($\rho_l$), length of fiber to diameter of fiber ($l_f/d_f$) and fibre content by volume ($V_f$).

At the next, the new formulae based on SR was presented and performance analysis was undertaken for confirmation of this formulae. It can be concluded that the SR formulation based model gives more exactness than others. As a final point, the main objective of this study was developing precise equation to assess the shear strength of SFRC beams. The new proposed formulae can be used by practical engineering applications.

REFERENCES


