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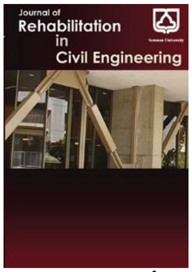

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Stepwise Regression for Shear Capacity Assessment of Steel Fiber Reinforced Concrete Beams

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ABSTRACT

Inclusion of steel fibers to concrete progresses the flexural and tensile capacities of concrete. Consequently the shear capacity of concrete flexural members improve. Predicting the shear capacity of concrete beams containing steel fiber is an important issue not only in structural design but also to retrofitting of existing structures. Since there are several variables to assess the shear capacity of steel fiber reinforced concrete (SFRC) beams, presenting a suitable equation is a complicated task. The aim of the present paper is to evaluate an empirical formulae based stepwise regression (SR) method for shear capacity of SFRC beams. A series of reliable experimental data has been provided from literatures for model development. The obtained results based SR model were compared with experimental data in training and testing state. A practical formulae based SR method has been developed for shear capacity assessment of SFRC beams. Besides, several equations based models also presented to compare with the equation based SR model. The comparison showed the SR formulae gives the most exact accuracy than others in terms of shear capacity assessment of SFRC beams.

1. Introduction

In practical applications concrete has been consider as a brittle material which is not able to resist tensile and shear forces. Reinforced concrete beams prone to suffer shear stresses as subjected to external forces. The tensile and shear failures of such members are the most essential concerns of practical engineers. Incorporating of steel fibers to concrete mixtures increase the tensile capacity and accordingly the shear strength of reinforced concrete beams. Flexural reinforced concrete members contained web reinforcement to prevent the sudden shear failure. Incorporating steel fibers in concrete mixtures act as crack prevention, and consequently improve the

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shear capacity. A large numbers of researches which done in current and past years acknowledged the effectiveness of steel fibers as shear reinforcement [1–30]. However, it is very difficult to predict the shear capacity of flexural members contained steel fibers accurately. Presenting an exact model to assess the shear capacity of SFRC flexural members is a challenging action in reason of existing several influences parameters. In order to predict the shear capacity of SFRC flexural members a series of empirical formulations were introduced [1-34], but it needs an accurate and simple formulae for employing the practical engineers.

Intelligent methods give a very strong a reliable procedure to develop formulation for comprehensive and explicit functions consisting of several variables. In the current research, a method based SR approach has been developed to present an empirical formulation for predicting the shear capacity of SFRC beams. A series of 239 experimental data was gathered from the literatures in order to develop the SR model. It was considered six main effective variables that affect the shear capacity of SFRC beams. Compressive strength of concrete (f_c'), effective depth (d), shear span to depth ratio (a/d), longitudinal reinforcement ratio (ρ_l), length of fiber to diameter of fiber (l_f/d_f) and fibre content by volume (V_f) have been candidate for input variables. It should be mentioned that the only output variable is shear capacity of abovementioned beams (v_{frc}). In order to show the accuracy of the obtained formulae based SR method, a comparison between the existing recommendation formula and the presented equation has been implemented. Besides a parametric investigations also were conducted to predict the capability of presented formulation as a function of several input variables. The existing formulations based methods have been summarized in Table 1.

Table 1. Existing formulations based methods for FRC beams.

Researchers	Predictive equation for ultimate shear strength(MPa)
Al–Ta'an and Al-Fee [2]	$v_{frc}=e\left(0.17\sqrt{f_c'}+106\rho_1\frac{d}{a}\right)+1.128F$ $e=1$ if $a/d>2.5$; $e=2.5d/a$ if $a/d\le 2.5$
Mansur *et al*. [7]	$v_{frc}=0.41\left(\tau V_f\frac{l_f}{d_f}\right)+\left(0.16\sqrt{f_c'}+17.2\frac{\rho v d}{M}\right)$
Swamy *et al*. [30]	$v_{frc}=0.37\tau V_f\frac{l_f}{d_f}+v_c$ $v_c=0.167\sqrt{f_c'}$ Factor of 2 d/a to multiply v_c for $a/d<2$
Ashour *et al*. [10]	$v_{frc}=\left(0.7\sqrt{f_c'}+7F\right)\frac{d}{a}+17.2\rho\frac{d}{a}$
Khuntia *et al*. [15]	$v_{frc}=\left(0.167\alpha+0.25F_1\right)\sqrt{f_c'}$ α is the arch action factor

	$\alpha = 1$ for $a/d \geq 2.5$; $2.5d/a \leq 3$ for $a/d < 2.5$
Sharma [24]	$v_{frc} = kf_t \left(\frac{d}{a}\right)^{0.25}$ $k = 1$ if f_t is obtained by direct tension test $k = 2/3$ if f_t is obtained by indirect tension test $k = 4/9$ if f_t is obtained using modulus of rupture
Gandomi *et al*. [29]	$v_{frc} = \frac{2d}{a}(\rho f_c + 0.41\tau F) + \frac{d}{2a}\frac{\rho}{(288\rho - 11)^4} + 2$
Ilker Fatih Kara [28]	$v_{frc} = \left(\frac{\rho_1}{3.022\left(\frac{a}{d}\right)}\right)^3 + \frac{F_1 d^{1/4}}{2.289} + \sqrt{\frac{9.436 f_c'}{d}}$

f_t are the tensile strength of fiber reinforced concrete; F is the fiber factor equal to bond factor $v_f l_f / d_f$; M and V are the moment and shear force; τ is the fibermatrix interfacial bond stress, α is the actor for arch action; v_c is the concrete contribution toward shear strength.

2. Database

A series of 239 samples from reliable literatures was gathered to develop the SR model [1, 3–14, 16–21, 23–27, 31]. The statistics characteristics of parameters for developing the model tabulated in Table 2. Besides, to show the distributions of applied parameters frequency histograms have been also presented in Fig. 1.

Table 2. Statistics characteristics of parameters for developing the model.

Parameter	v_f	l_f/d_f	ρ_l	d	a/d	f_c'	$v_{frc}(Mpa)$
Mean	0.0074	74.3682	0.0268	228.5586	3.0031	44.8594	3.9494
Std. Error of Mean	0.0003	1.6171	0.0007	9.37072	0.0676	1.1265	0.1867
Median	0.0075	63	0.0282	175	3	40.2	3.0299
Mode	0.01	60.00	0.031	127	2.50	33.20	3.30
Std. Deviation	0.0041	24.9991	0.107	144.8679	1.0456	17.4151	2.8865
Variance	0	624.954	0	20986.690	1.093	303.285	8.332
Skewness	0.810	0.780	0.775	1.646	-0.168	1.657	2.455
Std. Error of Skewness	0.157	0.157	0.157	0.157	0.157	0.157	0.157
Kurtosis	0.525	-0.251	0.855	1.973	-0.575	2.593	7.520
Std. Error of Kurtosis	0.314	0.314	0.314	0.314	0.314	0.314	0.314
Range	0.018	114.00	0.0472	690.50	4.54	90.90	18.42
Minimum	0.002	19.00	0.01	76.00	0.46	20.60	0.77
Maximum	0.02	133.00	0.0572	766.50	5.00	111.50	19.19

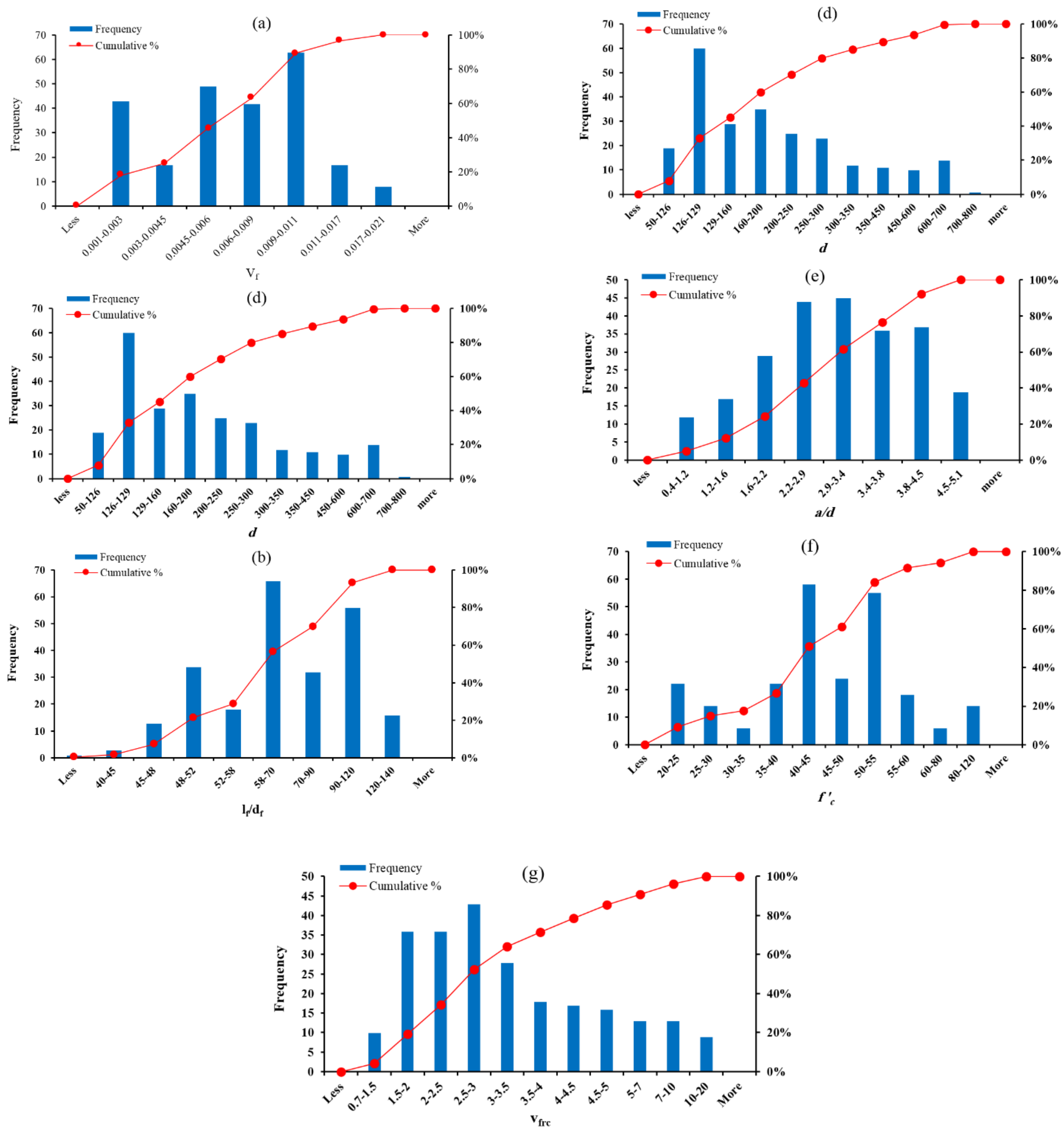


Fig. 1. Histograms of used parameters in the model development.

3. Stepwise Regression (SR) Model

SR is a powerful statistical method that obtain the relationship between independent variables and dependent variable. SR method examines the predictor variables which considered in a regression model. In other words, SR examines the best compositions of input parameters for assessment of dependent parameter (output) [35]. In order to find the best input variables combinations, it needs to find the best fit between input and output variables. This can be found by add or delete variables which affect the remaining sum of square [36]. The stepwise procedure, as consider either add and delete variables at a time, has been introduced as a strong method [37-39].

Forward SR selection of parameters selects the subset models using including one variable at a time to the earlier selected subset. This will be the variable that include the most value of correlation with dependent variable. On the other hand, backward eliminating of parameters selects the subset models using omitting one parameter whose elimination will cause the remaining sum of squares to increase the least. It should be kept in mind that neither forward selection nor backward elimination treats the influence which the including or elimination of a parameter is able to have on the contributions of other parameters. First, SR selecting variable managed using forward selection which rechecks at each state to examine the importance of introduced variables. When the convergence gives no enough minimum power, the selection method changes to backward elimination and parameters dropped one at a time till all existing parameters conclude the minimum power of convergence.

As one knows, SR selection of parameters needs more computing procedure than forward or backward selection. It is rational to expect SR selection to have a greater chance of selecting the best subsets in the sample data, but selection of the best subset for each subset size is not guaranteed.

The stopping criterion for SR selection of parameters employs both forward and backward elimination criteria. The parameter selection procedure finished when all parameters in the model conclude the rule to stay and no parameters out of the model conclude the rule to join. The rule for a parameter to join the model need not be the same as the rule for the parameter to keep.

3.1. Accuracy Measurements

In this study four measurements are applied to assess the strength of the offered ANN model, including: absolute percentage error (*Err*) for the ith output, mean absolute error (*MAE*), mean squared error (*MSE*) and correlation coefficient (*R*). *Err*, *MAE*, *MSE* and *R* have been presented as below:

$$Err_i = \frac{|y_i - t_i|}{t_i} \times 100 \tag{3}$$

$$R = \frac{\sum_{i=1}^{N}(y_i - \bar{y}_i)(t_i - \bar{t}_i)}{\sqrt{\sum_{i=1}^{N}(y_i - \bar{y}_i)^2 \sum_{i=1}^{N}(t_i - \bar{t}_i)^2}} \tag{4}$$

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(t_i - y_i)^2 \tag{5}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|y_i - t_i| \tag{6}$$

where *N* show the number of samples, t_i and show the exact and average of the exact outputs, respectively, and y_i and show the examined and average of the examined outputs, respectively (for the ith output).

3.2. Model Developing

As mentioned above, SR method employed to develop an empirical formulae to shear capacity assessment of SFRC beams. Therefore, it needs to select variables which may affect the shear capacity of SFRC beams. In order to present them, the variables that used in existing equations along with some others which may influence have been employed in a sensitivity analysis to investigate the powers of variables on the target (shear capacity). Thereafter, input and output vectors parameters of the developed model based on SR method consisted of six and one variables as follow:

$$Input = \left\{ V_f, \frac{l_f}{d_f}, \rho_1, d, \frac{a}{d}, f_c' \right\}$$

$$Output = \left\{ v_{frc} \right\}$$

where v_{frc} is the shear capacity of SFRC beams. V_f, l_f/d_f, ρ_l, d, a/d and f_c' are the most important parameters which affect the shear capacity of SFRC beams and have been used as input components. As mentioned previously, it is possible to use different functions to evaluate equations based SR method. Here, three different models based different functions were developed to progress practical formulations for shear capacity assessment of SFRC beams. Here, 80% and 20% of the specimens were used for training and testing sets, previously. The available forms for used independent variables in SR procedure has been given as below:

$$X_i, \ \frac{X_i}{X_j}$$

where Xi considered as the independent variables that shown in $\{Input\}$.

The applied models for developing of SR procedure are shown in Table 3. As it can be seen from Table 3, the applied functions for SR development are adjusted for two independent (X_1, X_2) and one dependent (y) variables. Consequently, all of developed equations based applied functions are shown in Table 4. In order to operate the SR process, all available data divided into two sets for training and testing subsets. 239 specimens divided into 191 and 48 specimens for training and testing analysis states. SPSS software application has been implemented for analysis process. From Table 4, it is evident that the developed equation consisting more function parameters gives more exact than others. For practical applications it is desirable to use simple yet accurate formula. Here, as mentioned above, three different equations based different input functions were investigated. As it is obvious the equation 3 gives more exact accuracy than others and beside this it can be calculated simple enough for practical engineers. Therefore, in order to assess the shear capacity of SFRC beams the equation 3 can be presented to engineers who deals with practical considerations.

Table 3. Models considered in SR process (inputs vs equations).

Model	Input	Equation
1	X_1, X_2	$y = b_0 + b_1 \times X_1 + b_2 \times X_2$
2	$X_1, X_2, X_1^2, X_2^2, X_1 \times X_2$	$y = b_0 + b_1 \times X_1 + b_2 \times X_2 + b_3 \times X_1^2 + b_4 \times X_2^2 + b_5 \times X_1 \times X_2$
3	$X_1, X_2, \frac{1}{X_1}, \frac{1}{X_2}, X_1 \times X_2$	$y = b_0 + b_1 \times X_1 + b_2 \times X_2 + b_3 \times \frac{1}{X_1} + b_4 \times \frac{1}{X_2} + b_5 \times X_1 \times X_2$

Table 4. Statistical details and equations of best subsets for each stepwise regression model.

Model	Equation	R
1	$v_{frc}=4.847+118.379\times V_f-0.02\times\frac{l_f}{d_f}+118.379\times\rho_l-0.002\times 340-1.369\times\frac{a}{d}+0.026\times f_c'$	0.7268
2	$v_{frc}=-9.331-210.61\times V_f\times\frac{a}{d}+1336.324\times V_f+371.091\times\rho_l-19313.74\times V_f^{\,2}-2.899\times\frac{l_f}{d_f}\times\rho_l+0.092\times\frac{l_f}{d_f}-3.431\times V_f\times\frac{l_f}{d_f}$	0.8441
3	$v_{frc}=-9.385+5.365\times\frac{d}{a}-4376.967\times V_f\times\rho_l+224.624\times\frac{1}{d}-3.031\times\frac{l_f}{d_f}\times\rho_l+0.491\times\rho_l\times f_c'+0.022\times\frac{l_f}{d_f}\times\frac{a}{d}+582.927\times V_f-125.526\times V_f\times\frac{a}{d}+570.546\times\rho_l-68.762\times\rho_l\times\frac{a}{d}+0.003\times d\times\frac{a}{d}-0.008\times d$	0.8893

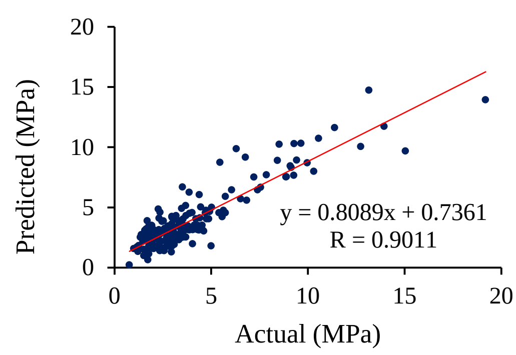
The proposed SR-based formulation of the shear capacity of SFRC beams is as given below:

$$
\begin{aligned}
v_{frc} &= -9.385+5.365\times\frac{d}{a}-4376.967\times V_f\times\rho_l+224.624\times\frac{1}{d}-3.031\times\frac{l_f}{d_f}\times\rho_l \\
&+0.491\times\rho_l\times f_c'+0.022\times\frac{l_f}{d_f}\times\frac{a}{d}+582.927\times V_f-125.526\times V_f\times\frac{a}{d}+570.546\times\rho_l \\
&-68.762\times\rho_l\times\frac{a}{d}+0.003\times d\times\frac{a}{d}-0.008\times d
\end{aligned}
\tag{7}
$$

3.3. Results Performance

The obtained results for v_{frc} using SR model development versus experimental have been tabulated in Figs. 2 (a) and (b) in training and testing states, respectively. As it can be found from the examination the R values for training and testing states are 0.9011 and 0.8512, respectively.

(a)



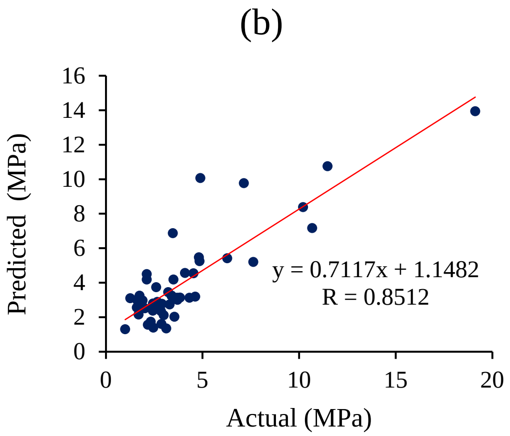


Fig. 2. SR model predictions for v_{frc} in (a) Training (b) testing states.

As mentioned previously, six input variables were considered as the affected parameters on the shear capacity assessment of SFRC beams. Thereafter, the SR formulae was introduced to apply with suitable accuracy. The statistics characteristics of presented formula and existing equations Al–Ta'an and Al Fee, Mansur et al, Swamy et al, Ashour et al, Khuntia et al, Sharma, Gandomi et al and Ilker Fatih Kara were tabulated in Table 5. The parameters of *R*, *MSE* and *MAE* are chosen to assess the performance of the presented formulation.

Table 5. Performance statistics of the models for v_{frc} prediction.

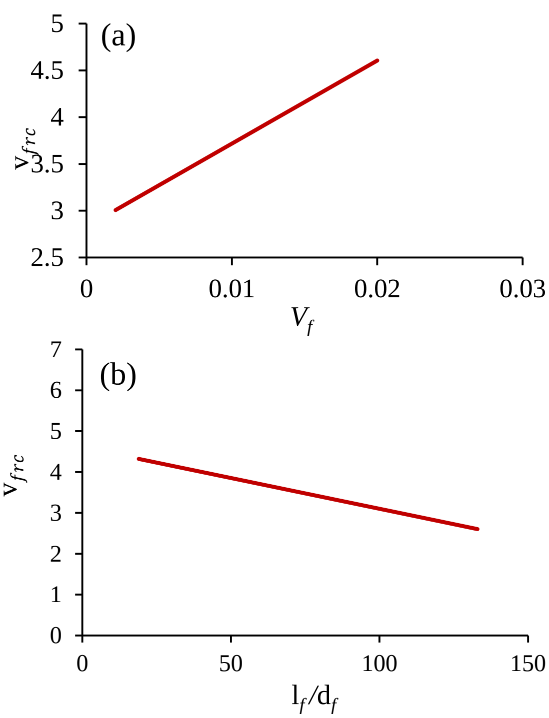
Method	Training			Testing		
	R	*MSE*	*MAE*	*R*	*MSE*	*MAE*
Proposed equation	0.9011	1.4775	0.9174	0.8512	2.7670	1.1742
Al–Ta'an and Al Fee [2]	0.6874	7.4450	1.4873	0.6319	7.4959	1.4915
Mansur et al. [6]	0.5481	8.5489	1.7568	0.4194	11.3510	1.8490
Swamy et al. [7]	0.4657	9.9899	1.9542	0.3501	12.7423	2.0543
Ashour et al. [12]	0.7017	5.4402	1.4814	0.6180	6.9434	1.5398
Khuntia et al. [17]	0.6663	7.9140	1.8041	0.5415	10.9933	1.9503
Sharma [20]	0.6253	7.8063	1.6485	0.4797	10.5404	1.7576
Gandomi et al	0.8072	2.7781	1.1221	0.6735	5.4844	1.3197
Ilker Fatih Kara [26]	0.4626	9.0383	1.7075	0.3667	11.6236	5.8020

For model validity, an accepted criteria introduced by Gandomi *et al*. [40]. This criteria expressed as follows:
1- If $|R| > 0.8$, a robust correlation obtained between the measured and predicted values.
2- If $0.2 < |R| < 0.8$, a correlation obtained between the measured and predicted values.
3- If $|R| < 0.2$, a weak correlation obtained between the measured and predicted values.

Moreover, it is clear that for a strong model, the error values have to be at minimum [42]. The evaluation measures presented in Table. 5 confirm that the proposed formulation (Eq. 7) is capable for prediction of v_{frc}. From Table 5, it is obvious the proposed formulae gives most exact accuracy than others.

4. Parametric Analyses

Moreover, in order to picture confirmation of the SR development model, a parametric analysis was accomplished based on the procedure that proposed in [41]. The mentioned procedure examines the response of the developed formulae to a set of assumed data. Based on this method, one input is changed while the other inputs are remained constant at their average. If this analysis yields conformed results to the underlying of problem, the strength of developed formulae is proved. For this study, the results of the mentioned parametric analysis show in Fig. 3. In fact, Fig. 3 illustrates the tendency of the shear capacity of SFRC beams to the variations of V_f, l_f/d_f, ρ_l, d, a/d and f_c'. Therefore, these results confirm that the proposed formulae is enough correct to be used by practical engineers.



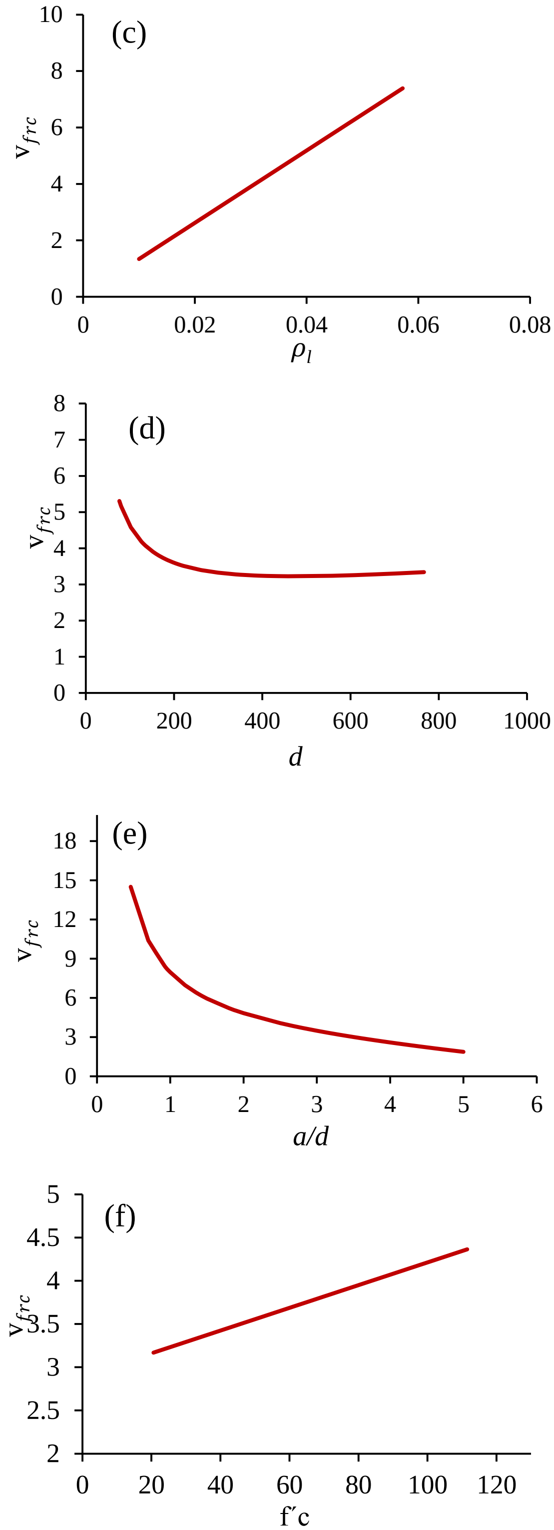


Fig. 3. Shear capacity analysis based SR model.

5. Conclusions

In the current study, the ANN was applied to assess the shear strength of SFRC beams. The experimental dataset including 239 specimens was obtained using reliable technical literatures for SR model development. Consequently, the SR model with six input parameters were considered including: compressive strength of concrete (f_c'), effective depth (*d*), shear span to depth ratio (*a*/*d*), longitudinal reinforcement ratio (ρ_l), length of fiber to diameter of fiber (l_f/d_f) and fibre content by volume (V_f).

At the next, the new formulae based on SR was presented and performance analysis was undertaken for confirmation of this formulae. It can be concluded that the SR formulation based model gives more exactness than others. As a final point, the main objective of this study was developing precise equation to assess the shear strength of SFRC beams. The new proposed formulae can be used by practical engineering applications.

REFERENCES

[1] Tan KH, Murugappan K, Paramasivam P. (1992), “Shear behavior of steel fiber reinforced concrete beams.” ACI Struct J, Vol. 89, Issue 6, pp. 3–11.

[2] Al-Ta’an, S. A., Al-Feel, J. R. (1990). “Evaluation of shear strength of fibre-reinforced concrete beams.” Cem Concr Compos, Vol. 12, Issue 2, pp. 87–94.

[3] Lim, T. Y., Paramasivam, P., Lee, S. L. (1987). “Shear and moment capacity of reinforced steel-fiber-concrete beams.” Mag Conc Res, Vol. 39, Issue 140, pp. 148–160.

[4] Naderpour, H., Alavi, S. A. (2017). “A proposed model to estimate shear contribution of FRP in strengthened RC beams in terms of Adaptive Neuro-Fuzzy Inference System.” Composite Structures, Vol. 170, pp. 215-227.

[5] Ilkhani, M., Moradi, E., Lavasani, M. (2017). “Calculation of Torsion Capacity of the Reinforced Concrete Beams Using Artificial Neural Network.” Soft Computing in Civil Engineering, Vol. 1, Issue 2, pp. 8-18.

[6] Hosseini, G. (2017). “Capacity Prediction of RC Beams Strengthened with FRP by Artificial Neural Networks Based on Genetic Algorithm.” Soft Computing in Civil Engineering, Vol. 1, Issue 1, pp. 93-98.

[7] Mansur, M. A., Ong, K. C. G., Paramasivam, P. (1986). “Shear strength of fibrous concrete beams without stirrups.” J Struct Eng ASCE, Vol. 112, Issue 9, pp. 2066–2079.

[8] Naderpour, H., Noormohammadi, E., Fakharian, P. (2017). “Prediction of Punching Shear Capacity of RC Slabs using Support Vector Machine.” Concrete Research, Vol. 10, Issue 2, pp. 95-107.

[9] Naderpour, H., Fakharian, P. (2017). “Predicting the Torsional Strength of Reinforced Concrete Beams Strengthened with FRP Sheets in terms of Artificial Neural Networks.” Journal of Structural and Construction Engineering, doi: 10.22065/jsce.2017.70668.1023

[10] Ashour, S. A., Hasanain, G. S., Wafa, F. F. (1992). “Shear behavior of high strength fiber reinforced concrete beams.” ACI Struct J, Vol. 89, Issue 2, pp. 176–184.

[11] Balazs, G. L., Kovacs, I. (2000). “Flexural behaviour of RC and PC beams with steel fibre.” In: Proceedings of the international workshop on structural application of steel fibre reinforced concrete, Politecncio of Milan, pp 85–92.

[12] Furlan, S., De Hanai, J. B. (1997). “Shear behaviour of fiber reinforced concrete beams.’ Cem Concr Compos, Vol. 19, Issue 4, pp. 359–366.

[13] Dinh, H. H., Parra-Montesinos, G. J., Wight, J. K. (2010). ‘Shear behavior of steel fiber reinforced concrete beams

without stirrup reinforcement." ACI Struct J, Vol. 107, Issue 5, pp. 597–606.

[14] Shin, S. W., Oh, J., Ghosh, S. K. (1994). "Shear behavior of laboratory sized high-strength concrete beams reinforced with bars and steel fibers, fiber reinforced concrete developments and innovations, SP 142. American Concrete Institute, Farmington Hills, pp 181–200.

[15] Khuntia, M., Stojadinovic, B., Goel, S. C. (1999). "Shear strength of normal and high strength fiber reinforced concrete beams without stirrups." ACI Struct J, Vol. 96, Issue 2, pp. 282–289.

[16] Greenough, T., Nehdi, M. (2008). "Shear behavior of fiber-reinforced self-consolidating concrete slender beams." ACI Mater J, Vol. 105, Issue 5, pp. 468–477.

[17] Imam, M., Vandewalle, L., Mortelmans, F. (1994). "Shear capacity of steel fibre concrete beams." In: Proceedings of ACI international conference on high-performance-concrete, Singapore, ACISP-149, pp. 227–243.

[18] Adebar, P., Mindess, S., St. Pierre, D., Olund, B. (1997). "Shear tests of fiber concrete beams without stirrups." ACI Struct J, Vol. 94, Issue 1, pp. 68–76.

[19] Kwak, Y. K., Eberhard, M. O., Kim, W. S., Kim, J. (2002). "Shear strength of steel-fibre-reinforced-concrete beams without stirrups." ACI Struct J, Vol. 99, Issue 4, pp. 530–538.

[20] Batson, G., Jenkins, E., Spatney, R. (1972). "Steel fibers as shear reinforcement in beams." ACI J Proc, Vol. 69, Issue 10, pp. 640–4.

[21] Cucchiara, C., Mendola, L. L., Papia, M. (2004). "Effectiveness of stirrups and steel fibers as shear reinforcement." Cem Concr Comp, Vol. 26, pp. 777–786.

[22] Adhikary, B. B., Mutsuyoshi, H. (2006). "Prediction of shear strength of steel fiber RC beams using neural networks." Constr Build Mater, Vol. 20, Issue 2, pp. 801–811.

[23] Cho, S., Kim, Y. (2003). "Effects of steel fibers on short beams loaded in shear." ACI Struct J, Vol. 100, Issue 6, pp. 765–74.

[24] Sharma, A. K. (1986). "Shear strength of steel-fibre-reinforced-concrete beams." ACI Struct J, Vol. 83, Issue 4, pp. 624–628.

[25] Rosenbusch, J., Teutsch, M. (2003). "Shear design with (σ- ε) method. In: Proceedings of the international RILEM workshop on test and design methods for steel fiber reinforced concrete." Bochum: RILEM Publications SARL; pp. 105–17.

[26] Hockenberry, T. and Lopez, M. M. (2012). "Performance of fiber reinforced concrete beams with and without stirrups." J Civil, Environmental and Architectural Engineering, Vol. 4, Issue 1, pp. 1–11.

[27] Hwang, J., Lee, D., Kim, K., Ju, H., Seo, S. (2013). "Evaluation of shear performance of steel fibre reinforced concrete beams using a modified smeared-truss model." Magazine of Concrete Research, Vol. 65, Issue 5, pp. 283–296.

[28] Fatih, K. I. (2013). "Empirical modeling of shear strength of steel fiber reinforcedconcrete beams by gene expression programming." Neural Comput & Applic, Vol. 23, pp. 823–834.

[29] Gandomi, A. H., Alavi, A. H., Yun, G. J. (2011). "Nonlinear modeling of shear strength of SFRC beams using linear genetic programming." Structural Engineering and Mechanics, Vol. 38, Issue 1, pp. 1–25.

[30] Swamy, R. N., Jones, R., Chiam, A. T. P. (1993). "Influence of steel fibers on the shear resistance of lightweight concrete I-beams." ACI Struct J, Vol. 90, Issue 1, pp. 103–114.

[31] Campione, G., La Mendola, L., Papia, M. (2006). "Shear strength of steel fiber reinforced concrete beams with stirrups.' Struct Eng Mech, Vol. 24, Issue 1, pp. 107–136.

[32] Nguyen-Minh, L., Marian, R. (2010). "New formula for the estimation of shear resistance of fibre reinforced beams.' Can J Civ Eng, Vol. 38, pp. 23–35.

[33] Slater, E., Moni, M., Alam, M. S. (2011). "Predicting the shear strength of steel fiber reinforced concrete beams." Constr Build Mater. doi: 10.1016/j.conbuildmat.2011.06.042

[34] ACI Committee 544.4R-88 (1988). "Design considerations for steel fiber reinforced concrete." American Concrete Inst, pp. 1–18 (reapproved in 1999).

[35] Cybenko, J. (1989). "Approximations by super positions of a sigmoidal function." Math Control Signal Syst, Vol. 2, pp. 303–314.

[36] Marquardt, D. (1963). "An algorithm for least squares estimation of non-linear parameters." J Soc Ind Appl Math, Vol. 11, pp. 431–41.

[37] Hagan, M. T., Menhaj, M. B. (1994). "Training feed forward networks with the Marquardt algorithm." IEEE Transactions on Neural Networks, Vol. 5, Issue 6, pp. 861-867.

[38] Khademi, F., Jamal, S. M., Deshpande, N., & Londhe, S. (2016). Predicting strength of recycled aggregate concrete using artificial neural network, adaptive neuro-fuzzy inference system and multiple linear regression. International Journal of Sustainable Built Environment, 5, Issue 2, pp. 355-369.

[39] Khademi, F., & Behfarnia, K. (2016). Evaluation of concrete compressive strength using artificial neural network and multiple linear regression models. Iran University of Science & Technology, 6, Issue 3, pp. 423-432.

[40] Gandomi, A. H., Tabatabaei, S. M., Moradian, M. H., Radfar, A., Alavi, A. H. (2011). "A New Prediction Model for the Load Capacity of Castellated Steel Beams." Journal of Constructional Steel Research, Vol. 67, pp. 1096-1105.

[41] Gandomi, A. H., Mohammadzadeh, S., Juan Luis Pérez-Ordó˜nezc, Alavi, A. H. (2014). "Linear genetic programming for shear strength prediction of reinforced concrete beams without stirrups." Applied Soft Computing, Vol. 19, pp. 112–120.