



## Damage Identification of Structures Using Second-Order Approximation of Neumann Series Expansion

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### ABSTRACT

In this paper, a new method proposed for structural damage detection from limited number of sensors using extreme learning machine (ELM). One of the main challenges in structural damage identification problems is the limitation in the number of used sensors. To address this issue, an effective model reduction method has been proposed. To condense mass and stiffness matrices, the second-order approximation of Neumann series expansion (NSEMR-II) has been used. Mode shapes and frequencies of damaged structures and corresponding generated damage states used as input and output to train extreme learning machine, respectively. To show the effectiveness of presented method, three different examples consists of a truss structure, irregular frame and shear frame have been studied. The obtained results show the ability of the proposed approach in identifying and estimating different damage cases using limited numbers of installed sensors and noisy modal data.

## 1. Introduction

Identification of damage in structures has received increasing attention in the last few decades. One of the many nondestructive evaluation approaches is based on the change of vibration parameters with a change in the structural properties [1].

One of the main challenges in structural damage identification problems is the limitation in the number of used sensors. To address this issue, different model reduction

methods has been proposed [2-3]. Hosseinzadeh et al. [4] have been developed a damage identification method using cuckoo search algorithm and a new model condensation method (iterated improved reduction system (IIRS)). In other work, Kourehli et al. [5] proposed a new damage detection approach based on reduced stiffness matrices and pattern search optimization algorithm. Li et al. [6] presented a damage identification method for offshore jacket structures based in incomplete modal

data. Also, Rasouli et al. [7] used Guyan's static reduction method and particle swarm evolutionary algorithm to detect damage in structures. Also, some researchers used the mode shape expansion approaches for structural damage detection [8]. Ghannadi et al. [9] developed a novel method for damage identification based on expanded mode shape data and artificial neural network. To expand mode shapes, SEREPa expansion method has been used. In this study, space truss and plane truss used to show the effectiveness of presented method.

Nowadays, different learning machines (LM) have been used to identify damage in structures. Most of these methods used modal data or static responses of structures as input data to predict damage existence and severity as output. Recently, Djemana et al. [10] developed a damage identification method using extreme learning machine (ELM). In this paper, piezoelectric sensors have been used. Results showed that ELM can be used as a tool to predict of a single damage in structures. The obtained results reveal that ELM is an effective tool to identify structural damages. Also, Gökdağ [11] presented a method to identify crack in beams under moving vehicle. In this paper an objective function formulated using dynamic responses of beam structures and solved by the particle swarm optimization (PSO). The results reveal that the proposed method can predict cracks with depth ratio of 0.1. Also, different optimization techniques have been proposed in recent years to detect damage in structures [12-14]. Hoseini vaez et al. [15] used wavelet transform to identify damages in Koyna dam. Also, Hoseini vaez et al. [16] proposed a damage identification approach in post-tensioned slab using 2D wavelet transforms. Also, bagheri et al. [17] used discrete wavelet analysis to detect damage in structures under

earthquake excitation. In other work, Yazdanpanah et al. [18] presented a damage identification method based on new damage indicator. Results show a better performance of proposed indicator in comparison with other indicators. Also, Naderpour and Fakharian [19] identified modal parameters of structures using wavelet packet transform and peak picking method. Kourehli [20] used modal data of damage plate structures as input and damage states as output to train ELM. To show the performance of the presented method, a cantilever and a plate with four-fixed have been used. Also, Kourehli [21] used artificial neural network and the Guyan reduction method to identify damages. Recently, Ghadimi et al. [22] presented a novel approach to predict cracks in beam structures using ELM.

In this paper, a new structural damage identification method is presented based on limited number of sensors data and ELM. In this approach, NSEMR-II is used to condense mass and stiffness matrices, while ELM is used to predict damage. To evaluate the effectiveness of proposed method, three different examples, namely a truss bridge, frame structure and 15-story shear frame containing single damage or several damages have been used. The obtained results reveal that the proposed method is viable method to identify structural damages.

## **2. Neumann series Expansion-Based Model Reduction**

One of the main challenges in structural damage identification problems is the limitation in the number of used sensors. To address this issue, different model reduction methods have been proposed in literature [23-28]. In this study, a highly accurate condensation model, namely, NSEMR-II is

used to reduce the finite element model. The background of NSEMR-II approach is briefly summarized in the following.

The analytical model of a given structure can be divided to master and slave DOFs as follows

$$\begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{Bmatrix} \phi_j^m \\ \phi_j^s \end{Bmatrix} = \lambda_j \begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix} \begin{Bmatrix} \phi_j^m \\ \phi_j^s \end{Bmatrix} \quad (1)$$

where the superscripts “*m*” and “*s*” denote the master and slave DOFs, respectively.

So, the reduced mass ( $M_r$ ) and stiffness ( $K_r$ ) matrices based on NSEMR-II can be expressed as [24]:

$$M_r = T^T M T \quad (2)$$

$$K_r = T^T K T \quad (3)$$

Where

$$T = \begin{bmatrix} I \\ -[B_1 + K_{ss}^{-1}M_{ss}(A_1A_4 + A_1A_5)]^{-1} \times \\ [B_2 + K_{ss}^{-1}M_{ss}(A_1A_2 + A_1A_3)] \end{bmatrix} \quad (4)$$

and

$$A_1 = K_{ss}^{-1}M_{ss}K_{ss}^{-1}K_{sm}M_{mm}^{-1} \quad (5)$$

$$A_2 = K_{mm}M_{mm}^{-1}K_{mm} \quad (6)$$

$$A_3 = K_{ms}M_{ss}^{-1}K_{sm} \quad (7)$$

$$A_4 = K_{mm}M_{mm}^{-1}K_{ms} \quad (8)$$

$$A_5 = K_{ms}M_{ss}^{-1}K_{ss} \quad (9)$$

$$B_1 = I + A_1K_{ms} \quad (10)$$

$$B_2 = K_{ss}^{-1}K_{sm} + A_1K_{mm} \quad (11)$$

In this paper, the NSEMR-II is used to condense mass and stiffness matrices to measured master DOFs.

### 3. Extreme Learning Machine (ELM)

The ELM is an extremely fast single-hidden layer feedforward neural network which was originally proposed by Huang et al. [29]. In ELM, the weights of the output layer optimize by Moore-Penrose generalized inverse. The structure of ELM can be seen in Fig. 1. See more details in refs. [22, 29].

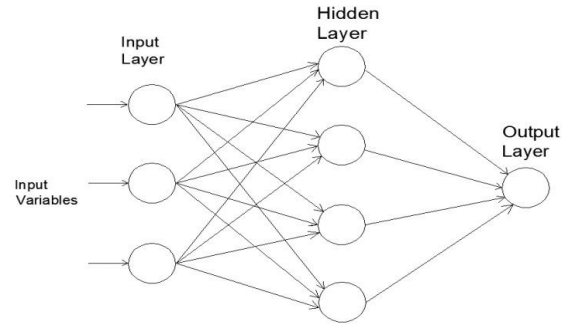


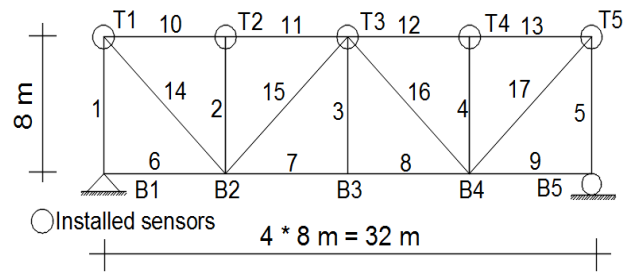
Fig. 1. The structure of ELM.

### 4. Numerical Examples

To show the performance of the proposed method using frequencies and incomplete mode shapes, three different numerical examples have been studied. A truss structure, irregular frame and shear frame are modeled using finite element method. In this paper, the effect of damages, simulated by reduction in elasticity modulus of structural finite elements. See more details in ref. [21]. Then, different damage scenarios considered and natural frequencies and incomplete mode shapes of the damaged structure are obtained and used as the input data to train ELM.

#### 4.1. Truss Structure

The first example is a plane truss structure with 17 elements and 8 nodes (see Fig. 2). The finite element model of studied structures was simulated by using MATLAB software.



**Fig. 2.** Plane truss structure.

Table 1 shows the characteristics of different elements in studied steel truss bridge structure. In this paper, the first three vibrating modes are utilized for damage detection.

To compare the performance of different model reduction approaches, three different model reduction approaches namely, Guyan's approach, first-order and NSEMR-II has been studied. Table 2, presented the first ten natural frequencies obtained by various reduction approaches for truss structure. It

can be seen that the NSEMR-II has less errors and is more accurate, which is utilized in this paper.

**Table 1.** Characteristics of truss elements.

Element s	Element s length (m)	Cross- sectiona l area (cm <sup>2</sup> )	Sections
B1B3	16.0	181.0	IPB360
B1T1	8.0	72.7	IPB360
B2T1	11.3	181.0	IPB360
B2T2	8.0	143.0	IPBL360
B2T3	11.3	373.0	IPBV300+2PL350* 10
B3T3	8.0	72.7	IPE360
T1T3	16.0	373.0	IPBV300+PL350*1 0

To evaluate presented method, three different damage cases are considered by reduction in elasticity modulus of truss elements. The considered reduction factors in different truss elements listed in Table 3. As it can be seen from Fig. 2, sensors installed at joints T1, T2, T3, T4, T5 and selected as measured DOFs.

**Table 3.** Considered reduction factors in different truss elements.

Case 1		Case 2		Case 3	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
12	0.1	6	0.2	3	0.2
-	-	16	0.1	7	0.2
-	-	-	-	15	0.1

In real cases, the presented approach uses mode shapes and frequencies of the damaged structure in measured DOFs. So, in the numerical examples using the finite element

modeling, some hypothetical damage scenarios have been used to obtain the mode shapes and frequencies.

**Table 2.** The first ten natural frequencies obtained by various reduction approaches for truss structure.

Natural frequencies (Hz)	f1	f2	f3	f4	f5	f6	f7	f8	f9	f10
Unreduced damaged model	11.087	15.975	28.716	49.970	63.458	67.732	76.509	91.382	91.815	100.452
Guyan's method	11.144	16.160	29.939	52.987	66.046	74.179	88.795	118.944	206.513	215.519
Guyan's method's errors (%)	0.512	1.157	4.262	6.038	4.078	9.517	16.059	30.162	124.922	114.549
The first order approximation	11.088	15.976	28.725	50.230	64.254	68.806	83.631	101.090	107.575	157.033
The first order approximation errors (%)	0.006	0.007	0.034	0.520	1.254	1.586	9.310	10.624	17.165	56.326
The second order approximation	11.088	15.976	28.719	50.027	63.711	68.043	79.267	91.854	102.117	116.250
The second order approximation errors (%)	0.005	0.006	0.011	0.114	0.398	0.459	3.605	0.517	11.220	15.727

To train ELM, incomplete mode shape and frequencies (modal data) of damaged structures used an input and corresponding damage severity (DS) used as output.

Training data consists of modal data of truss structures with different DS values equal to 0%, 10%, 20% and 0, 10 % for elements numbered 2,4,6,8,10,12 and elements numbered 1, 3, 5, 7, 9, 11, 13, 14, 15, 16, 17 respectively. In this case, only 10000 random combinations of the assigned DSs used to train and test ELM. Table 4 show the performance of the ELM for detecting and estimating damage. It can be see that the low values of MSE has been achieved.

**Table 4.** Performance of the ELM in truss bridge.

	Sample Numbers	MSE
Training	9000	5.910E-06
Testing	1000	7.797E-06

Finally, the effectiveness of the presented approach has been studied using three damage cases. Fig. 3 shows that the presented approach is robust and effective in spite of limited number of measurements which may be noisy data.

## 4.2. Frame Structure

The second example is an irregular steel frame with 7 column elements, 8 beam elements and 11 free nodes, as shown in Fig. 4. The steel material properties are mass density  $\rho=7850 \text{ kg/m}^3$ , elasticity modulus  $E=200 \text{ GPa}$ . For columns, mass per unit length is  $m=117.75 \text{ kg/m}$ , moment of inertia is  $I=3.3 \times 10^{-4} \text{ m}^4$ , and cross-sectional area is  $A=1.5 \times 10^{-2} \text{ m}^2$ . while for beams this properties are  $m=119.32 \text{ kg/m}$ ,  $I=3.69 \times 10^{-4} \text{ m}^4$  and  $A=1.52 \times 10^{-2} \text{ m}^2$  [31]. In this example, the measured DOFs are 11 translational DOFs to identification of damage in frame structure.

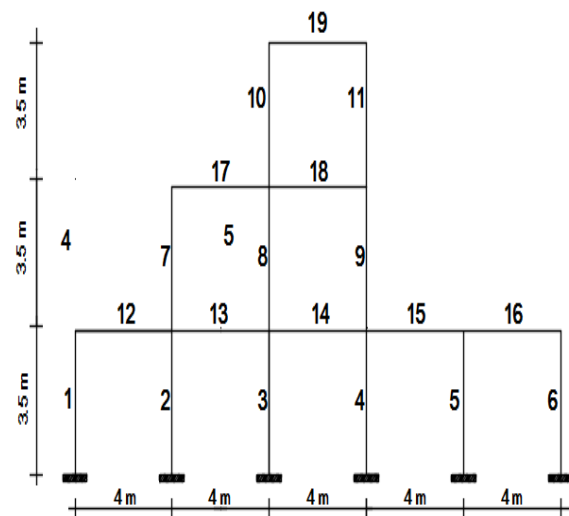
**Fig. 4.** Irregular plane steel frame.

Table 5, presented the first ten natural frequencies obtained by various reduction approaches for frame structure. It can be see that the NSEMR-II is more accurate and utilized in this paper.

Also, the considered reduction factors (DSs) in different frame elements listed in Table 6.

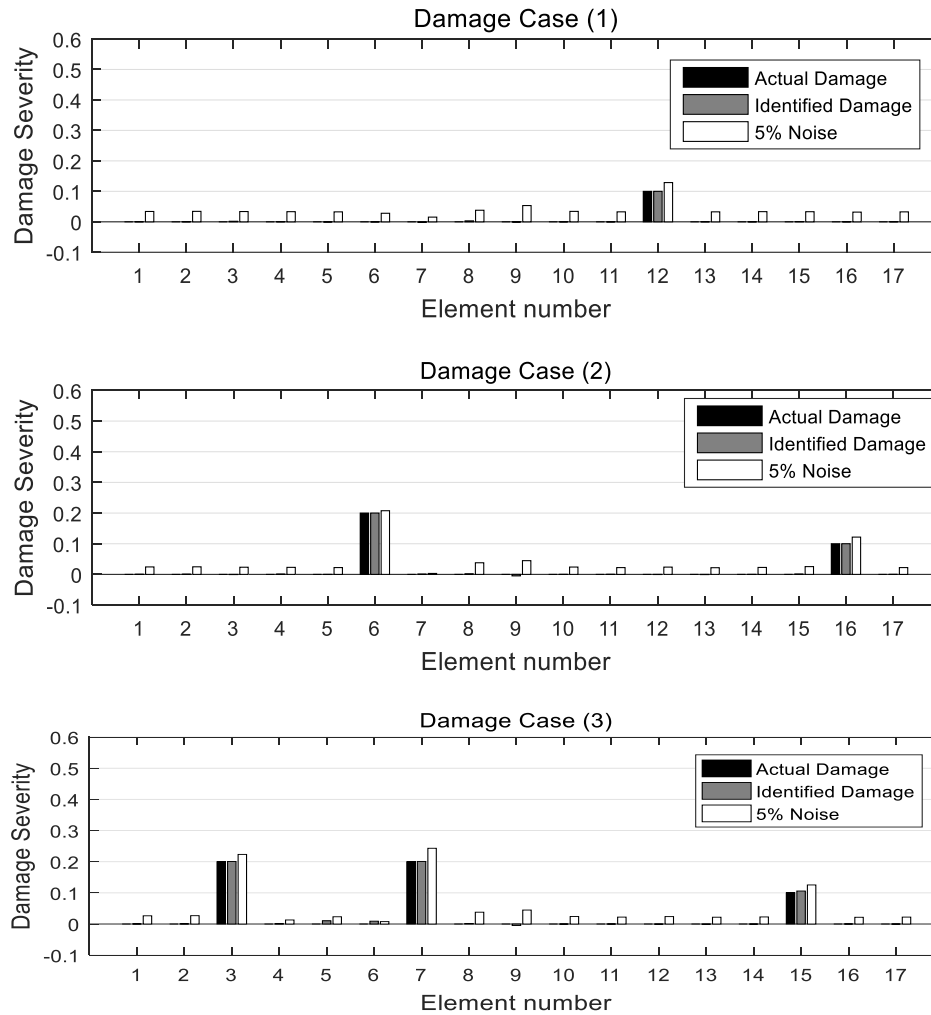


Fig. 3 Obtained results for three different damage cases using noise free data and 5% noisy data in truss structure.

Table 6. Considered reduction factors in frame structure elements.

Case 1		Case 2		Case 3	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
10	0.1	2	0.2	4	0.1
-	-	19	0.1	11	0.2
-	-	-	-	18	0.1

To generate training patterns, DS values equal to 0%, 10%, 20% for elements numbered 2,8,14,19 and 0, 10 % for other elements, were considered for every element in the structure. In this case, only 10000 random combinations of the assigned DSs used to train and test ELM. Table 7 show the efficiency of the ELM with low values of MSE.

**Table 7.** Performance of the ELM in frame structure.

	Sample Numbers	MSE
Training	9000	1.158E-06
Testing	1000	2.945E-06

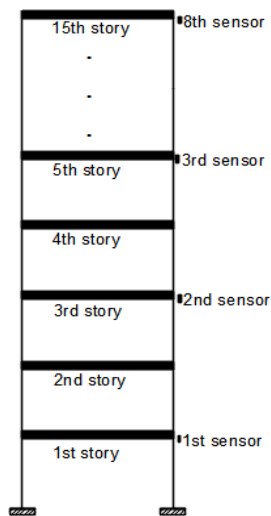
Fig. 5 show the performance of the presented method for detecting and estimating damage severities in different elements using only translational DOFs measurements for frame structure which may be noisy data.

**Table 5.** The first ten natural frequencies obtained by various reduction approaches for frame structure.

Natural frequencies (Hz)	f1	f2	f3	f4	f5	f6	f7	f8	f9	f10
Unreduced damaged model	11.901	27.032	42.759	81.507	98.516	103.270	106.222	117.252	126.583	132.295
Guyan's method	11.904	27.059	42.802	107.307	202.603	224.161	299.904	368.990	389.269	436.242
Guyan's method's errors (%)	0.028	0.102	0.099	31.653	105.656	117.063	182.336	214.699	207.520	229.750
The first order approximation	11.904	27.058	42.799	107.440	116.671	160.047	198.985	209.526	214.299	260.222
The first order approximation errors (%)	0.026	0.095	0.094	31.816	18.429	54.980	87.329	78.698	69.295	96.698
The second order approximation	11.903	27.055	42.799	83.228	100.720	106.537	119.607	129.951	144.247	163.513
The second order approximation errors (%)	0.023	0.086	0.093	2.111	2.237	3.164	12.600	10.831	13.954	23.597

### 4.3. Fifteen Story Shear Frame

The fifteen story shear frame is shown in Fig. 6. For this case, 8 sensors installed on the 1, 3, 5, 7, 9, 11, 13, 15 stories (see fig. 6). Also, the stiffness and mass in different stories are shown in Table 8.

**Fig. 6** 15-story shear frame.**Table 8.** The characteristics of the shear frame.

Story number	Mass (ton)	Stiffness (MN m <sup>-1</sup> )
1-5	50	300
6-10	50	200
11-15	50	100

Table 9, presented the first ten natural frequencies obtained by various reduction approaches for frame structure. It can be seen that the NSEMR-II is more accurate and utilized in this paper.

To show the performance of presented method, three different damage cases are considered. The considered reduction factors in different shear frame story's stiffness listed in Table 10.

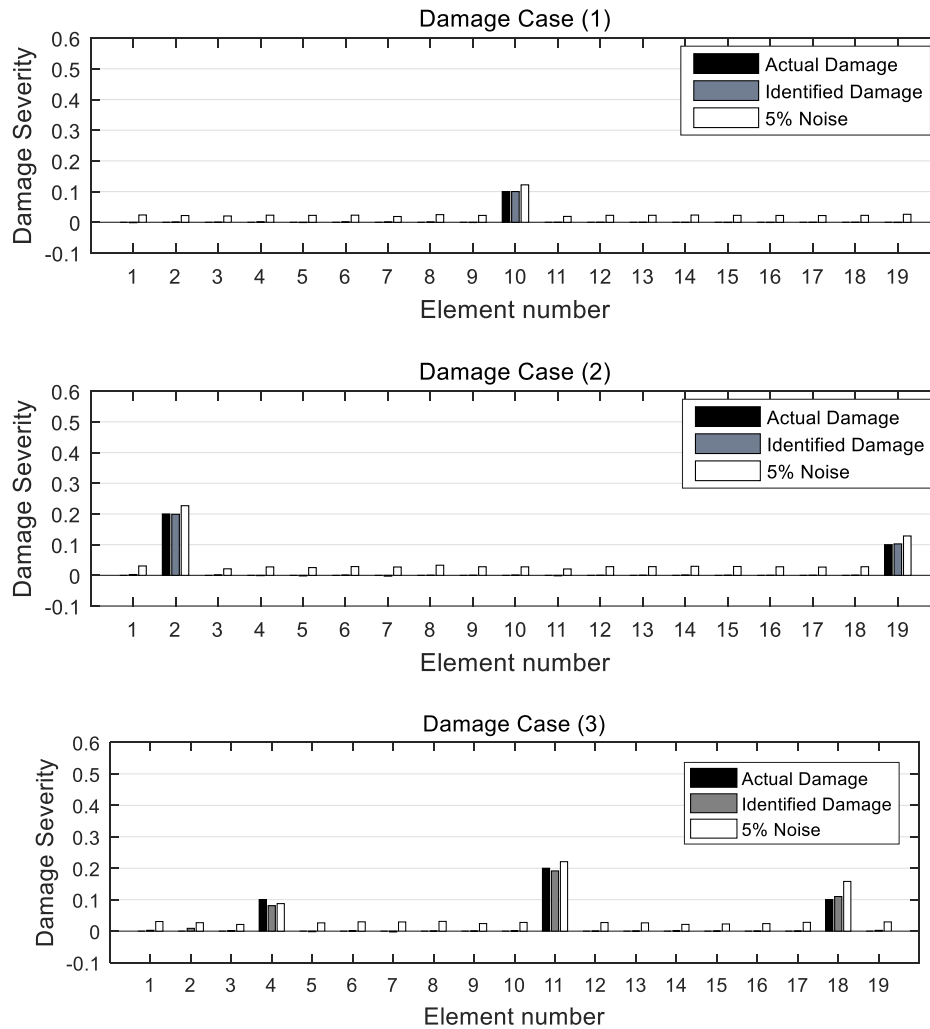


Fig. 5 Obtained results for three different damage cases using noise free data and 5% noisy data in frame structure.

Table 10. Considered reduction factors in different shear frame story's stiffness.

Case 1		Case 2		Case 3	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
2	0.1	9	0.2	7	0.1
-	-	13	0.1	8	0.2
-	-	-	-	14	0.2

To generate training patterns, DS values equal to 0%, 10%, 20% for elements numbered 1,3,5,7,9,11,13,15 and 0, 10 % for other elements, were considered for every element in the structure. In this case, only

10000 random combinations of the assigned DSs used to train and test ELM. Table 11 shows the low values of MSE in training and testing stages.

Table 11. Performance of the ELM in shear frame.

Sample Numbers	MSE	
Training	9000	4.7189E-07
Testing	1000	6.148E-07

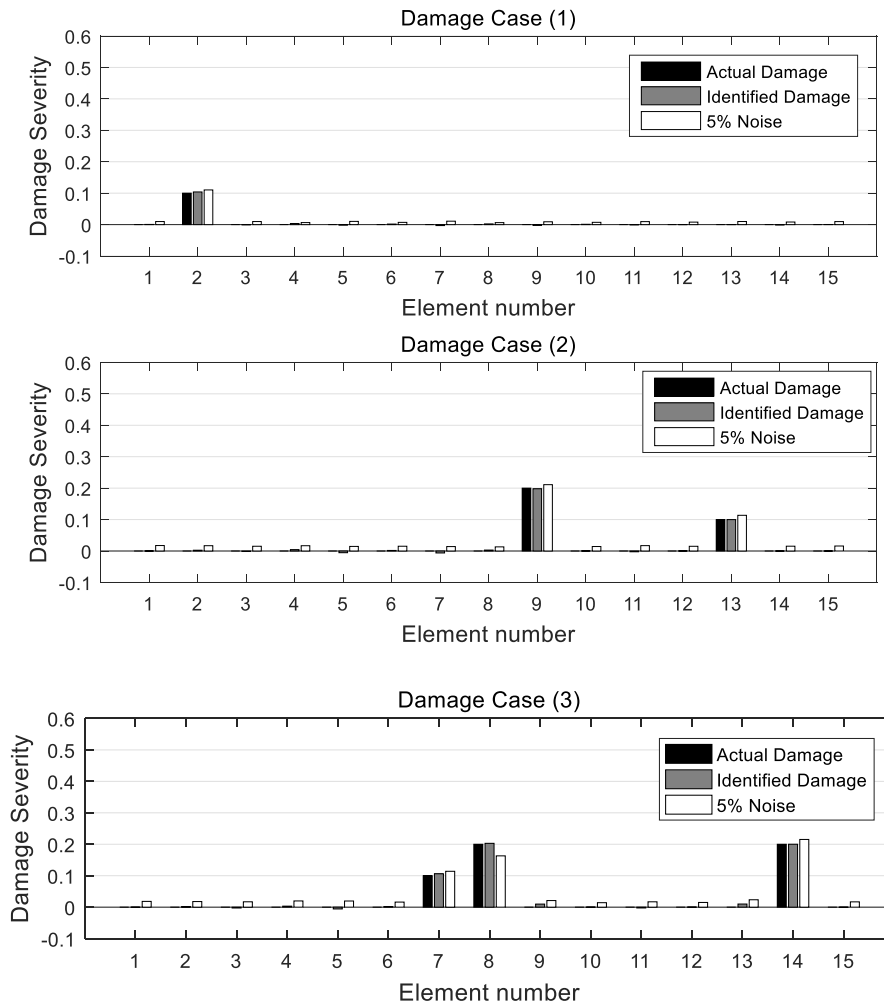


Fig. 7 show that the performance of the proposed approach for detecting and estimating damage severities in different elements using incomplete measurements

noisy data. The obtained results indicate that the proposed approach is promising in identification of different damage cases.

**Table 9.** The first ten natural frequencies obtained by various reduction approaches for 15-story shear frame.

Natural frequencies (Hz)	f1	f2	f3	f4	f5	f6	f7	f8
Unreduced damaged model	1.084	2.751	4.675	6.275	8.205	9.646	11.239	12.232
Guyan's method	1.086	2.780	4.802	6.597	8.641	10.333	12.729	16.684
Guyan's method's errors (%)	0.222	1.061	2.725	5.131	5.312	7.124	13.260	36.392
The first order aproximation	1.084	2.751	4.676	6.291	8.240	9.779	11.870	16.136
The first order aproximation errors (%)	0.000	0.001	0.032	0.257	0.427	1.379	5.615	31.909
The second order aproximation	1.084	2.751	4.675	6.276	8.209	9.694	11.621	15.535
The second order aproximation errors (%)	0.000	0.000	0.001	0.017	0.042	0.499	3.406	26.996



**Fig. 7.** Obtained results for three different damage cases using noise free data and 5% noisy data in 15-story shear frame.

## 5. Conclusions

In the presented study, damage identification problem was investigated using extreme learning machine and incomplete measurements. To condense mass and stiffness matrices, the NSEMR-II has been used. The ELM which is an extremely fast learning machine, used to predict damage. The performance of the presented approach was evaluated by using three examples, e.g., a truss structure, irregular frame and 15-story shear frame. Also, the performance of the presented approach has been studied using noisy data (5% noise). Results reveal that the presented approach is robust and promising using sparse sensor measurement and noisy data.

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