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Numerical Study on the Flexural Behaviour of Concrete Beams Reinforced by GFRP Bars

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ABSTRACT

Enhancement of the response of reinforced concrete (RC) beams applying fiber-reinforced polymer (FRP) reinforcement bars has become a popular structural technique over the past two decades as a result to the well-known advantages of FRP composites including their high strength-to-weight ratio and excellent corrosion resistance. This study presents numerical investigation of 20 concrete beams internally reinforced with GFRP bars without web reinforcement. The accuracy of the non-linear finite element model in ABAQUS software is first validated against experimental data from the literature. The study presents an investigation into the behaviour of FRP reinforced concrete beams including the evaluation of geometrical properties effects. In particular, the study is focused on the effects of span/depth ratio, the reinforcement ratio and the effective depth of the beam, aiming to correct deficiencies in this area in existing knowledge. It's been revealed that the finite element model is capable of accurately simulating the flexural behaviour of FRP reinforced beams. It was able to predict, with high accuracy, the force-displacement response the beam. Results manifested that FRP reinforcement is a proper solution in order to boost the ductility of RC beam members. Moreover, although that increasing in the span/depth ratio of the beam decreases beam's rigidity, however; it also postpones the yielding point in the beam's flexural response and leads to a higher level of displacement ductility for the beam.

1. Introduction

The application of fiber-reinforced polymer (FRP) composite materials has had a dramatic impact on civil engineering

techniques over the past three decades. FRPs are the ideal material for structural applications where high strength-to-weight and stiffness-to-weight ratios are required. The use of FRP composites for the

rehabilitation of beams and slabs has begun about 30 years ago with the pioneering research performed at the Swiss Federal Laboratories for Materials Testing and Research, or EMPA [1]. Afterward, FRP materials have been widely employed as a solution to boost the ductility of beam members in RC structures. Due to the high cost of FRP materials in the past, most applications of these materials were for rehabilitation purposes and externally bonding of members in RC structures. However, as the frequency of FRP materials usage scale increased, nowadays it led to the decrease of cost and more and more applications of these materials in constructions and rehabilitation projects in the future. Past earthquakes reconnaissance indicated that beam members in some structures have performed poorly as a result to corrosion of steel reinforcement in concrete and probable brittle-type failures with low ductility [2]. Where FRP composites are applied as reinforcement in the reinforced concrete (RC) beams, they increase the strength (ultimate limit state) and the stiffness (serviceability limit state) of the structure. Structural design of beams with FRP reinforcement is thus motivated by requirements for earthquake strengthening, higher service loads, smaller deflections, or simply the requirement to complement deficient steel reinforcement [3]. In the past two decades, researchers have performed various investigations in order to develop proper methods for designing steel reinforced beams that have ductile behaviour while providing high bending and shear capacity. On the other hand, many studies focused on strengthening and repairing RC beams employing other methods such FRP reinforcing. Regarding the mechanical properties of the FRP reinforcement, the

main differences in comparison with steel, are a lower modulus of elasticity and a linear elastic behaviour up to rupture, which implies the lack of plasticity in the behaviour of FRP [4]. From among research studies conducted on flexural behavior and serviceability performance of concrete beams reinforced with FRP bars one can refer to [5-15]. Among the research studies conducted on pullout behavior of GFRP bars in concrete and bond stress-slip behavior of GFRP bars in concrete respectively one can refer to [16] and [17], and Studies conducted on shear behavior of concrete beams reinforced with GFRP bars include references [18, 19]. The present study focuses on investigating the effectiveness of FRP reinforcing on the pushover behaviour of RC beams applying FE modeling technique. The FE meshes, boundary conditions and nonlinearity implementation methods have been calibrated/validated by comparing the predictions of the available experimental data. Subsequently, effects from FRP reinforcing on the bending response of RC beams were inquired. Therefore, two groups of FRP and steel reinforced beams, with same reinforcement ratio, have been selected to investigate the effect of FRP reinforcement on the moment capacity of RC beams. Geometrical and material nonlinearities in the concrete material, steel reinforcements and also FRP reinforcement have been taken into consideration. In the study effects from the variation of span/depth ratio, the reinforcement ratio and the effective depth of the beam are that the new issues that have been addressed.

2. Numerical Modeling

The software package used for FE modeling in this study was the general-purpose

nonlinear finite element package ABAQUS, which offers a comprehensive material constitutive law for simulation of concrete material. This section describes the modelling approach employed for the finite element analyses.

3. Geometry and Mesh

The configurations for four-point flexural tests were simulated in the numerical model. Figure 1 depicted the schematic geometry of the model and the finite element mesh used for the analysis of the model. Geometrical properties of specimens in the current study are presented in Table 1. Eight node three dimensional reduced integration elements with a Gaussian integration point in the element C3D8R have been used for simulating the concrete medium in the numerical model. Applying lower integration points can be beneficial when it comes to reduce the time of the analysis. ABAQUS uses a small artificial stiffness in order to prevent severe flexibility and no straining at the integration points [5]. Steel and FRP reinforcements are modeled using truss elements T3D2 and positioned in the exact locations as in the experimental works. Adjacent nodes have then been coupled employing embedment constraint. The beam cross section is discretized to elements with the dimension equal to 50 mm (15% of section width approximately). In the longitudinal direction, the solid elements located in middle regions had the same dimension. This caused the mesh in the concrete to be fine enough to capture the local cracks and failures. To optimize the computational efforts, a relatively coarser mesh up to 30% of the section width was adapted for the rest of the beam. Compatible meshes were also deliberated for steel and

FRP reinforcements. Figure 1.b illustrates the employed mesh in the finite element model.

4. Material Constitutive Laws

ABAQUS offers three types of concrete constitutive models to define the behaviour of material regarding its elasticity, plasticity and damage mechanisms; Smeared Cracking, Concrete Damage Plasticity (CDP), and Brittle Cracking. Among the available models, CDP model provides more control for the post-peak descending branch (degradation) of material. CDP model was used in this study to define the mechanical properties of concrete in the model. In order to define the compressive stress-strain behaviour of concrete material, empirical equations proposed by Hognestad et. al. was used. The stress-strain curve is illustrated in Figure 2 [6].

$$f_c = \begin{cases} f_c'' \left[\frac{2\varepsilon_c}{\varepsilon_0} - \left(\frac{\varepsilon_c}{\varepsilon_0} \right)^2 \right] & 0 \leq \varepsilon_c \leq \varepsilon_0 \\ f_c'' \left[1 - 0.15 \left(\frac{\varepsilon_c - \varepsilon_0}{0.0038 - \varepsilon_0} \right) \right] & \varepsilon_0 \leq \varepsilon_c \leq 0.003 \end{cases} \quad (1)$$

The tension strength of concrete (f_t) is also calculated from equation below according to the CEB-FIB 2010 [7]:

$$f_t = 0.3 f_c^{2/3} \quad (2)$$

The ratio of initial biaxial compressive yield stress to initial uniaxial compressive yield stress ($\sigma_{b0} / \sigma_{c0}$) was considered equal to 1.16. The ratio of the second deviatoric stress invariant on the tensile meridian to that on the compressive meridian (K_c) was applied equal to 0.67 for the analyses [8]. Pursuant to Dey [9], the dilation angle of concrete material was also set equal to 30 degree. For steel reinforcements, isotropic hardening plasticity was contemplated through allocating the yielding point and the ultimate strength of the steel bar, as well as a failure strain. FRP reinforcement is considered to be

orthotropic and transversely isotropic, i.e. the mechanical properties are the same in any direction perpendicular to the fibers. In this study, FRP material was modelled by defining lamina type of elasticity in accordance with the failure sub option

offered by ABAQUS. The mechanical properties of GFRP reinforcement was selected to be the same as the materials used by Andermatt and Lubell, which is summarized in Table 2 [2].

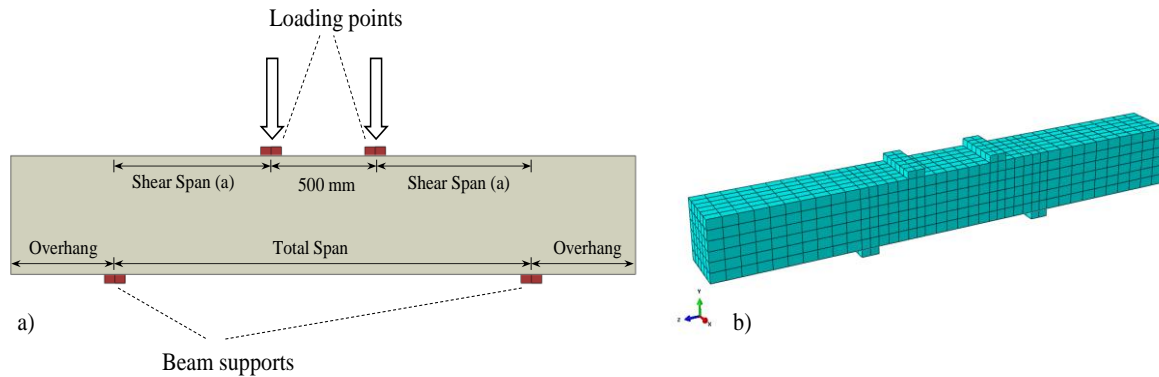


Fig. 1. a) Schematic geometry of the model, b) Finite element mesh.

Table 1. Geometrical properties of specimens.

No.	Reinforcement Material	ρ (%)	Width (mm)	Height (mm)	Shear Span (mm)	Shear Span/ Depth (a/d)	Overhang (mm)	Beam Span(mm)	Total Length(mm)
1	FRP	1.5	300	500	750	1.7	500	2000	3000
2		1.5	300	500	1000	2.2	250	2500	3000
3		1.5	300	500	1000	2.2	750	2500	4000
4		1.5	300	500	1250	2.8	500	3000	4000
5		1.5	300	500	1250	2.8	1000	3000	5000
6		1.5	300	500	1500	3.3	750	3500	5000
7		1.5	300	500	1500	3.3	1250	3500	6000
8		1.5	300	500	1750	3.9	1000	4000	6000
9		1.5	300	500	2000	4.4	1750	4500	8000
10		1.5	300	500	2250	5.0	1500	5000	8000
11		1.5	300	300	1250	5.0	500	3000	4000
12		1.5	300	400	1250	3.6	500	3000	4000
13		1.5	300	600	1250	2.3	500	3000	4000
14		1	300	500	1250	2.8	500	3000	4000
15		2	300	500	1250	2.8	500	3000	4000
16		2.5	300	500	1250	2.8	500	3000	4000
17	Steel	1.5	300	500	1250	2.8	500	3000	4000
18		1	300	500	1250	2.8	500	3000	4000
19		2	300	500	1250	2.8	500	3000	4000
20		2.5	300	500	1250	2.8	500	3000	4000

5. Reinforcement-Concrete Interaction

The bond interface between the concrete material and reinforcement can have a significant effect on the performance of reinforced concrete members. Two modelling

approaches are common for simulation of bond interface. In the first approach, a group of spring describing the behaviour of short embedment lengths in pull-out tests has to be employed for predicting the behaviour of longer embedment lengths. In the second approach, a full bond can be considered between the concrete medium and reinforcement by coupling the shared nodes between reinforcement and concrete elements. In terms of the bending capacity of

beams, by neglecting the effect from pull out of the reinforcement, both approaches revealed good compromise between numerical and experimental results. Thus, for the sake of simplicity, full bond situation is deliberated for the reinforcing bars. For this, the longitudinal and reinforcement bars are modelled with 3D truss elements T3D2 as embedded elements in concrete block of the beam.

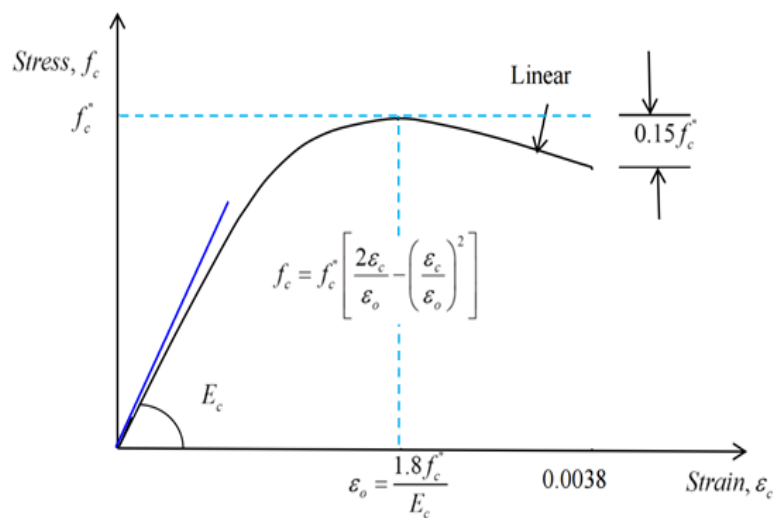


Fig. 2. Hognestad stress-strain curve for concrete material [6].

Table 2. FRP Material Properties.

Parameter	Value
Tensile Strength (MPa)	709
Elastic Modulus (MPa)	41100
Rupture Strain (%)	1.72

6. Numerical Analysis

To perform a quasi-static pushover analysis, either a static or a dynamic analysis (Explicit) could be used. Due to the higher stability of the response in static analyses, this analysis type in ABAQUS is selected in this study. Displacement controlled analysis was defined by applying a small increment through the analysis in the way that devote

an appropriate convergence for the analysis. The other analysis type (Dynamic/Explicit) was also previously employed by other researchers and was found to be able to achieve reasonable results, by performing sensitivity analysis for the specific problem [10, 11].

7. Model Validation

In this section, the validation of the numerical model for accurate following of the response of FRP reinforced beams under the four node flexural test has been performed by comparing the simulation results with available experimental data. Recently, few researchers validated the capability of ABAQUS to simulate the flexural response of RC beams systems subjected to four-node type loads [12, 13]. In order to ensure precision of the modelling approach, validation of numerical predictions against the results of four experimental test specimens by Andermatt and Lubell were performed. The tests were carried out for examining flexural capacity of FRP reinforced deep beams with different reinforcement ratios [2]. Four beam specimens A1N, A2N, A3N and B2N from the tests Andermatt and Lubell with reinforcement ratio equal to 1.49%, 1.47%, 1.47% and 1.71% respectively were selected to be applied in the model validation stage [2]. Table 3 portrayed the geometrical

properties of four test specimens, as well as the compressive strength of concrete poured in each specimen. Modelling approach presented in previous section was used for simulation and analysis of the aforementioned specimens. Figure 3 illustrates the comparison between numerical predictions and experimental data. As it is evident in the figure, the numerical model can reasonably predict the lateral response of FRP reinforced beams. A quantitative comparison between the results reveals a difference between the peak loads to be 0.6%, 1.1%, 0.7% and 2.6% for specimens A1N, A2N, A3N and B2N respectively. Small discrepancies between the results might be because of some uncertainties in material strength and also effect of some residual stresses due to the probable imperfections. These effects have not been contemplated in the numerical model. The validated numerical model is then utilized to perform a parametric study on bending capacity of FRP-reinforced beams.

Table 3. Test specimens used for validation of the numerical model.

Specimen	ρ (%)	Height (mm)	Width (mm)	Shear Span (mm)	f_c (MPa)
A1N	1.49	306	310	276	40.2
A2N	1.47	310	310	376	45.4
A3N	1.47	310	310	527	41.3
B2N	1.71	606	300	743	39.9

8. Results of Numerical Analysis

Four different levels of section height to width ratio with different reinforcement ratios were investigated. Both steel and FRP reinforcement were considered in the study.

The general overview of the models results in the following:

- Reinforcement ratio $\rho = 1\%$, 1.5% , 2% and 2.5% for FRP reinforcement
- Reinforcement ratio $\rho = 1\%$, 1.5% , 2% and 2.5% for steel reinforcement

- Section effective depth $d = 250\text{mm}$, 350mm , 450mm and 550mm
- Shear span to effective depth ratio $a/d = 1.7$, 2.2 , 2.8 and 3.3

Figure 4 presents a typical contour of tensile damage in the concrete material of beam from the numerical analysis, which shows the failure pattern of the specimen 9.

9. Reinforcement Ratio Effects

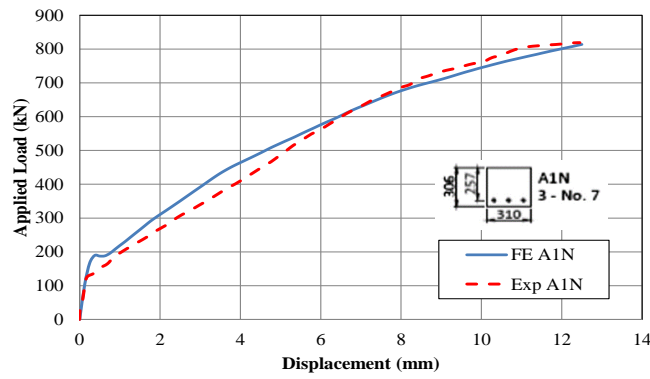
In this section, in order to evaluate the effectiveness of applying FRP reinforcement on the RC beam bending response, pushover responses of the studied beam are presented. Load-displacement pushover curves for both FRP reinforced and steel reinforced beam having various reinforcement ratio are highlighted in Figure 5. In general, FRP reinforced beams seem to have a more ductile behavior than the steel reinforced ones. All the steel reinforced beams with different reinforcement ratio have similar yield point close to $\delta/L=0.005$ and afterward, a plateau trend in the curves is observed. Although the FRP reinforced beams displayed less rigidity in their elastic response in comparison with the steel reinforced beam, increase in the reinforcement ratio created more major effects on the stiffness of the beams reinforced with FRP bars. Moreover, by increasing the reinforcement ratio, the yielding point in the curves are shifted a bit to imply more rigidity and less deflection corresponds to the yield point. By comparing the responses of FRP reinforced beams, it is evident that beams with reinforcement ratio less than 2.0% have a full ascending response, while in beams with reinforcement ratio more than 2.0%, more stiffness comes with a plateau in the post yield region of beams responses. In consonance with some guidelines and available literature, the

minimum reinforcement ratio for FRP reinforced beam is too low in comparison with steel reinforcement. However, results of the current study revealed that more reinforcement ratio is needed for cases that a plateau trend in the beam response is required. A quantitative comparison between results is presented in Table 4. Evaluating the results shows that FRP reinforced beams had a hardening trend ranging from 5% to 48% after the yielding point (24% average), while the hardening behaviour in the steel reinforced beams was in the range 2% to 11% after the yielding point (6% average).

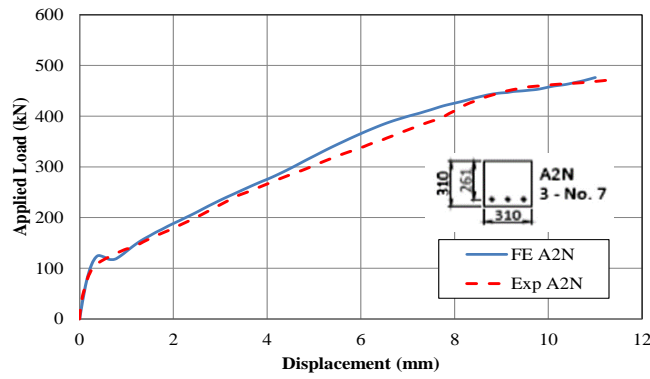
10. Effects of Beam Effective Depth

Pursuant to design guidelines, the effective depth is equal to the total depth minus the distance from the centroid of the tension reinforcement to the extreme tension concrete fibers, which depends on the number of layers of the steel bars. This geometrical parameter has a major role in the bending capacity of the beams which also affects the general response of the beam. In this section, a comparison between the responses of FRP reinforced beams with same section width and reinforcement ratio is performed (specimens 4, 11, 12 and 13) through load-displacement curves as presented in Figure 6. As it is observed in the pushover curves, by increasing the effective depth of the beam reinforced by FRP bars, more force is required to be applied on the beam to reach a certain deflection. Considering the specimen with $d = 250\text{mm}$ as the control specimen, by increasing the effective depth by 40%, 80% and 120%, the force corresponds to the yielding point will be increased by 47%, 119% and 176% respectively. Same comparison for the maximum force leads to 69%, 157% and 239% increase, respectively. This shows the

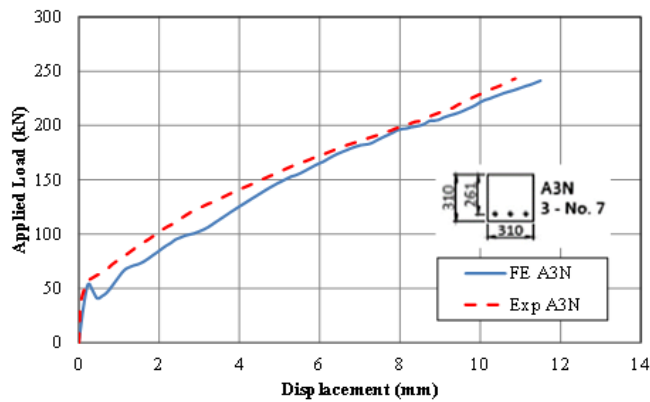
significant effect of the effective depth (i.e. beam depth) on the beam flexural response.



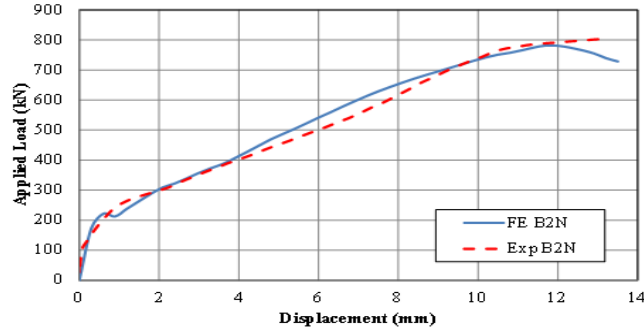
(a)



(b)



(c)



(d)

Fig 3. Verification of numerical model against four series of available experimental data tested by Andermatt and Lubell (2013) [2], a) A1N, b) A2N, c) A3N, d) B2N.

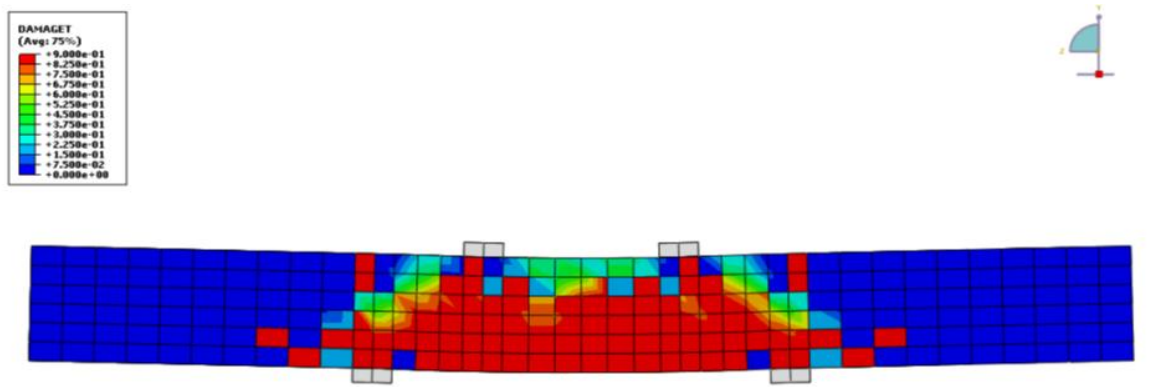


Fig. 4. Tensile damage in the FRP reinforced beam.

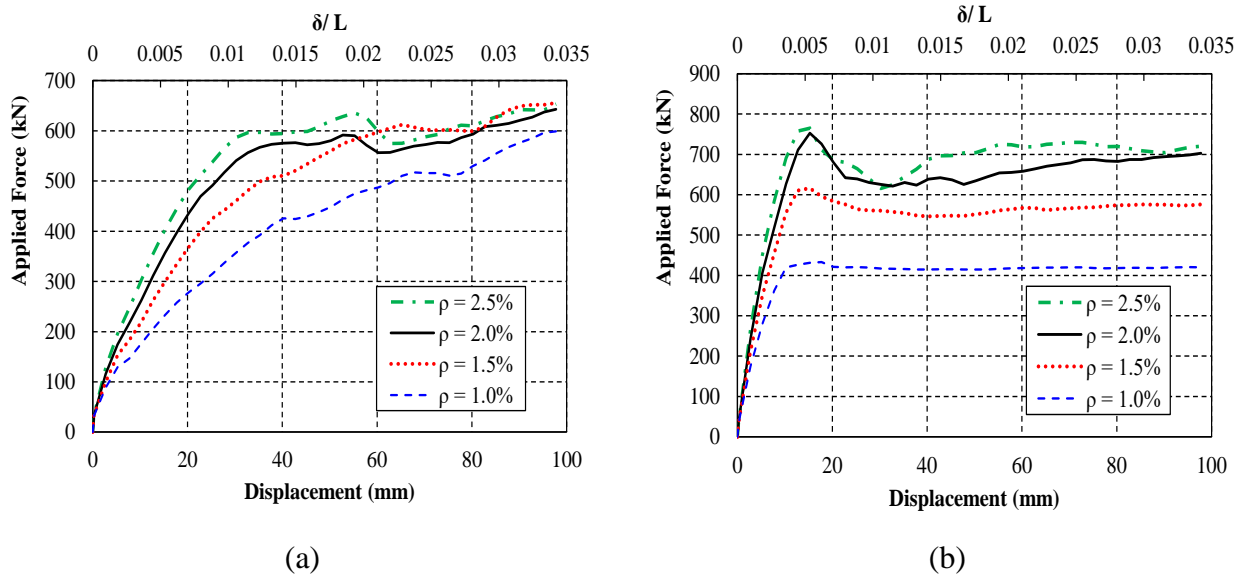


Fig. 5. Pushover response of beams with different reinforcement ratio, a) FRP reinforced beams, b) steel reinforced beams.

11. Effects of Shear Span to The Effective Depth Ratio

A wide range of the shear span to effective depth ratio (a/d), starting from 1.7 up to 5.0, was investigated in the current study. Figure 6 portrayed the force-displacement results beams with different a/d ratio. As it is evident in the figure, by increasing the shear span to effective depth ratio, the initial stiffness of the beam is dropped significantly. For presenting the results in a more tangible way, the curves are presented in two parts in Figure 6.a and 6.b. As it is depicted in the figures, beams with a/d ratio up to 2.8, have a

bi-linear response with a clear yielding point in their curve. For beams with a/d ratio more than 3.0 up to 5.0, a more continuous ascending trend is observed. In fact, the behaviour of beams with high shear span length seems to be more elastic and the damaged area and degradation of materials are too limited in these specimens. The initial alternative in the slope of the response curve is more because of the initial cracks in the outer tensile fibers of concrete till the point that tensile reinforcement takes full action in the beam's flexural response.

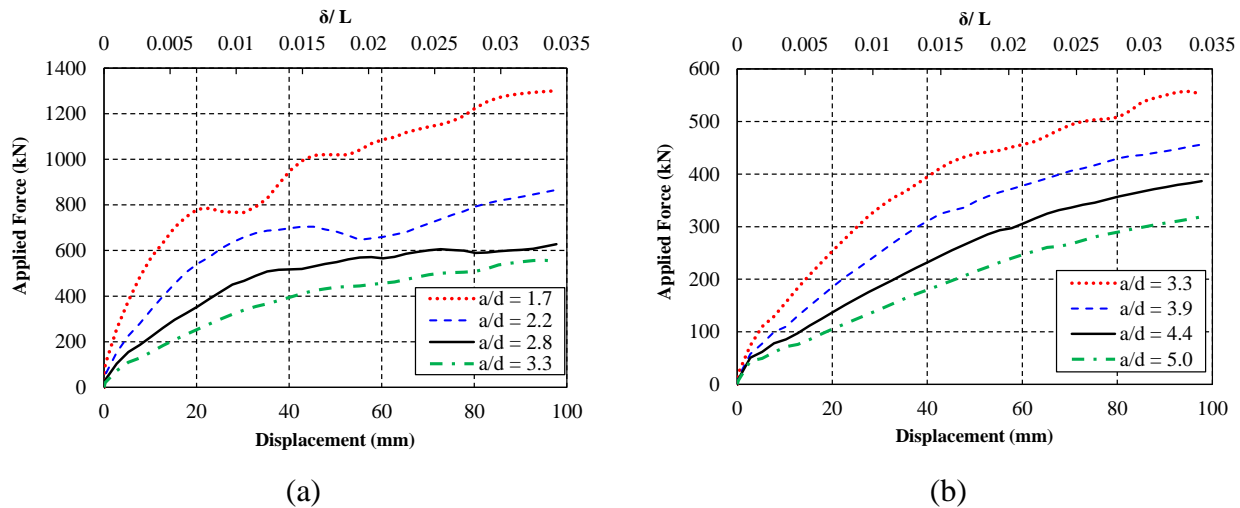


Fig. 6. Effects of shear span to effective depth ratio (a/d), a) $1.7 \leq a/d \leq 3.3$, b) $3.3 \leq a/d \leq 5.0$

12. Conclusion

Due to the growing demand of employing FRP reinforcing in RC beam members, the response of FRP-reinforced beams in the four node flexural test was investigated in the current study, using nonlinear FE analyses. Pushover analyses of beams were performed, and results revealed that:

- The numerical modelling approach presented in the current study is able to predict the flexural capacity FRP

reinforced beams with a very high level of accuracy. The total load-deformation curve from the FE analysis is in good agreement with available experimental data.

- The behaviours of FRP reinforced beams are more ductile than the steel reinforced ones.
- FRP reinforced beams have more hardening trend than steel reinforced beams; 5% to 48% (24% average) and 2% to 11% (6% average), respectively.

- Increase in the effective depth of the beam reinforced by FRP bars not only affects the beam yielding force, but also significantly increases the amount of maximum force resisted by the beam.
- A bi-linear response with a clear yielding point is observed for FRP reinforced beams with shear span to effective depth ratio up to 2.8, while for beams ratio more than 3.0 up to 5.0, a more continuous ascending trend is observed which mainly consists of elastic deformations.

In further studies, effect of FRP-reinforcing on the flexural response of non-rectangular beams with different geometries and rebar configurations can be evaluated. Furthermore, effect of FRP reinforcing on the shear response of beams can also be inquired.

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