Compressive Strength Prediction Using the ANN Method for FRP Confined Rectangular Concrete Columns

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ABSTRACT
Fiber Reinforced Polymer (FRP) was extensively employed as external confinement to strengthen the RC structures. Extensive studies were carried out to assess a more exact formula for measuring the strength enhancement of such strengthened concrete columns. A database from several experimental tests was gathered. A comparison between the experimental values and existing formulae showed an urgent need for a more exact formula. This study investigated to develop an exact formula based artificial neural networks (ANNs), to present the strength enhancement. The ANN-based method was simulated based on the collected database and an exact formula generated. The proposed formula was compared to current formulae employing the gathered database. The results demonstrated that the new formula based ANN gives the best accuracy than others. A sensitivity analysis based on Garson’s algorithm was generated for indicating the value of each used variable.

1. Introduction
Retrofitting of the RC columns utilizing FRP as confined reinforcement, showed the enhancement of the capacity and performance of structures columns. A series of different strength models to present the performance of RC columns retrofitted using FRP were introduced recently [1-9]. The most majority of the proposed formulae that are based on the Richart et al. [10] model, apply for circular concrete sections that estimate the enhancement in terms of capacity, the thickness and number of used FRP, and the section diameter. Little studies have investigated the FRP application for rectangular sections than the circular ones [11-13]. The pressure in reason of FRP confinement of a rectangular column around its perimeter is not uniform. For this reason,
it is difficult to formulate the distribution of stress exactly. The proposed formulae were the same for circular and rectangular sections, previously. Currently to modify the non-uniform stress distribution a ratio as shape factor has been introduced. Therefore, presenting an exact formula to assess the performance of a rectangular section is an interesting concern for the present paper. Equation 1 present the enhanced compressive strength of FRP-confined RC columns.

$$f'_c = f'_c \left(1 + a \frac{f_c}{f'_c} \right)$$

Equation 1

$f'_c$: Strength enhancement of FRP-confined RC column; $f'_c$: the compressive capacity of the RC column; $f_c$: The confinement lateral strength, $f_c = 2f_{FRP}/d$; $a$: the factor for confinement effectiveness; $f_{FRP}$: The FRP strength; $t$: The FRP thickness; $d$: The column diameter.

Nowadays, ANN has been utilized in the simulating of several civil engineering problems by researchers. The application of conventional methods to derive a formula to measure the capacity of FRP-confined RC columns, in reason of unknown multivariable and noisy database is not suitable. In this study, the ANN is applied for the estimation of the capacity of the FRP confined concrete columns. ANN was employed for input-output models. In this model, the experimental dataset is used for training the system. If using the dataset obtains appropriate information on the problem, therefore the trained ANN model will contain sufficient information on the output and the model can be present as a reliable model. In several published currently papers, ANN has been used for different applications in engineering [14-32]. As mentioned, collecting an exact experimental database is an essential activity in first. Therefore, a large numbers specimens on the FRP confined concrete columns were gathered. Besides, all of the important existing formulae for strength enhancement of FRP-confined RC columns were collected. The existing formulae to measure the strength of FRP-confined RC columns were tabulated in Table 1. A new formula based ANN approach was managed here.

### Table 1. The Models used for FRP-Confined Concrete Compressive Strength.

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Equations for $f'_c$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirmiran et al.</td>
<td>$f'_c = 1 + 6 \left( \frac{2c}{D} \right) \left( \frac{f'_c}{f'_c} \right)$</td>
<td>[33]</td>
</tr>
<tr>
<td>ACI</td>
<td>$f'_c = -1.254 + 2.254 \sqrt{1 + \frac{7.947f_c}{f'_c}} - \frac{2}{k_f} \frac{f_c}{f'_c}$</td>
<td>[34]</td>
</tr>
<tr>
<td>Lam and Teng</td>
<td>$f'_c = 1 + 3.3 \left( \frac{A_k}{A_r} \right) \left( \frac{f_c}{f'_c} \right)$</td>
<td>[11]</td>
</tr>
<tr>
<td>Al-Salloum</td>
<td>$f'_c = 1 + 3.13k \left( \frac{b}{D} \right) \left( \frac{f_c}{f'_c} \right)$</td>
<td>[35]</td>
</tr>
<tr>
<td>Restrepo and De Vino</td>
<td>$f'_c = \alpha_1 \alpha_2 \left( \frac{f_c}{f'_c} \right)$</td>
<td>[36]</td>
</tr>
</tbody>
</table>

$$\alpha_1 = 1.25(1.8 - (2.254 - \frac{7.947f_c}{f'_c} - 2k_f \frac{f_c}{f'_c}))$$

$$\alpha_2 = \left[ 1.4 \frac{f_c}{f_{c,\mu}} - 0.6(\frac{f_c}{f_{c,\mu}})^{0.8} \sqrt{\frac{f_c}{f_{c,\mu}}} + 1 \right] \left( \frac{f_c}{f_{c,1.25D}} \right) + k_f$$
2. Experimental Database

In this article, the experimental dataset for using in the ANN model was obtained from reliable technical literatures [11, 35, 37, 39-47]. The statistics of these variables that applied for the development of the ANN structure, are shown in Table 2. Moreover, to illustrate the distribution of these parameters, the frequency histograms are shown in Fig. 1. Fig. 1 presents the suitable used database that can be employed trustworthy.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Std. Error of Mean</th>
<th>Median</th>
<th>Std. Deviation</th>
<th>Variance</th>
<th>Skewness</th>
<th>Std. Error of Skewness</th>
<th>Kurtosis</th>
<th>Std. Error of Kurtosis</th>
<th>Range</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>b(mm)</td>
<td>158.7</td>
<td>3.12</td>
<td>150</td>
<td>43.01</td>
<td>1849</td>
<td>1.72</td>
<td>0.18</td>
<td>4.17</td>
<td>0.35</td>
<td>226</td>
<td>79</td>
<td>305</td>
</tr>
<tr>
<td>h(mm)</td>
<td>183</td>
<td>3.90</td>
<td>150</td>
<td>53.80</td>
<td>2894</td>
<td>1.11</td>
<td>0.17</td>
<td>0.24</td>
<td>0.35</td>
<td>205</td>
<td>100</td>
<td>305</td>
</tr>
<tr>
<td>r(mm)</td>
<td>28.9</td>
<td>0.93</td>
<td>30</td>
<td>12.80</td>
<td>163</td>
<td>0.67</td>
<td>0.18</td>
<td>0.39</td>
<td>0.35</td>
<td>55</td>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>$f_{r0}$</td>
<td>33.46</td>
<td>0.74</td>
<td>33</td>
<td>10.23</td>
<td>104</td>
<td>0.53</td>
<td>0.18</td>
<td>-0.42</td>
<td>0.35</td>
<td>36.90</td>
<td>18.30</td>
<td>55.2</td>
</tr>
<tr>
<td>$f_{c0}$</td>
<td>3.53</td>
<td>0.05</td>
<td>3.34</td>
<td>0.75</td>
<td>0.57</td>
<td>3.92</td>
<td>0.18</td>
<td>17.5</td>
<td>0.35</td>
<td>4.92</td>
<td>0.12</td>
<td>5.04</td>
</tr>
<tr>
<td>$f_{s0}$</td>
<td>3486.5</td>
<td>87.56</td>
<td>3788</td>
<td>1206.93</td>
<td>1456677</td>
<td>-1.624</td>
<td>0.18</td>
<td>1.42</td>
<td>0.35</td>
<td>4289</td>
<td>230</td>
<td>4519</td>
</tr>
<tr>
<td>$E_{frp}$</td>
<td>201</td>
<td>4.87</td>
<td>229</td>
<td>67.16</td>
<td>4510</td>
<td>-1.864</td>
<td>0.18</td>
<td>1.84</td>
<td>0.35</td>
<td>243</td>
<td>14</td>
<td>38207</td>
</tr>
<tr>
<td>$E_{fr0}$</td>
<td>47.5</td>
<td>1.07</td>
<td>45.6</td>
<td>14.80</td>
<td>219</td>
<td>0.58</td>
<td>0.18</td>
<td>0.38</td>
<td>0.35</td>
<td>73.7</td>
<td>17.2</td>
<td>90.9</td>
</tr>
</tbody>
</table>

Table 2. The variables descriptive statistics.
3. Artificial Neural Networks

In the present research ANN is used to investigate the practical formula for measuring the capacity of the FRP-retrofitted RC columns. The relation of the output (compressive strength of FRP-confined RC column) and inputs variables was generated by ANN procedure. The most reputable method of the ANN was introduced as multilayer perceptron (MLP).
The MLP applies feed-forward procedure for generation. The MLP has been introduced as one of the powerful network for examination each continuous function in each desired accuracy \[48, 49\]. The feed-forward process examines one or several variable/s as output/s using foreteller inputs variables (see Fig. 2). The network process of feed-forward is based on the layers formation and connected synapses. A value that named weight is labeled to each synapse, demonstrates the effect of its neuron and consequently input. Besides, a supervised algorithm that is used for multilayers networks was employed. Back-propagation (BP) process compares the obtained output from algorithm to real value and adapt the results until the specified error obtained. The ANN generates using a trial and error process that each input variables specified by a weight. Then the input nodes is introduced as:

\[
net_j = \sum_{i=1}^{n} w_{ij} x_i + b_j
\]  

(2)

where net\( _j \) identified as the set data for neuron; and \( x_i \), \( b_j \) and \( w_{ij} \) are the input, the bias and the weight for each parameters, respectively. Finally, the output was evaluated using the function that generated for present ANN. There are several transfer functions including Sigmoid, Hyperbolic tangent and Gaussian functions. The outputs function was identified as below:

\[
out_j = f\left(net_j\right)
\]

(3)

where \( out_j \) and \( f \) identify as the output and transfer function, respectively.

In the current study the Levenberg-Marquardt (LM) algorithm is performed. The LM is an optimized algorithm that in the least apaches than other algorithms convergence \[50\].

3.1. Performance Measures

Four criteria were employed to investigate the efficiency of the established ANN based model. The following parameters introduced as absolute percentage error (Err), mean absolute error (MAE), mean squared error (MSE) and correlation coefficient (R) as below:

\[
Err_i = \left(\frac{|y_i - t_i|}{t_i}\right) \times 100
\]

(4)

\[
MAE = \frac{1}{N} \sum_{i=1}^{N} |y_i - t_i|
\]

(5)

\[
MSE = \frac{1}{N} \sum_{i=1}^{N} (t_i - y_i)^2
\]

(6)

\[
R = \frac{\sum_{i=1}^{N} (y_i - \overline{y})(t_i - \overline{t})}{\sqrt{\sum_{i=1}^{N} (y_i - \overline{y})^2 \sum_{i=1}^{N} (t_i - \overline{t})^2}}
\]

(7)

where \( N \) show the number of samples, \( t_i \) and \( \overline{t} \) show the exact and average of the exact outputs, respectively, and \( y_i \) and \( \overline{y} \) show the examined and average of the examined outputs, respectively (for the \( i^{th} \) output).
3.2. ANN Structure for Predicting $f_{cc}$

The experimental dataset was applied for developing a new formula to measure the compressive strength of the FRP-confined RC columns. The recognition of the variables influence the $f_{cc}$ of FRP-confined RC columns is difficult and moreover the effective parameters are not independent from each other and some of them may be strongly related together. After an initial survey the input and output parameters for model generating consist seven and one variables:

$$input = \{b, h, r, f_{co}, nt, F_{FRP}, E_{FRP}\}$$

$$output = \{f_{cc}\}$$

where $f_{co}$: The compressive capacity of FRP-confined RC columns. $b$, $h$, $r$, $f_{co}$, $nt$, $F_{FRP}$, and $E_{FRP}$ were identified that affect the compressive strength. As noted, nodes numbers influence the model network, the ideal hidden nodes number is specified by trial-and-error process. By enhancement of neurons number in the hidden layer, accuracy of network increases. On the other hand, it is clear that each network by more neurons in the hidden layer yields a tedious and lengthy equation. Therefore, by considering the accuracy of the model, in the current paper a model by two hidden layer nodes was generated. The R and MSE of the generated network consisting of the different nodes are tabulated in Figs. 3 and 4.

A serious problem that happens during the training process is over-fitting. The mentioned problem happens as the addition of the new value to the generating network causes the error becomes remarkable. The early-stopping procedure was employed to overbear the over-fitting problem. As mentioned, the database was distributed in three groups concluding, training, validating, and testing. While the error in validating set increases the training state is regenerated and the weights invalidating state were come back. In other words, the validating set is utilized to avoid the over-fitting process, which causes network
estimates suitable estimations of the other samples than those applied in the training set (See Fig. 5).

![Image of Fig. 5](image-url)

**Fig. 5.** The error of validation set increases from the specified point. [5].

Data distributed to three groups entitled training, validation, and testing. The distribution percent values in each set are arranged as 70, 15, and 15 for training, validation and testing sets model. Thus, from 190 data for the estimation of $f_{cc}$, 132 specimens were applied in training set, 29 specimens are applied for validating, and testing of model. It is an important issue to introduce the amount of the used specimens in training set of the proposed model, and it is mainly related to the reliability of the model [52]. Frank and Todeschini [53] investigated that the optimized model achieves as the number of database specimens to used variables exceed from 3. Moreover, they recommend the ratio 5 for a reliable model. Here, the noted ratio adjusted as $190/7 = 27.14$. Standardization of input and output values for model simulation made the developed model more optimized as below:

$$X_{si} = \frac{X_i - Mean}{SD} \quad (7)$$

where $X_i$ is variable values, $Mean$ and $SD$ identified as the mean and standard deviation of variables, respectively. Therefore, the input and output layers comprised seven and one neuron, respectively. Consequently, an ANN model concluding one layers with two neurons in hidden layer modified as LM/BP learning algorithm was established. The Log-Sigmoid transfer function was applied to derive the formulae in explicit form. The predicted results of $f_{cc}$ values by ANN model are illustrate in Fig. 6. It is shown in Fig. 6 that training, validation and testing data sets yield a good correlations between actual and predicted values.

![Image of Fig. 6](image-url)

**Fig. 6.** Results of predicted $f_{cc}$ based on the ANN (a) Training, (b) validation, and (c) testing.
3.3. Formula Assessment

Closed form equation can be established based on the constructed ANN model for predicting the $f_{cc}$ of concrete columns. The output of each network can be stated as follow:

$$ output = f\left(W_2 \times f\left(W_1 \times X + b_1\right) + b_2\right) $$ (8)

where $W_1$ and $W_2$ show the first and second layer weight matrix, respectively, moreover $b_1$ and $b_2$ show the first and second bias of layer. Consequently, The formula based ANN method for assessing the $f_{cc}$ is generated as:

$$ output = f\left(W_2 \times f(W_1 \times X)\right) $$

where

$$ X = [1 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7]^T \]

$$ W_1 = \begin{bmatrix} b_{1,1} & W_{1,1,1} & W_{1,2,1} & W_{1,3,1} & W_{1,4,1} & W_{1,5,1} & W_{1,6,1} & W_{1,7,1} \\ b_{1,2} & W_{1,1,2} & W_{1,2,2} & W_{1,3,2} & W_{1,4,2} & W_{1,5,2} & W_{1,6,2} & W_{1,7,2} \end{bmatrix} $$

$$ W_2 = \begin{bmatrix} b_{out,1} & W_{A,1,1} & W_{A,1,2} \\ b_{out,2} & W_{A,2,1} & W_{A,2,2} \end{bmatrix} $$

$$ W_2 = [3.1816 \ 211.8431 \ -214.9806] $$

Finally, a formula based on the ANN method to assess the compressive strength is shown as follows:

$$ f_{cc} = \left(94.6115 + \frac{3135.8}{1 + e^{-\beta_1}} - \frac{3182.2}{1 + e^{-\beta_2}}\right) $$ (9)

$$ \beta_1 = 0.0002 \times b + 0.0125 \times (h) - 0.0358 \times (r) + 0.0498 \times (f_{cc}) - 0.1363 \times (nr) - 0.0009 \times (E_{frp}) - 0.0035 \times (E_{frp}) + 2.544 $$ (10)

$$ \beta_2 = 0.0003 \times b + 0.0128 \times (h) - 0.0362 \times (r) + 0.0481 \times (f_{cc}) - 0.2495 \times (nr) - 0.0009 \times (E_{frp}) - 0.0037 \times (E_{frp}) + 2.6576 $$ (11)

4. Model Accuracy

The accuracy of the proposed formula based ANN-model is tabulated in Table 2. The parameters of $R$, $MSE$ and $MAE$ are chosen to assess the performance of the presented formulation. For model validity, an accepted phenomena introduced by Gandomi et al. [54]. This criteria expressed as follows:

1- For $|R| > 0.8$, a strong correlation between the obtained and evaluated values happens.
2- For $0.2 < |R| < 0.8$, a correlation between the obtained and evaluated values happens.
3- For $|R| < 0.2$, a weak correlation between the obtained and evaluated values happens.

| Table 3. Accuracy of the existing models for $f_{cc}$ evaluations. |
|-------------|-----------------|-----------------|-----------------|
| Method      | Training $R$    | Validation $R$ | Testing $R$     |
|             | $MSE$           | $MAE$           | $MSE$           | $MAE$           | $MSE$           | $MAE$           |
| Proposed equation | 0.9566 | 35.5571 | 5.0567 | 0.9392 | 51.1020 | 5.9233 | 0.9215 | 43.6775 | 5.5226 |
| Mirmiran et al. | 0.8240 | 85.8963 | 7.1749 | 0.8716 | 66.4226 | 7.0488 | 0.7482 | 93.5278 | 7.7700 |
| Institue(ACI) Commit American Concrete | 0.8645 | 542.4004 | 21.1494 | 0.9150 | 506.4664 | 21.1398 | 0.8437 | 482.8296 | 19.9262 |
| Restrepo and De Vino | 0.8682 | 520.314 | 20.1107 | 0.9137 | 474.4924 | 19.9398 | 0.8367 | 509.6434 | 20.2659 |
| Lam and Teng | 0.8189 | 306.3745 | 14.7545 | 0.8464 | 314.4418 | 15.2415 | 0.9118 | 183.6548 | 12.2308 |
| Al-Salloum | 0.7901 | 357.5065 | 16.2591 | 0.8249 | 340.8241 | 15.9330 | 0.8768 | 224.6613 | 13.3079 |
| Shehata et al. | 0.7452 | 113.5239 | 7.7576 | 0.8000 | 114.5554 | 7.419 | 0.6436 | 126.4965 | 8.1558 |
| Kumutha et al. | 0.7528 | 104.1631 | 7.4875 | 0.8068 | 106.7285 | 7.2831 | 0.6593 | 115.0554 | 7.7131 |
Moreover, it is clear that for a strong model, the error values have to be at minimum. The evaluation measures presented in Table 3 confirm that the proposed formulation (Eq. 9) are capable for prediction of $f_{cc}$ in the concrete columns as confined using FRP. Furthermore, for more comparison the obtained results of proposed formula and existing formulations have been plotted as shown in Figs. 7-13. In this study the least dimension of column sections ($b$), long side of column sections ($h$), corner radius of a section ($r$), unconfined concrete strength ($f_{co}$), total thickness of FRP ($nt$), tensile strength of FRP ($F_{FRP}$) and elastic modulus of FRP ($E_{FRP}$) are presented as important effective variables for assessing the $f_{cc}$.

Accuracy of the ANN formula, and those evaluated from Mirmiran et al., Institute (ACI) Committee American Concrete, Lam and Teng, Al-Salloum, Restrepo and De Vino, Shehata et al. and Kumutha et al. is examined and given in Table 3.

Fig. 7. Mirmiran et al. and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 8. The ACI and proposed formula comparison: (a) training, (b) validation data, (c) testing data.
Fig. 9. The Lam & Teng, and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 10. The Al-Salloum and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 11. The Restrepo & De Vino, and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 12. The Shehata et al. and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 13. The Kumutha et al and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.

5. Sensitivity Analysis

Sensitivity analysis was assessed for determining of the strengthen of each
variable. Garson’s algorithm [56] was performed to compute the significance of the input variables. Fig. 14 illustrates the procedure of this algorithm. An example to show the Garson’s algorithm is clarified as below:

1- The contribution of inputs through input-hidden-output linkage evaluates (e.g., $C_{Aa}=W_{Aa} \times W_{OA}$).

2- The inputs relative contributions evaluates (e.g., $r_{Aa}=|C_{Aa}|/|C_{Aa}+C_{Ab}+C_{Ac}|$).

3- The input relative contributions are summed (e.g., $S_a=r_{Aa}+r_{Ba}$).

4- The relative importance of each input ($r_I$) evaluates (e.g., $r_{Ia}=S_a/(S_a+S_b+S_c)$).

Fig. 14. Garson’s algorithm used in ANN.

Fig. 15 shows the significance of each parameters. It can be found that the $(F_{FRP})$ and $(h)$ shown the most influence on the compressive capacity of FRP-confined RC columns.

5. Conclusions

In the current research, the ANN was applied to assess the compressive capacity of the FRP-confined RC columns. The experimental dataset including 190 specimens was obtained using 12 reliable technical literatures for ANN model development. Consequently, the ANN model containing two hidden layer neurons is constructed and seven input parameters were considered including: the least dimension of column sections $(b)$, long side of column sections $(h)$, corner radius of a section $(r)$, unconfined concrete strength $(f_{cc})$, FRP thickness $(nt)$, FRP strength $(F_{FRP})$ and elastic modulus of FRP $(E_{FRP})$.

At the next, the new formula based on ANN was presented and performance analysis was undertaken for confirmation of this formulæ. It can be concluded that the ANN formulation based model give more exactness than other existing formulæ. At the end, the importance of the used parameters was determined using Garson’s algorithm and it was found that the $(F_{FRP})$ and $(h)$ exert dominant influences on the compressive capacity of FRP-confined RC columns, respectively. As a final point, the essential aim of the present research was developing precise equation to evaluate the compressive capacity of FRP-confined RC columns. The new proposed formulæ can be used by practical engineering applications.

REFERENCES


