Fiber Reinforced Polymer (FRP) was extensively employed as external confinement to strengthen the RC structures. Substantial studies were carried out in order to assess a more exact formula for measuring the strength enhancement of such strengthened concrete columns. A database from several experimental tests was gathered. A comparison between the experimental values and existing formulae called for an urgent need for a more exact formula. Therefore, the aim of this paper is to develop an exact formula based on artificial neural networks (ANNs), and to present the strength enhancement. The ANN-based method was simulated in consonance with the collected database and an exact formula generated. The proposed formula was compared to current formulae employing the gathered database. The results revealed that the new formula based ANN gives the best accuracy than others. A sensitivity analysis based on Garson’s algorithm was generated for indicating the value of each applied variable.

1. Introduction

Retrofitting of the RC columns applying FRP as confined reinforcement, exhibited the enhancement of the capacity and performance of structures columns. A series of different strength models to demonstrate the performance of RC columns retrofitted using FRP were conducted recently [1-9]. The most majority of the proposed formulae that are pursuant to the Richart et al. [10] model, apply for circular concrete sections that estimate the enhancement in terms of capacity, the thickness and number of used FRP, and the section diameter. To our knowledge, there were several attempts inquiring the FRP application for rectangular sections than the circular ones [11-13]. Since the pressure in reason of FRP confinement of a rectangular column around its perimeter is...
not uniform, it is challenging to formulate the distribution of stress exactly. The proposed formulae were the same for circular and rectangular sections, previously. Currently, in order to modify the non-uniform stress distribution a ratio as shape factor has been introduced. Therefore, it would be of special interest to present an exact formula in order to assess the performance of a rectangular. Equation 1 present the enhanced compressive strength of FRP-confined RC columns.

\[ f'_{cc} = f'_{cc} \left(1 + a \frac{f_l}{f'_{cc}} \right) \]  

(1)

\( f'_{cc} \): Strength enhancement of FRP-confined RC column; \( f'_{cc} \): the compressive capacity of the RC column; \( f_l \): The confinement lateral strength, \( f_l = 2t_{FRP}/d \); \( a \): the factor for confinement effectiveness; \( f_{FRP} \): The FRP strength; \( t \): The FRP thickness; \( d \): The column diameter.

Nowadays, ANN has been applied in the simulating of several civil engineering problems by researchers. The application of conventional methods to derive a formula to measure the capacity of FRP-confined RC columns, in reason of unknown multivariable and noisy database is unlikely suitable. In this study, the ANN is applied to anticipate the capacity of the FRP confined concrete columns. ANN was employed for input-output models. In this model, the experimental dataset is used for training the system. If applying the dataset collects appropriate information on the problem, consequently the trained ANN model will contain sufficient information on the output and the model can be present as a reliable model. In several published currently papers, ANN has been used for different applications in engineering [14-32]. As mentioned above, collecting an exact experimental database is an essential activity in first. Therefore, a large numbers specimens on the FRP confined concrete columns were congregated. Besides, all of the important existing formulae for strength enhancement of FRP-confined RC columns were collected. The existing formulae to measure the strength of FRP-confined RC columns are tabulated in Table 1. A new formula based ANN approach was conducted here.

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Equations for ( \frac{f'<em>{cc}}{f'</em>{co}} )</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirmiran et al.</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = 1 + 6 \left( \frac{2r}{D} \right) \left( \frac{f_{o.7}}{f'_{cc}} \right) )</td>
<td>[33]</td>
</tr>
<tr>
<td>ACI</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = -1.254 + 2.254 \sqrt{1 + \frac{7.94k}{f_{o}\cdot f'<em>{cc}} - 2 \cdot k \cdot f_l / f'</em>{cc}} )</td>
<td>[34]</td>
</tr>
<tr>
<td>Lam and Teng</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = 1 + 3.3 \left( \frac{A}{A} \right) \left( \frac{f_l}{f'_{cc}} \right) )</td>
<td>[11]</td>
</tr>
<tr>
<td>Al-Salloum</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = 1 + 3.13k \left( \frac{b}{D} \right) \left( \frac{f_l}{f'_{cc}} \right) )</td>
<td>[35]</td>
</tr>
<tr>
<td>Restrepo and De Vino</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = \alpha_1 \alpha_2 ) ( \alpha_1 = 1.25 \left( \frac{f_{o.8}}{f_{o.8}} \right) \left( \frac{1 + 7.94}{f_{o.8}} - 1.6 \cdot f_{o.8} - 1 \right) ) ( \alpha_2 = \left[ 1.4 \cdot \frac{f_{o.8}}{f_{o.8}} - 0.6 \left( \frac{f_{o.8}}{f_{o.8}} \right)^2 - 0.8 \right] \left( \frac{f_{o.8}}{f_{o.8}} \right) + 1 )</td>
<td>[36]</td>
</tr>
</tbody>
</table>
2. Experimental Database

In this article, the experimental dataset for using in the ANN model was obtained from reliable technical literatures [11, 35, 37, 39-47]. The statistics of these variables that applied for the development of the ANN structure, are depicted in Table 2. Moreover, in order to illustrate the distribution of these parameters, the frequency histograms are highlighted in Fig. 1. Fig. 1 presents the suitable used database that can be employed trustworthy.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>b(mm)</th>
<th>h(mm)</th>
<th>r(mm)</th>
<th>f_{te}(Mpa)</th>
<th>ntl(mm)</th>
<th>E_{FRP}(Mpa)</th>
<th>f_{E}(Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>158.7</td>
<td>183</td>
<td>28.9</td>
<td>33.46</td>
<td>0.53</td>
<td>3486.5</td>
<td>201</td>
</tr>
<tr>
<td>Std. Error of Mean</td>
<td>3.12</td>
<td>3.90</td>
<td>0.93</td>
<td>0.74</td>
<td>0.05</td>
<td>87.56</td>
<td>4.87</td>
</tr>
<tr>
<td>Median</td>
<td>150</td>
<td>150</td>
<td>30</td>
<td>33</td>
<td>0.34</td>
<td>3788</td>
<td>229</td>
</tr>
<tr>
<td>Mode</td>
<td>150</td>
<td>150</td>
<td>30</td>
<td>35.3</td>
<td>0.34</td>
<td>3500</td>
<td>230</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>43.01</td>
<td>53.80</td>
<td>12.80</td>
<td>10.23</td>
<td>0.75</td>
<td>1206.93</td>
<td>67.16</td>
</tr>
<tr>
<td>Variance</td>
<td>1849</td>
<td>2894</td>
<td>163</td>
<td>104</td>
<td>0.57</td>
<td>145,677</td>
<td>4510</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.72</td>
<td>1.11</td>
<td>0.67</td>
<td>0.53</td>
<td>3.92</td>
<td>-1.624</td>
<td>-1.864</td>
</tr>
<tr>
<td>Std. Error of Skewness</td>
<td>0.18</td>
<td>0.17</td>
<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.17</td>
<td>0.24</td>
<td>0.39</td>
<td>-0.42</td>
<td>17.5</td>
<td>1.42</td>
<td>1.84</td>
</tr>
<tr>
<td>Std. Error of Kurtosis</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Range</td>
<td>226</td>
<td>205</td>
<td>55</td>
<td>36.90</td>
<td>4.92</td>
<td>4289</td>
<td>243</td>
</tr>
<tr>
<td>Minimum</td>
<td>79</td>
<td>100</td>
<td>5</td>
<td>18.30</td>
<td>0.12</td>
<td>230</td>
<td>14</td>
</tr>
<tr>
<td>Maximum</td>
<td>305</td>
<td>305</td>
<td>60</td>
<td>55.2</td>
<td>5.04</td>
<td>4519</td>
<td>38207</td>
</tr>
</tbody>
</table>
3. Artificial Neural Networks

In the present research ANN is applied to inquire the practical formula for measuring the capacity of the FRP-retrofitted RC columns. The relation of the output (compressive strength of FRP-confined RC column) and inputs variables was generated by ANN procedure. The most reputable method of the ANN was introduced as multilayer perceptron (MLP). The MLP
applies feed-forward procedure for generations. The MLP has been inaugurated as one of the powerful network for examination each continuous function in each desired accuracy \[48, 49\]. The feed-forward process examines one or several variable/s as output/s applying foreteller inputs variables (see Fig. 2). The network process of feed-forward is based on the layer’s formation and connected synapses. A value that named weight is labeled to each synapse, demonstrates the effect of its neuron and consequently input. Besides, a supervised algorithm that is applied for multilayers networks was employed. Back-propagation (BP) process compare the obtained output from algorithm to real value and adapt the results until the specified error obtained. The ANN generates using a trial and error process that each input variables specified by a weight. Subsequently, the input nodes is inaugurated as:

$$net_j = \sum_{i=1}^{n} w_{ij}x_i + b_j$$

where \(net_j\) identified as the set data for neuron; and \(x_i\), \(b_j\) and \(w_{ij}\) are the input, the bias and the weight for each parameters, respectively. Finally, the output was evaluated applying the function that generated for present ANN. There are several transfer functions including Sigmoid, Hyperbolic tangent and Gaussian functions. The outputs function was identified as below:

$$out_j = f(net_j)$$

where \(out_j\) and \(f\) identify as the output and transfer function, respectively.

In this paper, the Levenberg-Marquardt (LM) algorithm is performed. The LM is an optimized algorithm that in the least apaches than other algorithms convergence \[50\].

3.1. Performance Measures

Four criteria were employed to investigate the efficiency of the established ANN based model. The following parameters introduced as absolute percentage error \(Err\), mean absolute error \(MAE\), mean squared error \(MSE\) and correlation coefficiient \(R\) as below:

$$Err_i = \frac{|y_i - t_i|}{t_i} \times 100$$

$$MAE = \frac{1}{N} \sum_{i=1}^{N} |y_i - t_i|$$

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (t_i - y_i)^2$$

$$R = \frac{\sum_{i=1}^{N} (y_i - \bar{y})(t_i - \bar{t})}{\sqrt{\sum_{i=1}^{N} (y_i - \bar{y})^2 \sum_{i=1}^{N} (t_i - \bar{t})^2}}$$

where \(N\) show the number of samples, \(t_i\) and \(\bar{t}\) show the exact and average of the exact outputs, respectively, and \(y_i\) and \(\bar{y}\) show the examined and average of the examined outputs, respectively (for the \(i^{th}\) output).
3.2. ANN Structure for Predicting $f_{cc}$

The experimental dataset was applied for developing a new formula to measure the compressive strength of the FRP-confined RC columns. The recognition of the variables influences the $f_{cc}$ of FRP-confined RC columns is difficult and moreover, the effective parameters are not independent from each other and some of them may be strongly related together. After an initial survey the input and output parameters for model generating consist seven and one variables:

$$\text{input} = \{b, h, r, f_{co}, nt, F_{FRP}, E_{FRP}\}$$

$$\text{output} = \{f_{cc}\}$$

where $f_{co}$: The compressive capacity of FRP-confined RC columns. $b$, $h$, $r$, $f_{co}$, $nt$, $F_{FRP}$, and $E_{FRP}$ were identified that affect the compressive strength. As noted, nodes numbers influence the model network, the ideal hidden nodes number is specified by trial-and-error process. Through enhancement of neurons number in the hidden layer, accuracy of network increases. On the other hand, it is clear that each network by more neurons in the hidden layer yields a tedious and lengthy equation. Therefore, by considering the accuracy of the model, in the current paper a model by two hidden layer nodes was generated. The R and MSE of the generated network consisting of the different nodes are protrayed in Figs. 3 and 4.

An ill-posed problem that happens during the training process is over-fitting. This happens as the addition of the new value to the generating network causes the error becomes remarkable. The early-stopping procedure was employed to rectify the over-fitting problem. As mentioned above, the database was distributed in three groups concluding, training, validating, and testing. While the error in validating set increases the training, state is regenerated and the weights invalidating state were come back. In other words, the validating set is applied as an alternative approach, in order to avoid the over-fitting process, which causes
network estimates suitable estimations of the other samples than those applied in the training set (See Fig. 5).

Fig. 5. The error of validation set increases from the specified point. [5].

Data distributed to three groups entitled training, validation, and testing. The distribution percent values in each set are arranged as 70, 15, and 15 for training, validation and testing sets model. Thus, from 190 data for the estimation of $f_{cc}$, 132 specimens were applied in training set, 29 specimens are applied for validating, and testing of model. It is a straightforward issue to introduce the amount of the used specimens in training set of the proposed model, and it is mainly related to the reliability of the model [52]. Frank and Todeschini [53] investigated that the optimized model achieves as the number of database specimens to applied variables exceed from 3. Moreover, they recommend the ratio 5 for a reliable model. Here, the noted ratio adjusted as $190/7 = 27.14$. Standardization of input and output values for model simulation made the developed model more optimized as below:

$$X_{i} = \frac{X_{i} - Mean}{SD}$$  \hspace{1cm} (7)

where $X_{i}$ is variable values, $Mean$ and $SD$ identified as the mean and standard deviation of variables, respectively. Therefore, the input and output layers comprised seven and one neuron, respectively. Consequently, an ANN model concluding one layers with two neurons in hidden layer modified as LM/BP learning algorithm was established. The Log-Sigmoid transfer function was applied to derive the formulae in explicit form. The predicted results of $f_{cc}$ values by ANN model are illustrated in Fig. 6. As it is exhibited in Fig. 6, training, validation and testing data sets yield a good correlations between actual and predicted values.

Fig. 6. Results of predicted $f_{cc}$ based on the ANN (a) Training, (b) validation, and (c) testing.
3.3. Formula Assessment

Closed form equation can be established based on the constructed ANN model for predicting the $f_{cc}$ of concrete columns. The output of each network can be stated as follow:

$$output = f(W_2 \times f(W_1 \times X + b_1)) + b_2$$

(8)

where $W_1$ and $W_2$ show the first- and second-layer weight matrix, respectively, moreover $b_1$ and $b_2$ display the first and second bias of layer. Consequently, the formula-based ANN method for assessing the $f_{cc}$ is generated as:

$$output = f(W_2 \times \left[ \begin{array}{c} 1 \\ f(W_1 \times X) \end{array} \right])$$

where

$$X = \left[ \begin{array}{cccccc} 1 & x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \right]^T$$

$$W_1 = \left[ \begin{array}{cccccc} b_{a1} & W_{1a1} & W_{1a2} & W_{1a3} & W_{1a4} & W_{1a5} \\ b_{a2} & W_{2a1} & W_{2a2} & W_{2a3} & W_{2a4} & W_{2a5} \end{array} \right]$$

$$W_2 = \left[ \begin{array}{ccc} b_{out} & W_{A1, out} & W_{A2, out} \end{array} \right]$$

Finally, a formula based on the ANN method to assess the compressive strength is presented as follows:

$$f_{cc} = \left( \frac{94.6115 + 3135.8 - 3182.2}{1 + e^{-\beta_1}} \right)$$

(9)

$$\beta_1 = 0.0002 \times (b) + 0.0125 \times (h) - 0.058 \times (r) + 0.0498 \times (f_{nt}) - 0.1363 \times (mt) - 0.0009 \times (E_{frp}) - 0.0035 \times (E_{frp}) + 2.544$$

(10)

$$\beta_1 = 0.0003 \times (b) + 0.0128 \times (h) - 0.0362 \times (r) + 0.0481 \times (f_{nt}) - 0.2495 \times (mt) - 0.0009 \times (E_{frp}) - 0.0037 \times (E_{frp}) + 2.6576$$

(11)

4. Model Accuracy

The accuracy of the proposed formula-based ANN-model is depicted in Table 2. The parameters of $R$, $MSE$ and $MAE$ are selected to assess the performance of the presented formulation. For model validity, an accepted phenomenon introduced by Gandomi et al. [54]. This criteria expressed as follows:
1- For $|R| > 0.8$, a strong correlation between the obtained and evaluated values happens.
2- For $0.2 < |R| < 0.8$, a correlation between the obtained and evaluated values happens.
3- For $|R| < 0.2$, a week correlation between the obtained and evaluated values happens.

<table>
<thead>
<tr>
<th>Method</th>
<th>Training</th>
<th>Validation</th>
<th>Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>$MSE$</td>
<td>$MAE$</td>
</tr>
<tr>
<td>Proposed equation</td>
<td>0.9566</td>
<td>35.5571</td>
<td>5.0567</td>
</tr>
<tr>
<td>Mirmiran et al.</td>
<td>0.8240</td>
<td>85.8963</td>
<td>7.1749</td>
</tr>
<tr>
<td>Institute(ACI) Committe</td>
<td>0.8645</td>
<td>542.4004</td>
<td>21.1494</td>
</tr>
<tr>
<td>American Concrete</td>
<td>0.8189</td>
<td>306.3745</td>
<td>14.7545</td>
</tr>
<tr>
<td>Restreto and De Vino</td>
<td>0.8682</td>
<td>520.314</td>
<td>20.1107</td>
</tr>
<tr>
<td>Lam and Teng</td>
<td>0.8189</td>
<td>306.3745</td>
<td>14.7545</td>
</tr>
<tr>
<td>Al-Salloum</td>
<td>0.7901</td>
<td>357.5065</td>
<td>16.2591</td>
</tr>
<tr>
<td>Shehata et al.</td>
<td>0.7452</td>
<td>113.5239</td>
<td>7.7576</td>
</tr>
<tr>
<td>Kumutha et al.</td>
<td>0.7528</td>
<td>104.1631</td>
<td>7.4875</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$R$</th>
<th>$MSE$</th>
<th>$MAE$</th>
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<tr>
<td></td>
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<td>Training</td>
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<tr>
<td></td>
<td>$R$</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>Testing</td>
<td></td>
</tr>
</tbody>
</table>
Moreover, it is clear that for a strong model, the error values have to be at minimum. The evaluation measures presented in Table 3 confirm that the proposed formulation (Eq. 9) are capable for prediction of $f_{ec}$ in the concretes columns as confined applying FRP. Furthermore, for more comparison the obtained results of proposed formula and existing formulations have been plotted as presented in Figs. 7-13. In this study, the least dimension of column sections ($b$), long side of column sections ($h$), corner radius of a section ($r$), unconfined concrete strength ($f_{co}$), total thickness of FRP ($nt$), tensile strength of FRP ($F_{FRP}$) and elastic modulus of FRP ($E_{FRP}$) are presented as major effective variables for assessing the $f_{ce}$.

Accuracy of the ANN formula, and those evaluated from Mirmiran et al., Institute (ACI) Committee American Concrete, Lam and Teng, Al-Salloum, Restrepo and De Vino, Shehata et al. and Kumutha et al. is examined and portrayed in Table 3.

Fig. 7. Mirmiran et al. and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 8. The ACI and proposed formula comparison: (a) training, (b) validation data, (c) testing data.
Fig. 9. The Lam & Teng, and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 10. The Al-Salloum and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 11. The Restrepo & De Vino, and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 12. The Shehata et al., and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.
Fig. 13. The Kumutha et al and proposed formula comparison: (a) training data, (b) validation data, (c) testing data.

5. Sensitivity Analysis

Sensitivity analysis was employed for determining the strengthen of each
variable. Garson’s algorithm [56] was performed to compute the significance of the input variables. Fig. 14 illustrates the procedure of this algorithm. An example to demonstrate the Garson’s algorithm is clarified as below:

1- The contribution of inputs through input-hidden-output linkage evaluates (e.g., \( C_{Aa} = W_{Aa} \times W_{0A} \)).

2- The inputs relative contributions evaluate (e.g., \( r_{Aa} = |C_{Aa}|/(C_{Aa} + C_{Ab} + C_{Ac}) \)).

3- The input relative contributions are summed (e.g., \( S_a = r_{Aa} + r_{Ba} \)).

4- The relative importance of each input (\( r_I \)) evaluates (e.g., \( r_{Ia} = S_a/(S_a + S_b + S_c) \)).

Fig. 14. Garson’s algorithm used in ANN.

Fig. 15 reveals the significance of each parameters. It can be found that, the \( (F_{FRP}) \) and \( (h) \) demonstrated the most influence on the compressive capacity of FRP-confined RC columns.

5. Conclusions

In the current research, the ANN was applied to assess the compressive capacity of the FRP-confined RC columns. The experimental dataset including 190 specimens was obtained using 12 reliable technical literatures for ANN model development. Consequently, the ANN model containing two hidden layer neurons is constructed and seven input parameters were deliberated, including: the least dimension of column sections \( (b) \), long side of column sections \( (h) \), corner radius of a section \( (r) \), unconfined concrete strength \( (f_{cc}) \), FRP thickness \( (nt) \), FRP strength \( (F_{FRP}) \) and elastic modulus of FRP \( (E_{FRP}) \). Next, the new formula based on ANN was presented and performance analysis was undertaken for confirmation of this formulae. It can be concluded that, the ANN formulation-based model give more exactness than other existing formulae. At the end, the importance of the applied parameters was determined using Garson’s algorithm and it was found that the \( (F_{FRP}) \) and \( (h) \) exert dominant influences on the compressive capacity of FRP-confined RC columns, respectively. As a final point, the ultimate aim of the present research was developing precise equation to evaluate the compressive capacity of FRP-confined RC columns. The new proposed formulae can be applied by practical engineering applications.
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