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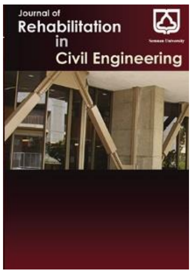

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Dynamic Analysis of Tapered Tall Buildings with Different Tube Structural Systems

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ABSTRACT

The purpose of this study is to obtain one of the important dynamical properties, namely natural frequency (ω) for a number of tall buildings with tube and tapered tube systems. Furthermore, it presents an approximate method to analyze the free vibration of tall buildings by tube, tube-in-tube, bundled tube, and tapered tube structures. The method we have proposed would enable us to compute the natural frequency of tubular vertical and tapered tall buildings by the help of computer programming. The models were analyzed by finite element and analytical methods. The results indicate that the investigated analytical method correctly calculates the natural frequency and is in decent accord with the finite element results and has better compatibility with tapered structures without angle with higher altitude. The resulting computational error is very low. Also, this analytical method has the least error for tube systems and the highest error is for tube-in-tube systems.

1. Introduction

To evaluate a tall bulding's response to wind or earthquake, it is necessary to estimate the natural frequency of such buildings. As a consequence, it seems that developing new and simple methods for free vibration analysis and specifying the natural frequency and mode shape functions are essential. The mass and stiffness of the structure change along the height in the current tall structures. In recent years, tubular building has been acknowledged as an economic and advanced structural system. In this paper, a new and simple analytical approach is suggested for approximate analysis of tall structures with a combined system of framed tube, tube-in-tube structures, bundled tube, and tapered tube systems. Therefore, a tall structure is modeled by a cantilevered beam with variable stiffness and mass along the height, hence, in order to calculate the natural frequencies, the governing partial

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differential equation is solved along with its variable coefficients. The tall structure behavior is equivalent to a cantilevered beam with variable hollow box cross-sections (Fig. 1 and Fig. 2).

Equation (1): For tapered tube system

$$\begin{aligned} K_s(x) &= [K_s(x)]_I \\ K_B(x) &= [K_B(x)]_I \\ m(x) &= [m(x)]_I \\ N(x) &= \int_x^H g\, m(x)dx \end{aligned} \tag{1}$$

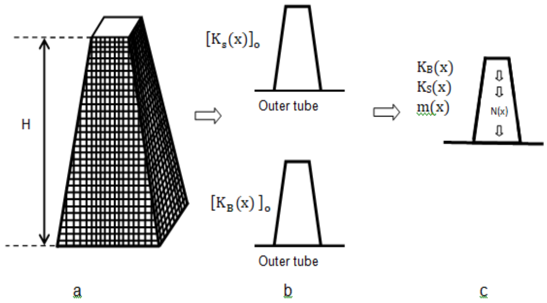

Fig. 1. Modeling of tall structure: (a) tapered tube system, (b) equivalent bending and shear beam, (c) equivalent beam of whole structure with varying shear and bending stiffness and variable mass in height due to variable axial force.

Equation (2): For tube-in-tube system and bundled tapered tubes:

$$\begin{aligned} K_s(x) &= [K_s(x)]_I + [K_s(x)]_O \\ K_B(x) &= [K_B(x)]_I + [K_B(x)]_O \\ m(x) &= [m(x)]_I + [m(x)]_O \\ N(x) &= \int_x^H g\, m(x)dx \end{aligned} \tag{2}$$

where in Equations (1) and (2), Figs (1) and (2), H is the height of the structure, subscript F is the tube system, subscript W is the shear wall, subscript I is the inner tube, and subscript O is the outer tube. The bending stiffness of $K_B(x)$, shear stiffness $K_s(x)$, the mass of the beam is equal to $m(x)$, g is the gravity acceleration, and the axial force of the beam is equal to $N(x)$.

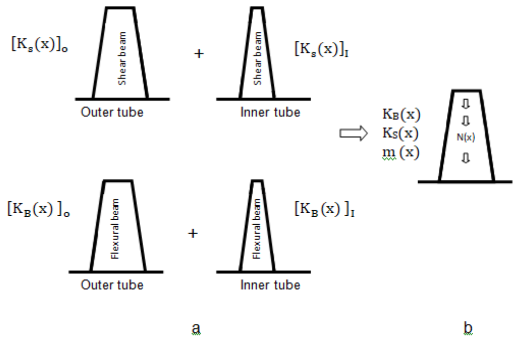

Fig. 2. Modeling of a tall structure: (a) tube-in-tube system and bundled tapered tube with equivalent bending and shear beams, (b) the equivalent beam of the whole structure with shear and flexural stiffness and variable mass in height due to variable axial force.

One of the important parameters in tapered slimming structures is the calculation of the vibrational natural frequency of the structure (ω, radians per second). A new method is presented for this calculation by dynamical relations of structures. Compared to software computations, it was concluded that due to their structural nature, tall buildings are usually flexible and prone to dynamic loads. Basically, analyses including more than one dimension such as finite element analysis (FEM) is employed in order to obtain the precise dynamic behavior of structures. The tubular systems are also one of the most suitable structural systems for high-rise towers and buildings. The tube system can be considered as a system in which a hollow canister column is console from the ground, so that the external side of the building represents tubular behavior against lateral forces of the tube. In other words, it can be considered as a three-dimensional rigid frame around the exterior part of the building which is designed for resistance against all lateral and vertical forces. The column dimensions and distances and bending strength of the perimeter beams directly affect the tubular frame system performance. Tapered structure system gets narrower with increasing height and decreases with lowered windshield plan level on upper floors of the building. As a result, the intensity of the wind and subsequent excess wind pressure will also decrease. The gradient-form buildings and slimming tapered ones towards up has an architectural and symbolic form of the integration of the architecture and structure. Due to inward gradient of the frontage, the narrowing of the form reduces the impact level of the wind on upper floors of the building and also reduces the intensity of the wind and the surplus of wind loads that can even be more critical than earthquake forces [1].

The narrowing of the building at higher altitudes can take the following forms: 1) gradual, conical, and tapered reduction, and 2) breakdowns that are effective ways to reduce perpendicular reactions against winds in building design [2,3]. A dynamic stiffness method has been developed and employed to analyze and investigate the free vibration properties of tapered beam [4].

1.2. Kinds of tubular systems:

(a) Framed tube structure

A tubular system is a system where structural elements are designed in such a way that enables the system to be resistant against the loads on the structure including the general as well as the lateral loads. The most salient feature of this system is the use of peripheral columns in close distances and these members are connected to each other by deep beams. The distance between outer columns is about two or three meters. So, the whole building is considered as a huge vertical cantilever that resists against the overturning moment [5,6].

(b) Tube-in-tube structure

Another type of framed tube including an internal and an external tube is tube-in-tube structure. Together, these internal and external tubes are working in a way to withstand lateral and gravity loads on the buildings. External tube with inner shear cores is combined in tube-in-tube form which increases stiffness and makes it easier to create taller buildings. It can be used in structures having more than 100 floors [5,6].

(c) Bundled tube system

Having two or more tubes connected to each other makes bundled tudbe system capable of being used for creating a multi-cell tube in which frames can withstand shear along the lateral loads. Whereas the wing frames tolerate turning moments, this system allows heights up to 110 floors and larger areas. In this system, the internal networks reduce the shear lag on the wings of the beam [5,6].

2. Literature review

For preliminary design stage, approximate methods presented for free vibration analysis of the structures in tubular buildings are suitable solutions. A lot of research has investigated free vibration of tall structures using various methods. Using finite element analysis (FEA), an orthotropic membrane analogy has also been developed for simplified analysis of framework panels [6]. In order to calculate the natural frequency of tube-in-tube structures in tall buildings a formula has been proposed working directly from the fourth-order Sturm-Liouville differential equations [7]. A study conducted using free vibration analysis in tubular frame structures for different frequencies and the effect of mass and lateral stiffness of structures on natural frequencies and its displacement under free vibration has been studied and discussed [8]. Translational mode is taken as the main mode of diagrid tube-in-tube structure. The first and second modes have a significant influence on structure, while the vibration mode has little influence on the structure [9]. To analyze the free vibration of tube-in-tube tall buildings, Lee formulated an approximate solution [10-12]. Approximate formulae are proposed in order to calculate the fundamental frequency of structures. As a prismatic cantilever flexural-shear beam, tubular structures are idealized in a way assuming the lower most elevation to be fully fixed. Consequently, natural frequency expressions are derived through an energy method [13]. The tapered beam considered in [14] is restrictive in its choice of beam cross-sections in that the area variation is assumed linear whereas the second moment of area variation is considered cubic in terms of the beam length parameter [14]. An analytical model for the dynamical analysis of tall buildings [15]. Having calculated the natural frequencies of flexural, axial and torsion vibration of the beams, he has converted the governing differential equations into weak form integral equations. Tall structure modeling by a cantilevered beam with variable stiffness and mass under effects of variable axial force caused by the structure weight may provide realistic conditions for an accurate structural analysis [16]. The first natural frequency of tall buildings with a combined system of framed tube has been calculated [17]. An analytical approach based on energy principles has been developed for computing the natural frequencies of buildings constructed by framed tube systems [18,19]. The fundamental frequency of tall buildings has been determined framed tube systems vary in size along the height of the structure [20]. The effective impact of the core to improve the behavior of tubular structures was investigated and the such on the shear delay of the tubular system was estimated approximately [21]. The free vibration analysis has been solved using the DQM method governing differential equation for free vibration of coupled shear walls [22,23]. The weak-form integral equations have been developed for free vibration analysis and calculated the natural frequencies of non-prismatic beams [24,25]. Natural frequencies of tapered Timoshenko beam are presented for different combinations [26]. They converted the equations to its weak-form integral equations. Here, we present a new and simple solution to calculate the natural frequency of framed tube, tube-in-tube structures, bundled tube, and tube tapered systems. Bending moment function approximation is proposed instead of mode shape function approximation. The proposed method is much simpler in mathematical computation steps. Shear stiffness, bending stiffness, and mass vary with height throughout the structural unit. The effect of structural weight on its differing amounts of natural frequency is examined by using variable axial force proposed by Mohammadnejad and Haji-Kazemi (2018) [27]. Analysis and design of tall structures, explanation of analysis methods of different types of structures of tall buildings have been introduced using precise computer and approximate methods [28]. Some articles on the subject of tall buildings with reference to shear wall structures, basic design criteria, and a variety of structural forms for initial and final analysis have been published [29]. The framed tubular structural system is formed by creating rigid connections between several columns in close proximity to each other and high deep beams in the perimeter of the structure [30]. The tubular structures are formed using a continuous method in which the two beams are modeled separately by a tubular beam. These analyses are performed using the principle of minimum potential energy [31]. A simple analytical approach is provided by Takabatake et al. (1993). This simplified procedure is performed by replacing the tube with an equivalent beam [32]. Based on the continuous method, an analytical method is presented to analyze the frequency of buildings prepared by shear walls and narrow structures. According to this method, a general solution is obtained to determine the natural frequencies of the structures [33]. Based on the generalized continuous approach, the main hypothesis of tubular structure analysis is that in two different structural systems, deformation in shear and bending modes are confined to each other [34]. The study of the effects of reducing unequal lateral stiffness in seismic response and distribution of normal seismic responses has been predicted with appropriate curves [35].

3. Formulation and solution

3.1. Methodology: Weak form of differential equations

The differential equation for free vibration of beam with variable stiffness and mass is a partial differential equation with variable coefficients. Many mathematical methods may be used for numerical or approximate analysis of this equation. The proposed approach to convert the governing partial differential equation to a solvable equation is to transform the equation into its weak form. The weak form of the differential equation has many uses in place of the original equation [27,36-38].

3.2. Equivalent properties of the framed tube

For the analysis of framed tube structures, Kwan has proposed a helpful model. Using equivalent orthotropic plates, he has made a number of assumptions in his model to describe the framed tube system. Using his assumptions, the tall structure can be modeled as a cantilever beam with a variable cross-section in height. It is a common practice to fix the value of thickness of the membrane t such that the area of the membrane (*d.t*) is equal to the sectional area of the column (A_c) [10, 27,36].

$$A_C = dt \tag{3}$$

where d is center-to-center distance of the columns of the outer tube and subject to a lateral force Q. The lateral deflection may be computed as the sum of that due to bending Δ_b and due to shear Δ_s. The bending deflection Δ_b is given by the following formula [10, 27,36]:

$$\frac{\Delta_b}{Q} = \frac{(h - H_b)^3}{12E_m I_c} + \left(\frac{h}{d}\right)^2 \frac{(d - H_c)^2}{12E_m I_b} \tag{4}$$

where h, H_b, E_m, I_C, I_b, H_c are story height, height of beam, elastic modulus of the construction material, moments of inertia of the column, moments of inertia of the beam and height of the column, respectively. A_{sc} is the cross-sectional area of the column; A_{sb} is the cross-sectional area of the beam; Q is the unit subject to a lateral force on the structure. On the other hand, the shear deflection Δ_s is given by [10, 27,36]:

$$\frac{\Delta_s}{Q} = \frac{(h - H_b)}{G_m A_{sc}} + \left(\frac{h}{d}\right)^2 \frac{(d - H_c)}{G_m A_{sb}} \tag{5}$$

where A_{sb} and A_{sc} are effective shear areas of the beam and column, respectively, and G_m is the shear modulus of the material, G shear modulus. Equivalent shear modulus of the membrane is calculated as follows [10, 27,36]:

$$G = \frac{\frac{h}{st}}{\frac{\Delta_b}{Q} + \frac{\Delta_s}{Q}} \tag{6}$$

To solve the differential equation governing the free vibration of a beam with varying stiffness and mass, different mathematical techniques are presented. But here, the technique presented by Mohammadnejad and Haji-Kazemi (2018) [27] is compared with other methods. The differential equation of weak form governing the behavior of equivalent beam with variable stiffness and mass subject to the loading of q and the axial force of N is presented as follows (30-38):

$$\frac{\partial^2}{\partial x^2}\left[k_B(x)\frac{\partial^2}{\partial x^2}W(x,t)\right]-\frac{\partial}{\partial x}\left[k_s(x)\frac{\partial}{\partial x}W(x,t)\right]-\frac{\partial}{\partial x}\left[n(x)\frac{\partial}{\partial x}W(x,t)\right]+m(x)\frac{\partial^2}{\partial t^2}W(x,t)+q(x,t)=0 \quad (7)$$

In this equation, $W(x,t)$ is displacement; $m(x)$ is mass per unit height; $k_B(x)$ refers to the flexural stiffness; $k_s(x)$ is the shear stiffness; $n(x)$ stands for axial force, and $q\ (x,t)$ is lateral distribution of forces. The whole building is taken as a prismatic cantilever beam having shear stiffness *GA(x)*, bending stiffness *EI(x)* which depends on both shear modulus *G* and cross-sectional area *A(x)*. In free vibration mode, $q(x,t)$ is considered equal to zero. Assuming the harmonic vibration of $W(x,t)=W(x)e^{i\Omega t}$, $W(x)$ is considered the function of mod shape and *Ω* is natural frequency of the structure. By putting these values and values of Eq. (1) in Eq. (2) and using Equations 3 to 6, Eq. (7) is obtained as Eq. (8) and Eq. (9) [27-35].

$$\xi=\frac{x}{H}$$
$$k_B(\xi)=EI_0K_B(\xi)\,,\ k_s(\xi)=GA_0K_s(\xi)$$
$$n(\xi)=N_0N(\xi)\ ,m(\xi)=m_0m(\xi) \quad (8)$$
$$\beta^2=\frac{GA_0H^2}{EI_0}$$
$$\alpha^2=\frac{m_0\Omega^2H^4}{EI_0}$$
$$\gamma^2=\frac{N_0H^2}{EI_0}$$

$$\frac{d^2}{d\xi^2}\left[K_B(\xi)\frac{d^2}{dx^2}\omega(\xi)\right]-\frac{\partial}{\partial\xi}\left[\beta^2K_s(\xi)\frac{d}{d\xi}\omega(\xi)\right]-\frac{\partial}{\partial\xi}\left[\gamma^2N(\xi)\frac{d}{dx}\omega(\xi)\right]-\alpha^2\omega(\xi)=0 \qquad 0\le\xi\le1 \quad (9)$$

α^2 are the coefficients without dimension of mass, β^2 are the coefficients without dimension of shear stiffness, and γ^2 are the coefficients without dimension of axial force.

Equation (9) is a free vibration equation of tall structures in terms of a variable without ξ dimension. For finding natural frequency from Eq. (9), it can be integrated from the sides of four-load equation. But according to a method recently proposed by Mohammadnejad and Haji-Kazemi (2018) [27], Eq. (9) can be solved using integrating twice. This method is used in this study, and here is the formulation of this method.

Equation (10) is the result of two times integration of the sides of Eq. (9) obtained by applying the boundary conditions of the integral constants included in it.

$$\int_0^\xi h_1(\xi,s)M(s)ds+\int_0^1 h_2(\xi,s)M(s)ds+K_B(\xi)M(\xi)=0 \quad (10)$$

$$M=\frac{d^2\omega}{d\xi^2}\ ,\omega(\xi)=\int_0^\xi(\xi-s)M(s)ds$$
$$h_1(\xi,s)=\int_0^S[\beta^2K_s(s)+\gamma^2N(s)]ds-\int_0^\xi(\beta^2K_s(s)+\gamma^2N(s))ds-\frac{\alpha^2}{6}(\xi-s)^3$$
$$h_2(\xi,s)=\frac{\alpha^2}{2}(1-s)^2(\xi-1)-h_1(1,s)$$

In order to obtain the answer from Eq. (10) based on the method proposed by Mohammadnejad and Haji-Kazemi (2018) [27], a power series was proposed instead of *M (ξ)* function. This power series is given in Eq. (11).

$$M(\xi)=\sum_{r=0}^{R}C_r\xi^r \quad (11)$$

In Eq. (11), C_r constant is unknown and must be determined, and R is a positive value that determines the accuracy of calculations. By putting Eq. (11) in Eq. (10), a linear algebraic system will be obtained that can be solved.

4. Analysis and results

Tapered angle of building and type of tubular system were the variables of this research. For the investigation of these variables, it's necessary to change the tapered angle in different tubular structural systems and the results should be examined. For this purpose, three tubular structural systems were considered: 1) simple tubular system, 2) tube-in-tube, and 3) bundled tube system. For these three systems, three tapered angles of 0, 1.23, and 2.45 degrees from vertical deflection were considered according to the ratio of height to the diameter and based on the definition of tall buildings, angle changes, and careful comparison of the systems. So, nine models were investigated in this research. For the presentation of an approximate new model for primary design, tubular buildings (Fig. 3) , tube-in-tube, and tapered bundled tubular systems were considered equivalent with cantilever shear-bending beam with variable cross-sections. Also, the number of floors were considered fixed in homologous structural systems. For this purpose, the models with tubular system, tube-in-tube (Fig. 4) , and bundled tubular systems were considered with 40 and 70 floors (Fig. 5). The length of center-to-center openings were considered three meters on the ground level. The parameter of floor mass is one of the effective parameters in natural frequency of building. Since the tapered angle is different, the area of floor is not the same in different models.

4.1. Selection of basic assumptions, consumed construction materials, modeling, reason model selection and structural analysis, discuss about the results.

4.1.1. Basic assumptions considered:
Analyzing a tall structure by accurate consideration of behavioral issues of elements and construction materials, even if the properties of the materials and dimensions of the elements being clear, is practically impossible, and it is inevitable to apply simplistic assumptions to reduce the size of the problem. In this regard, the most common hypotheses are introduced.

4.1.2. Materials
Construction materials of structure elements have linear elastic behavior. This assumption makes it possible to combine the effects of forces, displacements, the combination of two bending and shear effects, and the use of linear analysis methods.

4.1.3. Effective elements on structure behavior
Only important early structural elements are involved in the overall behavior. Considering this assumption, the effects of secondary structural elements and non-structural elements are ignored conservatively.

4.1.4. Floor slabs
It is assumed that the floors are on a rigid plate. This assumption causes the horizontal displacement of all vertical elements at the floor level being dependent on the rotation and horizontal transfer of the floors.

4.1.5. Ignorable stiffness
Minor stiffness of elements are overlooked. The lateral bending stiffness of slabs, the sub-axis stiffness of the shear walls, and the torsional stiffness of the columns, beams,

and walls are some types of stiffness that can be ignored.

4.1.6. Ignorable deformations

Small, ineffective deformations are ignored. These deformations include: axial and shear deformations of beams, bending and shear deformations of slabs and axial deformations of the columns of short- and medium-sized buildings.

4.1.7. Continuous perimeter and its limitations

1) The distance between the columns and the beams along the height of the building is constant; 2) The dimensions of all beams and columns on each floor are the same; 3) Structural materials are linearly elastic, isotropic, homogeneous and obedient to Hooke's law; 4) The structural system is assumed symmetrical on all floors; 5) The properties of the building are uniform throughout the height of the structure. Considering these assumptions, the structure is modeled as a beam with a box section and a continuous perimeter.

4.2. Consumed construction materials

In the models of this study, in general, 3 types of materials have been used, and the specifications of the materials used to design the towers are St52 steel. The modulus of elasticity and shear of this type of steel is 200,000 and 77,000 MPa, respectively. The sections used for the beams and columns are made of plate girder and square boxes in the software, respectively. The specifications of the consumed materials have been defined based on concrete with modulus of elasticity of 2×10^4 MPa and a specific gravity of 2500 kg/cm^2, and the Poisson coefficient of steel and concrete is 0.3 and 0.2, respectively. Reinforced concrete is used to model the shear wall and ceiling aperture.

4.3. Type of analysis

Because the models are tall buildings, according to Regulation 2800, the analysis must be dynamic. Therefore, in this research, the models are initially analyzed by spectral dynamic method, and according to the results of the initial analysis of sections, and the ninth and tenth subjects of the National Building Regulations are controlled and re-analyzed. This process continues until all sections are confirmed, and the results of the final analysis are used to measure and compare the natural frequency of the models. Because the models are tall structures, the effects of P-Δ as well as the effect of large deformations must be considered in the analysis. The soil under the foundation is considered of type II and the construction area with moderate seismic risk.

4.4. Modeling details in software and the reason for model type selection

Prior to any structural modeling, it is necessary to ensure that the dimensions of the plan and the height and sections are appropriate Stafford Smithand and Coull in The Basics of Designing Tall Buildings [35] that from the point of view of structural architecture, when the ratio of height to diameter is more than π, the structure is considered as a tall building. Therefore, this criterion has been used in selecting the dimensions of the plan and height of building. The assumed three-dimensional building has a square plan measuring 30x30 meters, which has been studied in three different heights of 40, 55 and 70 floors. The height of the floors and the distance between the axis of the columns is equal to three meters which has been selected based on the specifications of tube-in-tube and bundled tapered tubular systems. Each of them has three angles for modeling, so that the slope of the tapered tubular system is external and up to the inner vertical tube and the inner tube is without slope and vertical. Due to the fact that the selected angles are up to the extension of the inner tube, the comparability of the systems and the responsiveness of the type of structural system have been selected according to the height of the floors. To create a tubular system in structures, the length of the openings of the tubular frames is considered three meters and the length of the other openings of the structure is considered six meters in this model. The connection of the components outside the tubular system to the components of the tubular system will be articulated type and all connections will be considered rigid type in tube frames. The support of all columns is considered fixed support type. For modeling the apertures, the weight of the concrete as well as the design details are ignored and are entered only to apply the load in the modeling. The length of the three-meter opening is fixed and is cut based on the angle of columns along the angle and height.

Therefore, all design steps, including drawing members, allocating loads and controlling sections, have been performed in ETABS software. The gravitational loads of a seismic mass are calculated and defined in accordance with subject 6 of the National Building Code and Regulation 2800. Lateral loads of wind and earthquake based on ASCE 7-10 are automatically calculated and applied in the software. The AISC 360-10 design regulation has been selected because the tenth subject of the National Building Regulations is derived from it and the seismic design criteria are controlled according to AISC 341-10. The type of special bending frame structural system is selected by the tubular system.

4.5. The used sections and profiles

This paper is a research project, so the criterion for selecting profiles is to meet the conditions mentioned in the ninth and tenth subject of the National Building Regulations. Accordingly, all profiles used in beams, columns and braces are considered as square cans. It is considered according to Table (1). Modeling of beam and column elements is done as linear elements in the software. Also, the general model of the structure is done in three-dimensional form in the software environment. In order to select a profile for each element, it is possible to optimize the sections in ETABS software and the final control is done by the software.

By applying monotonic and equal load in floors, the mass of floors will not be the same according to Standard 2800 [39], the total dead load (including the weight of structural components and the diaphragm) mass of each floor. Since the floor area varies, to calculate the mass of each floor, it is necessary to multiply the load per unit area by the floor area. Considering the wide dead load of 9500 N/m^2.Effective load to the floor of each story is considered about 9.5 KN/m^2 which was shown by q in computations. Also, the equivalent (including the weight of peripheral columns tube) load is about 7.5 KN/m^2 which was shown by q' (for story 40). To calculate the mass parameter in terms of x, the total of structure load, exerted load to the floors, and peripheral load should be considered. Since the height of each floor is three meters, the obtained average should be divided by three. Applying loads is in terms of the weight unit, so the total load should be divided by three to change to mass unit.

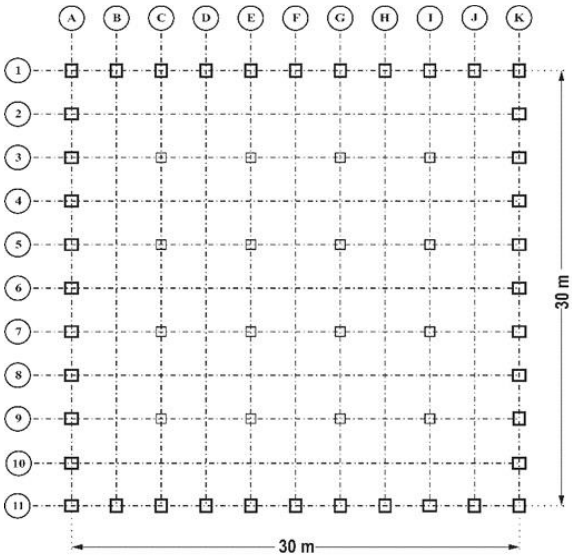

Fig. 3. Sectional plan view of tube tall building.

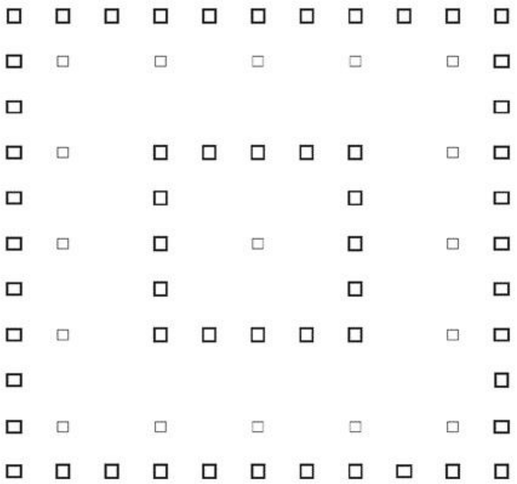

Fig. 4. Sectional plan view of tube-in-tube building.

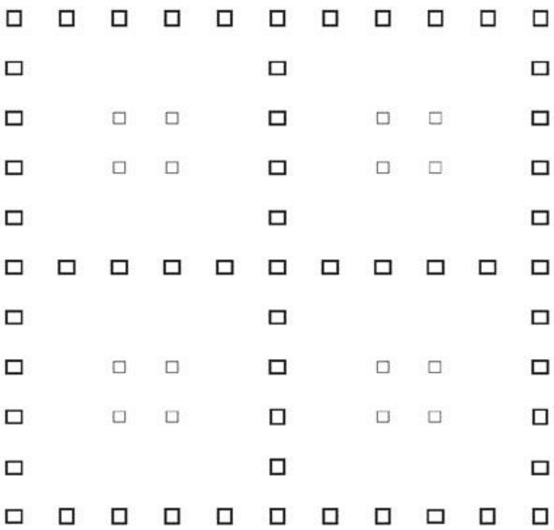

Fig. 5. Sectional view of bundled tube tall building.

5. Mathematical formulation

To verify the accuracy of the modeling, the natural frequency of models is calculated once again by a mathematical method. First, these equations are given for the tapered tube systems.

5.1. Tube system equations

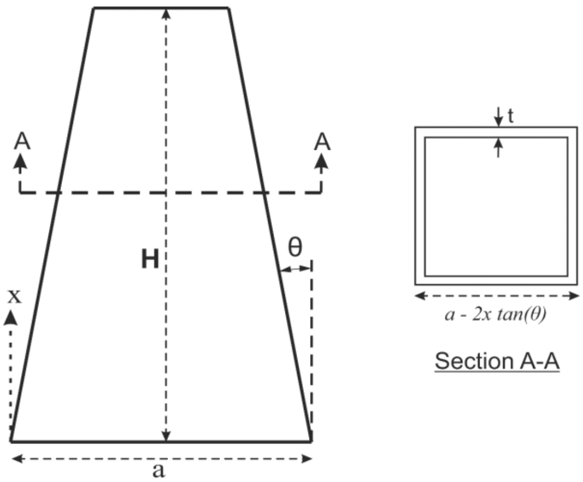

Fig. 6. Equivalent cantilever beam, with tapered angle and box cross-section.

Fig. 6 shows that the second moment inertia of beam surface is a function of height and varies with height. Given that the cross-sectional area is like a hollow square, so the relation of the second moment inertia of the surface in terms of x is defined as follows Eq. (12) :

$$I(x) = \frac{(a-2x\,tan\theta)^4}{12} - \frac{(a-2x\,tan\theta-2t)^4}{12} \qquad (12)$$

$$x = 0 \;\rightarrow\; I_0 = \frac{2ta^3}{3}$$

Using the parameters of $I\ (x)$ and I_0 obtained above, Eq. (8) can be rewritten for bending stiffness:

$$k_B(x) = EI(x) = E\,\frac{2t(a-2x\,tan\theta)^3}{3} = E\frac{2ta^3}{3}\left(1-\frac{2x\,tan\theta}{a}\right)^3 = EI_0\left(1-\frac{2x\,tan\theta}{a}\right)^3 \qquad (13)$$

With constitute $x = \xi H$ from Eq. (13) as ξ parameter, Eq. (14) is as follows:

$$k_B(\xi) = EI_0 K_B(\xi) = EI_0(1 - \frac{2\xi H\, tan\theta}{a})^3 \quad (14)$$

The cross-sectional area of the equivalent cantilever beam is regarded as a function of *x* which is defined as Eq. (15):

$$A(x) = (a - 2x\, tan\theta)^2 - (a - 2x\, tan\theta - 2t)^2 \quad (15)$$

$$x = 0 \;\rightarrow\; A_0 = (a)^2 - (a - 2t)^2$$

Using the parameters of *A (x)* and A_0 obtained from Eq. (15), Eq. (16) can be rewritten for shear stiffness:

$$k_s(x) = GA(x) = G((a - 2x\, tan\theta)^2 - (a - 2x\, tan\theta - 2t)^2) = G((a)^2 - (a - 2t)^2)(1 - \frac{2x\, tan\theta}{a-t}) = GA_0(1 - \frac{2x\, tan\theta}{a-t}) \quad (16)$$

With $x = \xi H$ from Eq. (16) as a function of ξparameter, the Eq. (17)is:

$$k_s(\xi) = GA_0 K_s(\xi) = GA_0\left(1 - \frac{2\xi H\, tan\theta}{a-t}\right) \quad (17).$$

The effective load on each floor is shown in the calculations with q. Also, the load equivalent to perimeter walls is shown with q' and $m(x)$ in the Eq. (18):

$$m(x) = \frac{q(a-2x\, tan\theta)^2 + 4q'(a-2x tan\theta)}{3g} \quad (18)$$

$$x = 0 \;\rightarrow\; m_0 = \frac{qa^2 + 4q'a}{3g}$$

$$m(x) = \frac{qa^2+4q'a}{3g}\left(\frac{(a-2x\, tan\theta)(4q'+aq-2qx tan\theta)}{a(4q'+aq)}\right) = m_0\frac{(a-2x\, tan\theta)(4q'+aq-2qx tan\theta)}{a(4q'+aq)}$$

Considering the value of q' equal to zero, the above equation is as follows Eq. (19):

$$m(x) = m_0(1 - \frac{2x\, tan\theta}{a})^2 \quad (19)$$

It is easy to see that the equations obtained for *m (x)* and *N (x)* in Eq. (19) and Eq. (20-21) in the previous section are the same for all structural systems, and only the difference in systems in the equations $k_B(x)$ and $k_S(x)$

5.2. Equations of a tube in tube system

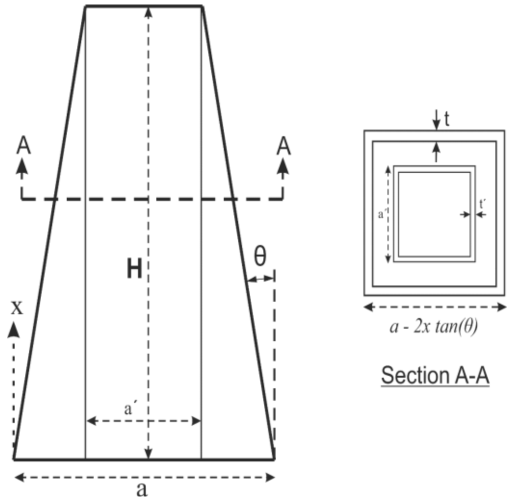

Fig. 7. Equivalent cantilever beam, with tapered angle and box cross-section for tube in tube system.

Equations 18 and 19 are used to obtain the axial force in the equivalent beam:

$$N(x) = \int_x^H g\, m(x)dx \quad (20)$$

$$= x^2\left(\frac{4q' tan\theta}{3} + \frac{2aq tan\theta}{3}\right) - H^2\left(\frac{4q' tan\theta}{3} + \frac{2aq tan\theta}{3}\right) + H\left(\frac{qa^2}{3} + \frac{4q'a}{3}\right) - x\left(\frac{qa^2}{3} + \frac{4q'a}{3}\right) + \frac{4H^3 q\, tan\theta^2}{9} - \frac{4x^3 q\, tan\theta^2}{9}$$

$$x = 0 \;\rightarrow\; N_0 = \frac{4Haq'}{3} - \frac{4H^2 q' tan\theta}{3} + \frac{4H^3 q\, tan(\theta)^2}{9} + \frac{Ha^2 q}{3} - \frac{2H^2 aq\, tan\theta}{3} \quad (21)$$

According to Fig. 7, it is possible to obtain the second moment inertia Eq. (22) of the surface as well as the area of the cross-sectional area of the equivalent beam for tube-in-tube structural system (The outer tube of the tapered and the inner tube are assumed to be fixed sections)

$$I(x) = \frac{(a-2x\, tan\theta)^4}{12} - \frac{(a-2x\, tan\theta-2t)^4}{12} + \frac{2t'a'^3}{3} \quad (22)$$

$$x = 0 \rightarrow I_0 = \frac{2ta^3}{3} + \frac{2t'a'^3}{3}$$

Using the parameters $I(x)$ and I_0 obtained above, we can rewrite the Eq. (23) for flexural stiffness:

$$k_B(x) = EI(x) = E\,\frac{2t(a-2x\,tan\theta)^3}{3} + \frac{2t'a'^3}{3} = EI_0\left(1 - \frac{8tx^3\tan(\theta)^3 - 12at\,x^2\tan(\theta)^2 - 6a^2tx\,tan\theta}{ta^3 + t'a'^3}\right) \quad (23)$$

By placing the value of $x = \xi H$ of Eq. (23), the following equation is considered as a function of the parameter of ξ, and Eq. (24)is as follows:

$$k_B(\xi) = EI_0K_B = EI_0\left(1 - \frac{8t(\xi H)^3\tan(\theta)^3 - 12at\,(\xi H)^2\tan(\theta)^2 - 6a^2t\xi H\,tan\theta}{ta^3 + t'a'^3}\right)$$

The cross-sectional area of the equivalent cantilever beam is the same as the second anchor of the surface and is a function of *x*, which is defined as follows Eq. (25):

$$A(x) = (a - 2x\,tan\theta)^2 - (a - 2x\,tan\theta - 2t)^2 + a'^2 - (a' - 2t')^2 \quad (25)$$

$$x = 0 \rightarrow A_0 = (a)^2 - (a - 2t)^2 + a'^2 - (a' - 2t')^2$$

Using the parameters A(x) and A_0 obtained above, the Eq. (26) can be rewritten for shear stiffness:

$$k_s(x) = GA(x) = G\big((a - 2x\,tan\theta)^2 - (a - 2x\,tan\theta - 2t)^2 + a'^2 - (a' - 2t')^2\big) = GA_0\left(1 - \frac{2tx\,tan\theta}{-t^2 + at - t'^2 + a't'}\right) \quad (26)$$

By placing the value of $x = \xi H$ of Eq. (26), the following equation is considered as a function of the parameter of ξ, and Eq. (27) is as follows:

$$k_s(\xi) = GA_0K_s(\xi) = GA_0\left(1 - \frac{2t\xi H\,tan\theta}{-t^2 + at - t'^2 + a't}\right) \quad (27)$$

5.3. Equations of the bundled tube system

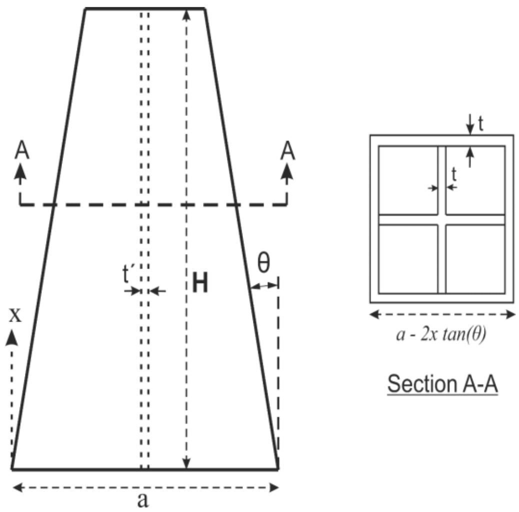

Fig. 8. Equivalent cantilever beam, with tapered angle and box cross-section for bundled tube system.

According to Fig. 8, it is possible to obtain the second moment inertia of the surface as well as the area of the cross-sectional area of the equivalent beam for categorized tube structural system:

$$I(x) = \frac{(a-2x\,tan\theta)^4}{12} - \frac{(a-2x\,tan\theta-2t)^4}{12} + \frac{t(a-2x\,tan\theta)^3}{12} + \frac{(a-2x\,tan\theta)t^3}{12} \quad (28)$$

$$x = 0 \rightarrow I_0 = \frac{2ta^3}{3} + \frac{ta^3}{12} + \frac{a^3t}{12}$$

Using the parameters of *I(x)* and I_0 obtained above Eq. (28), Eq. (29) can be rewritten for bending stiffness:

$$k_B(x) = EI(x) = E\left(\frac{2t(a-2x\,tan\theta)^3}{3} + \frac{t(a-2x\,tan\theta)^3}{12} + \frac{(a-2x\,tan\theta)t^3}{12}\right) == EI_0\left(\frac{(a-2x\tan\theta)(36x^2\tan(\theta)^2 + 9a^2 + t^2 - 36ax\,tan\theta)}{a(9a^2+t^2)}\right) \quad (29)$$

By placing the value of $x = \xi H$ of Eq. (29), the following equation is considered as a function of the parameter of ξ, and Eq. (30) is as follows:

$$k_B(\xi) = EI_0 K_B(\xi) = \quad (30)$$
$$EI_0\left(\frac{(a-2\xi H\tan\theta)(36(\xi H)^2\tan(\theta)^2+9a^2+t^2-36a\xi H\tan\theta}{a(9a^2+t^2)}\right.$$

The cross-sectional area of the equivalent cantilever beam is a function of *x,* which is defined as follows Eq. (31):

$$A(x) = (a-2x\,tan\theta)^2 - (a-2x\,tan\theta-2t)^2 + 2t(a-2x\,tan\theta) \quad (31)$$

$$x = 0 \;\rightarrow\; A_0 = (a)^2 - (a-2t)^2 + 2at$$

Using the parameters, *A (x)* and A_0obtained above, Eq. (32) can be rewritten for shear stiffness:

$$k_s(x) = GA(x) = G((a-2x\,tan\theta)^2 - (a-2x\,tan\theta-2t)^2 + 2t(a-2x\,tan\theta)) = GA_0(1-\frac{6x\,tan\theta}{3a-2t}) \quad (32)$$

By putting the value x = ξH from Eq. (32) of the above equation as a function of the parameter ξ, Eq. (33) is given as follows:

$$k_s(\xi) = GA_0K_s(\xi) = GA_0(1-\frac{6\xi H\,tan\theta}{3a-2t}) \quad (33)$$

6. Verification: Analysis with differential equations (mathematical method), finite element, and other articles

For natural frequency verification, two models of simple tube systems with a height of 120 meters or 40-story and with tapered angles of 0 and 2.45 degrees are obtained and compared by using mathematical and modeling methods in the software.

6.1. Modeling with finite element

Initially, the geometry of the structure and the location of beams and columns in ETABS [40] software environment are shown in Fig. 9 and Fig. 10 for both models with a tapered angle of 0 and 2.45 degrees

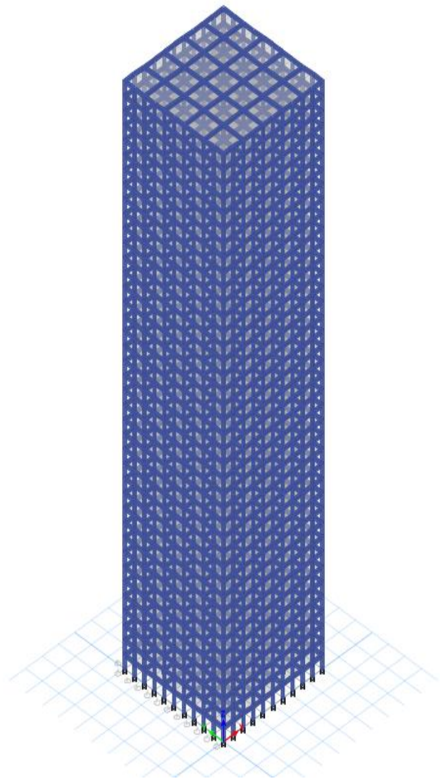

Fig. 9. 40-story tube system model with zero-degree tapered angle.

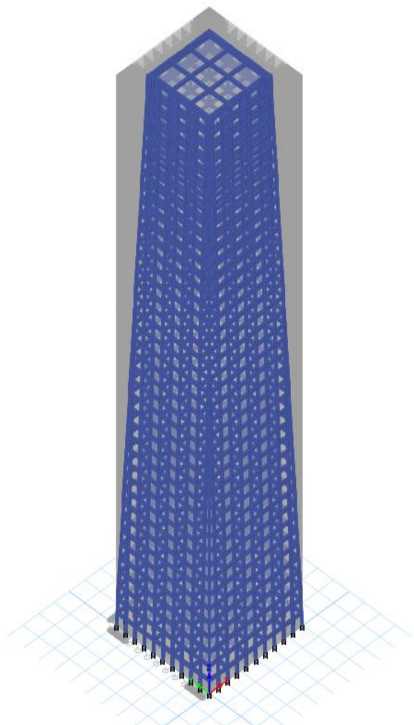

Fig. 10. 40-story tube system model with 2.45-degree tapered angle.

In order to determine the dimensions of elements of the beam and the column, an initial analysis and design of the structures are made to determine the sections used for different structural elements. To do this, ETABS automatic selection software was used. After designing and selecting the section sizes by the software, box section with a height of 500 mm and wall thickness of 50 mm were considered as the section of members like beams and columns in peripheral frames forming the tube system. After determining cross-sections of elements, modal analysis was done on the models. The results of the analysis showed that the model with zero tapered angle had a natural frequency of 2.04 radians per second. And the model with a tapered angle of 2.45 degrees had a natural frequency of 2.52 radians per second.To verify the validity of modeling, mathematical method was also used to calculate natural frequency of models. For this purpose, the technique presented by Mohammadnejad and Haji-Kazemi (2018) [27] was used, which was obtained in Eq. (3) to Eq. (21) in the upper section for tapered tube system. To obtain the natural frequency from mathematical method presented above, first, the properties of equivalent cantilever beam for tapered tube system should be obtained. According to the cross-section considered for beams and columns of metal structures (box section with a height of 500 mm and thickness of 50 mm), wall thickness, elastic and shear modulus of the equivalent cantilever beam can be obtained according to the Eq. (3) to Eq. (6).

$$t = 30mm$$

$$E = E_m = 2 \times 10^5 MPa$$

$$G = 1.174 \times 10^5 MPa$$

Using these parameters and inserting them in Eq. (1) to Eq. (21) for a tube system, and total equation of (1) to (21), natural frequency can be obtained for tube system. Accordingly, the main calculated frequency for a 40-story tube system with 2.45-degree tapered angle, and accuracy of R=1, in Equations (1) to (21), and using MATLAB [41] software, is equal to 2.48 radians per second. Also, the calculated natural frequency for the zero-degree tapered angle is equal to 2.029 radians per second.

7. Discuss about the results

7.1. Overview

In the previous section, the number of models and details of their modeling in software as well as how to calculate the natural frequency of structures using the theoretical method were presented. In this section, the results of computer modeling as well as the theoretical method for all models are presented and compared. It is emphasized that the scope of confirmation of the obtained results be confined to the selected models.

7.2. Preliminary analysis of models

Before examining the results and obtaining the natural frequency of the models, it is necessary to first identify the sections used in beams and columns. For this purpose, ETABS software was used to design models and identify the cross-sections. The results are shown in Table 1.

Table 1. Cross-sections used in metal structure models.

Stories and systems	Section beams and columns
Stories 40 with simple tube	box section with a height of 500×500 mm and thickness of 50 mm
Stories 55 with simple tube	box section with a height of 600×600mm and thickness of 100 mm
Stories 40 with tube in tube	box section with a height of 500×500 mm and thickness of 50 mm
Stories 70 with tube in tube	box section with a height of 600×600mm and thickness of 100 mm
Stories 40 with tube bundled	box section with a height of 500×500 mm and thickness of 50 mm
Stories 70 with tube bundled	box section with a height of 600×600mm and thickness of 100 mm

For better comparisons, a cross-section for beams and columns and a number of equal floors having different tapered angles have been used in models with a system. It should be noted that in the mathematical method presented by Mohammadnejad and Haji-Kazemi (2018) [27], the beam and column sections must be the same across all floors. In this research, cross-sections of beams and columns are considered the same.

The necessary parameters must first be calculated. According to the relations of Eq. (4-6), this method requires the calculation of the equivalent thickness of the tube wall *(t)*, the modulus of elasticity and the shear modulus in the equivalent beam (*E* and *G*) obtained by placing the values in Table 1. Accordingly, for 40-story models using 50×50cm cross-sections with 5cm wall thickness, the parameters are as follows:

$$t = \frac{A_c}{s} = 30mm \quad E = E_m = 2 \times 10^5 MPa$$

$$G = 1.174 \times 10^5 \qquad (34)$$

Also, for 70-story models using 60×60cm profile with 10cm wall thickness, the following parameters are obtained:

$$t = \frac{A_c}{s} = 66.667mm \; E = E_m = 2 \times 10^5 MPa$$

$$G = 1.535 \times 10^4 MPa \qquad (35)$$

7.3. The results of mathematical method for variety tubular structural systems

Considering the calculated parameters in Eqs. (34,35), using MATLAB and considering R = 1 in Eq. (11) and placement of geometric parameters of height (120,165 and 210 m), side length (30 m), angle of deviation (0 ,1.23 and 2.45°) as well as Eq. (1-33) calculated in the previous section, 18 natural frequencies can be obtained for 18 models with structural systems as shown in Table 2.

Table 2. Natural frequencies calculated by mathematical methods in models with different types of tube systems.

Systems \ Degrees	0	1.23	2.45
Stories 40 tube	2.029	2.242	2.480
Stories 55 tube	2.102	2.782	3.212
Stories 40 tube in tube	2.183	2.489	3.109
Stories 70 tube in tube	1.572	2.010	2.781
Stories 40 tube bundled	3.119	3.151	3.312
Stories 70 tube bundled	2.003	2.211	2.890

7.4. Results of finite element modeling method

As stated earlier in this section, the models were first analyzed once to approximate determination of the profiles used by the models. After the initial analysis, the values in Table 1 are assigned to the profiles and the models are re-analyzed. For structural systems, 18 natural frequencies were obtained as shown in Table 3.

Table 3. Natural frequency calculated by finite element method in models with different types of tube system.

Systems \ Degrees	0	1.23	2.45
Stories 40 tube	2.04	2.569	2.520
Stories 55 tube	1.938	2.761	3.156
Stories 40 tube in tube	2.210	3.962	2.071
Stories 70 tube in tube	1.549	2.082	2.748
Stories 40 tube bundled	3.433	4.165	3.225
Stories 70 tube bundled	1.613	2.023	2.847

7.5. Calculating the accuracy of the mathematical method

In order to accurately understand the mathematical method and its compatibility with different types of tube systems as well as different tapered angles, the results of this method should be compared with the finite element method. Therefore, to determine the effects of structural system variables, structural height and tapered angle were added to the accuracy of the presented mathematical method, and each of the variables is examined separately.

7.6. Accuracy of mathematical method in terms of structural system

For this purpose, the frequency differences obtained from both mathematical and finite element modeling methods are separated based on structural system orders. Then, the mean and standard deviation of the differences in responses are examined. In Table 4, the natural frequencies obtained from both methods are analyzed in terms of the structural system.

Table 4. Natural frequency obtained from mathematical and finite element methods in structural system.

System		Natural frequency						Standard deviations	Mean of errors
tube	Finite element	2.04	2.569	2.52	1.938	2.761	3.156		
	Mathematical	2.029	2.242	2.48	2.102	2.783	3.212		
	Error	0.011	0.327	0.04	-0.164	-0.022	-0.056	0.15060	-0.0055
Tube in tube	Finite element	2.21	3.962	2.071	1.549	2.082	2.748		
	Mathematical	2.183	2.489	3.109	1.572	2.01	2.781		
	Error	0.027	1.473	-1.038	-0.023	0.072	-0.033	0.7321912	0.002
Tube bundled	Finite element	3.343	4.165	3.225	1.613	2.023	2.847		
	Mathematical	3.119	3.151	3.312	2.003	2.211	2.89		
	Error	0.224	1.014	-0.087	-0.39	-0.188	-0.043	0.45248	-0.065

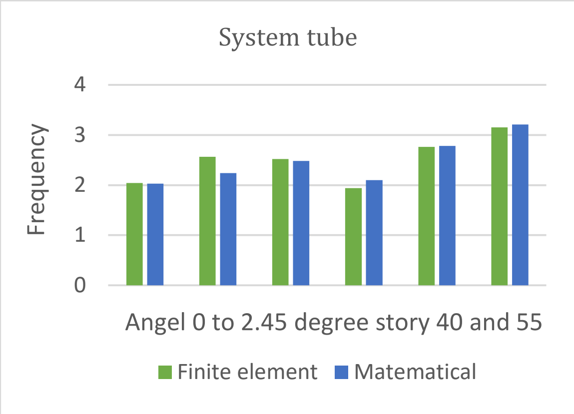

Fig. 11. The natural frequency obtained for 6 models with a tubular system.

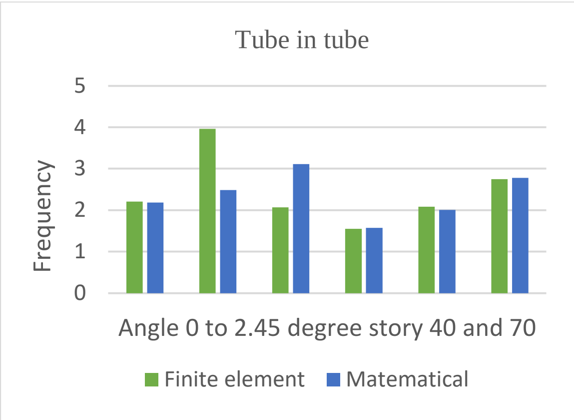

Fig. 12. The natural frequency obtained for 6 models with a tube in tube system.

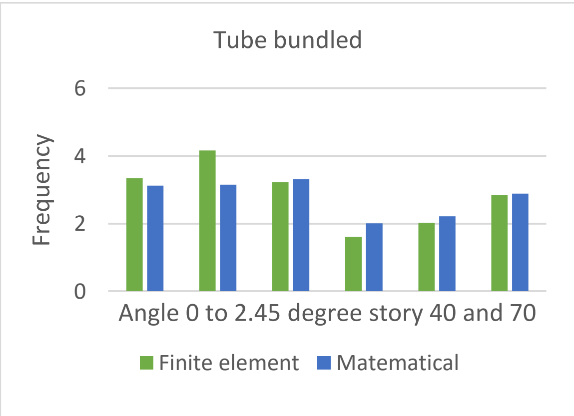

Fig. 13. The natural frequency obtained for 6 models with a tube bundled system.

Considering Table 4 and bar graph Fig.11 to Fig.13 as well as comparing the standard deviation of the errors, it can be seen that the scattering errors are lower for the tube system and are more for tube-in-tube system than the others. Also, by comparing the mean errors, it can be found that the mathematical method for the categorized tube system gives a higher frequency value than the real state.

The results of Table 4 show that the mathematical method is most compatible with the tube system. It also has the least compatibility with tube-in-tube structural system. In other words, tube structural systems, categorized tubes, and tube-in-tube systems are more in line with the mathematical method, respectively.

7.7. The accuracy of mathematical method in terms of the height of structure

For this purpose, the frequency differences obtained from both mathematical and finite element modeling methods are separated based on the number of floors. Then the mean and standard deviation of the differences in responses are examined. In Table 5, the natural frequencies obtained from both methods are analyzed in terms of the number of floors.

Also, in Fig. 11, the graphs of the number of floors according to the natural frequency of different models with the same number of floors are shown in both mathematical and numerical methods.

Table 5. Natural frequency obtained by numerical and mathematical methods in terms of number of floors.

Stories 40			Stories 55			Stories 70		
Finite element	Mathematical	Error	Finite element	Mathematical	Error	Finite element	Mathematical	Error
2.04	2.029	0.011	1.938	2.102	-0.164	1.549	1.572	-0.023
2.569	2.242	0.327	2.761	2.783	-0.022	2.082	2.01	0.072
2.52	2.48	0.04	3.156	3.212	-0.056	2.748	2.781	-0.033
2.21	2.183	0.027				1.613	2.003	-0.39
3.962	2.489	1.473				2.023	2.211	-0.188
2.071	3.109	-1.038				2.847	2.89	-0.043
3.343	3.119	0.224						
4.165	3.151	1.014						
3.225	3.312	-0.087						
Standard deviations		0.666837	Standard deviations		0.060538	Standard deviations		0.15005
Mean of errors		0.04	Mean of errors		-0.056	Mean of errors		-0.038

Considering Table 5 as well as Fig. 14 and comparing the standard deviation of the errors, it can be seen that the in 70-story buildings, error scattering is less than 40-story buildings. Also, by comparing the mean errors, it can be found that the mathematical method for buildings with 70 floors gives more frequency value than reality. This value is less for buildings with less than 40 stories. The results in Table 5 and Fig. 11 show that the mathematical method is most compatible with the number of floors. In other words, the higher the number of floors, the more the accuracy of the mathematical method.

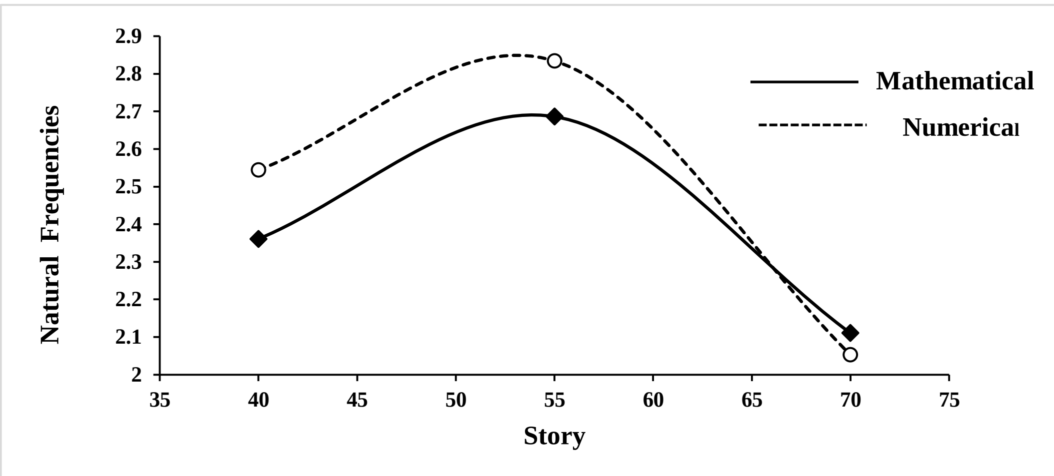

Fig. 14. Natural frequency means obtained by the number of floors in mathematical and numerical methods.

7.8. The accuracy of the mathematical method in terms of tapered angle

For this purpose, the frequency differences obtained from both mathematical and finite element modeling methods are separated based on tapered angle. Then, the mean and standard deviation of the differences in responses are examined. In Table 6, the natural frequencies obtained from both methods are analyzed in terms of tapered angle.

Table 6. Natural frequencies obtained by mathematical and numerical methods in terms of tapered angles.

Stories 40			Stories 55			Stories 70		
Finite element	Mathematical	Error	Finite element	Mathematical	Error	Finite element	Mathematical	Error
2.04	2.029	0.011	2.569	2.242	0.327	2.52	2.48	0.04
1.938	2.102	-0.164	3.962	2.489	1.473	2.071	3.109	-1.038
2.21	2.183	0.027	4.165	3.515	1.014	3.225	3.212	-0.087
1.549	1.572	-0.023	2.761	2.783	-0.022	3.156	2.781	-0.056
3.343	3.119	0.224	2.082	2.010	0.072	2.748	2.211	-0.033
1.613	2.003	-0.39	2.023	2.211	-0.188	2.847	2.89	-0.043
Standard deviations		0.18886	Standard deviations		0.598939	Standard deviations		0.375464
Mean of errors		-0.006	Mean of errors		0.1995	Mean of errors		-0.0495

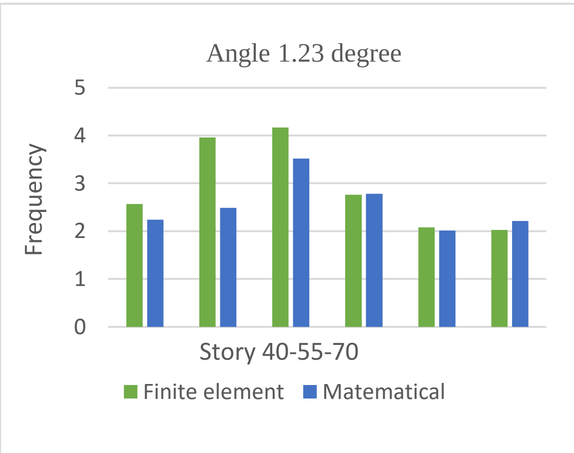

Fig. 15. The bar graph shows the natural frequency at different tapered angles.

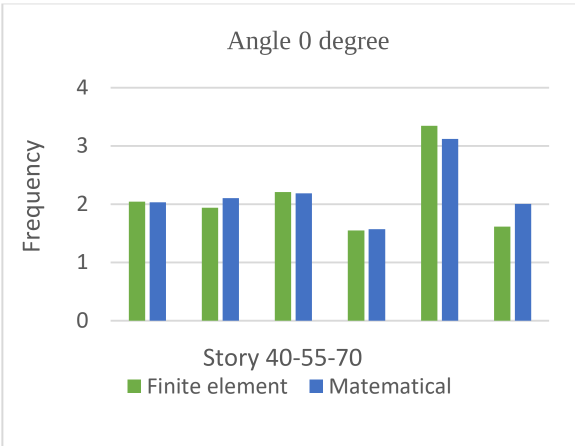

Fig. 16. The bar graph shows the natural frequency at different tapered angles.

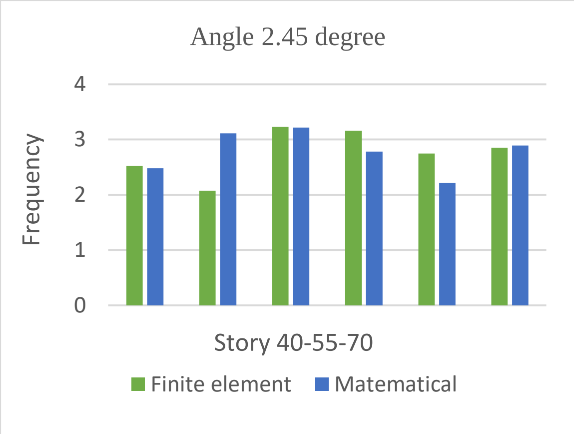

Fig. 17. The bar graph shows the natural frequency at different tapered angles.

Considering Table 6 , bar graph Fig.15 to Fig.17 and comparing the standard deviation of the errors, it can be seen that the scattering of the errors for the tapered angle of zero degree is lower than other angles. By comparing the mean errors, it can be found that the mathematical method for the tapered angle of 1.23 degrees offers a lower frequency value than the reality. And also, for the tapered angle of 2.45 degrees, this value is more than reality and for the tapered angle of zero degree there is no clear trend.

The results of Table 6 show that the mathematical method is most compatible with the zero-degree tapered angle. There is no clear trend for other angles. The reason for the error of the mathematical method for 1.23- and 2.45-degree angles is due to the decrease in the number of columns in height which causes a sudden change in the structural stiffness, but in the mathematical method the stiffness reduction is considered gradual.

7.9. Obtaining the relationships between variables

Finite element modeling results are used to obtain the relationships between variables that are more consistent with reality. For this purpose, the relationships between the variables of the tapered angle and the type of structural system are investigated with natural frequency. The relationship between the structure height and natural frequency is known, and when beam and column profile dimensions are constant, the higher the structural height, the less the natural the frequency.

7.10. Obtaining the relationships between variables

Limited element modeling results are used to obtain the relationships between variables that are more consistent with reality. For this purpose, the relationships of tapered angle variables and the type of structural system with natural frequency are examined. The relationship of the height variable of the structure with the natural frequency is clear, and if the dimensions of the profile of the beams and columns are constant, the higher the structure is, the more the natural frequency decreases.

7.11. The relationship between natural frequency and tapered angle

In order to obtain the relationship between the natural frequency and the tapered angle, the natural frequency must be examined separately in each structural system as well as in the number of floors. For this purpose, according to Fig. 18 to Fig. 20 the frequency diagram based on tapered angle has been obtained based on the structural system separation and the number of floors.

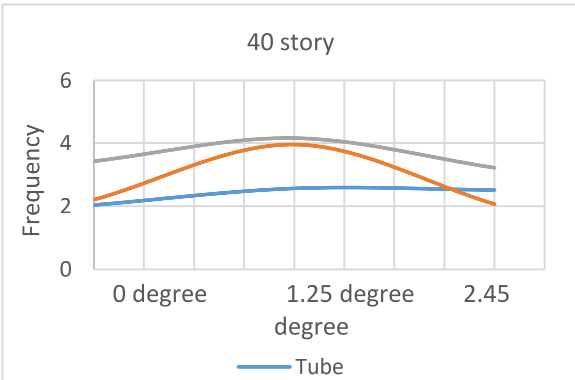

Fig. 18. Tapered angle diagram in frequency for Tube, Tube in Tube and Tube bundled systems.

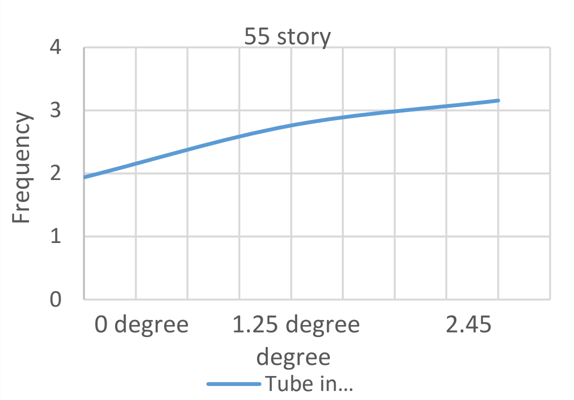

Fig. 19. Tapered angle diagram in frequency for Tube in tube structural systems.

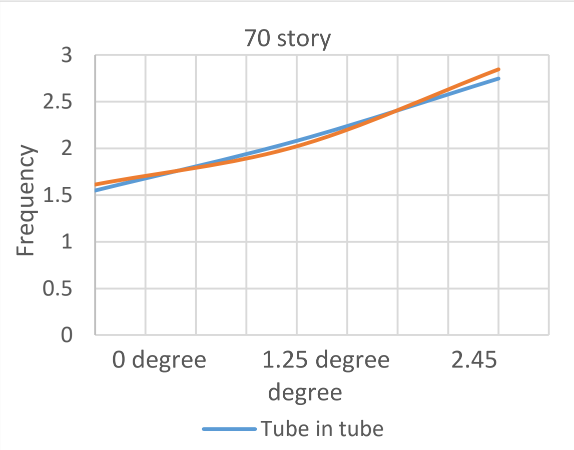

Fig. 20. Tapered angle diagram in frequency for Tube bundled structural systems.

For the 40-storey models, the natural frequency has lost its incremental trend at 2.45 degrees. In other words, the zero-degree angle has the lowest frequency and the 1.23-degree angle has the highest natural frequency.

The reason for the decrease in the natural frequency at the tapered angle of 2.45 can be attributed to the decrease in the number of columns and hard elements at the height of the structure.But the trend is quite rising in structures with 70 floors. In other words, for structure with 70 floors, the higher the tapered angle, the more the natural frequency.In sum, it can be said that in tapered structures, a rise in tapered angle increases the natural frequency. But on the other hand, as the floor level decreases in height, the hardening elements such as columns also decrease, which in turn reduces the natural frequency.

7.12. The relationship between natural frequency and type of structural system

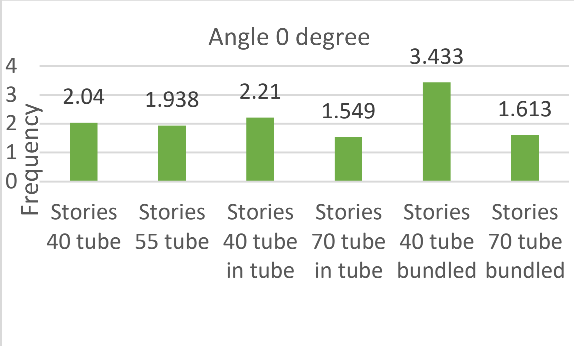

Fig. 21. Natural bar rod diagram with tapered angles 0 degree according to the structural system.

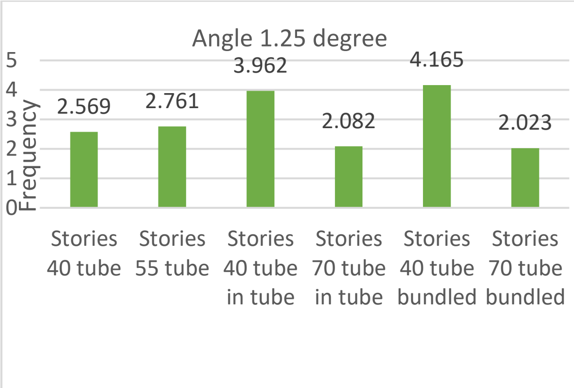

Fig. 22. Natural bar rod diagram with tapered angles 1.25 degree according to the structural system.

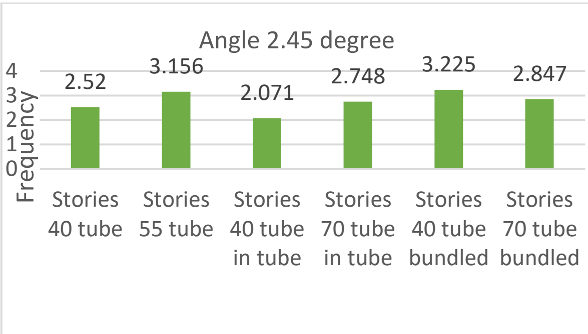

Fig. 23. Natural bar rod diagram with tapered angles 2.45 degree according to the structural system.

For this purpose, the mean of natural frequencies for different tapered angles have been calculated and bar rod diagram Fig.20 to Fig.23 compared based on the separation of the number of floors. Considering Fig. 21 to Fig. 23 it can be seen that for 40-storey models, the categorized tube system has the highest frequency, and then the tube system and tube-in-tube systems have the highest frequency, respectively. For models with 55 and 70 floors, tube system, tube-in-tube and bundled tube systems have the highest frequency, respectively. In the previous section, the results of numerical modeling as well as the results of the mathematical method were presented, and these results were compared and the errors of the mathematical method were analyzed. Also, the relationships between research variables were investigated and their relationship was obtained. This section now gives an overview of the present study as well as its conclusions.

8. Conclusion

In the previous section, the results of numerical modeling as well as the results of the mathematical method were given, and these results were compared and the errors of the mathematical method were analyzed. The relationships between the research variables were also examined and their relationship was obtained. One of the most important dynamic characteristics of high-rise buildings is tapered vibrational frequency. There are no proper classification criteria for the new class of high-rise tubular, and tubular tapered buildings in standard regulations. The results of this research are confined to the assumptions mentioned in the text. So, the results of the present research and also the summary of the previous sections and the above paragraph, can be deduced and summarized as some applicable rules:

- Using the dynamic relationships of structures for free vibration of tall tapered buildings, the proposed formulae are obtained and with the help of programming, the first frequency was obtained which is used in computational efficiency and initial design.
- The approximate analytical method provided for tapered structures is well adapted to the results of finite element modeling and the use of the natural frequency obtained from this method was confirmed. This method can be used for calculation.
- The offered mathematical method has an acceptable accuracy, which has the highest and least accuracy and consistency for the tubular and tube-in-tube systems, respectively. For the range of selected and studied models, the average error range is approximately 0.08 radians per second for tapered structural systems.
- The mathematical analytical method is more compatible with the tubular system than other systems. In other words, the tubular structural systems, bundled tubular, and tube-in-tube systems, respectively, due to having a lower error rate and acceptable accuracy, have more compatibility and adaptation with the presented analytical method.
- Studies show that the accuracy of the offered mathematical method has the most compatibility with the highest number of floors. In other words, with more floors the accuracy of the offered method will be higher. So, the higher the height of the structure, the greater the accuracy of the mathematical method, or in other words, the proposed method is more compatible with the highest number of floors for tapered structures.

- Studies show that the proposed analytical method, which gives very little error for tapered angled structures and is more compatible with the proposed mathematical method, is more appropriate. Using this method for tapered angled structures has a small amount of error in calculations.
- The above research shows that in conditions where the stiffness of the structure remains constant, when the tapered angle of a structure is higher (taking into account buildings with a fixed height), the natural frequency of the mathematical method will be more. On the other hand, increasing the tapered angle means reducing the number of hardening elements, such as columns, which reduces the stiffness of the structure and reduces the natural frequency in the finite element method, which must be considered.
- To find the relationship between the variables, the study shows that although increasing the tapered angle increases the natural frequency of the structure, there is no significant relationship between the natural frequency of the structure and the type of tapered structural system.

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