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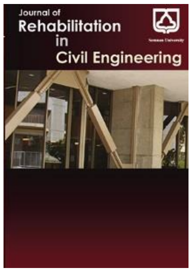

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A Study on Multiple Tuned Liquid Column Ball Dampers (MTLCBDs)

Asghar Veisi Chaman Esmaeili[1], **Hamid Toopchi-Nezhad**[2*]

1. Department of Civil Engineering, Kermanshah Branch, Islamic Azad University, Kermanshah, Iran
2. Department of Civil Engineering, Razi University, Kermanshah, Iran
Corresponding author: h.toopchinezhad@razi.ac.ir

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ABSTRACT

Tuned liquid column ball dampers (TLCBDs) are a relatively new type of liquid dampers in which the motions of liquid in a U-shaped tube counteracts the forces acting on the structure. Damping in the oscillating liquid is introduced through a steel ball rolling by the liquid passage. The tube and steel ball in a single TLCBD-system may acquire enormously large dimensions. One way to decrease the size, and perhaps the total costs, is to replace a single-TLCBD with a multiple (M)-TLCBDs of smaller dimensions. Current literature lacks to address the governing equations of an M-TLCBD and its application in wind response mitigation of tall buildings. In this paper, the governing equations of motion for an MTLCBD-system has been developed. Next, the dynamic response of a tall building, equipped with various MTLCBDs, to harmonic wind excitations is investigated. The influence of different design parameters such as mass ratio, length ratio, and the number of individual dampers on the response mitigation efficiency of MTLCBDs has been studied. Overall, the performance of a MTLCBD is found to be sensitive to the variations in the design parameters mentioned above.

Nomenclature

- A cross-sectional area of U-tube
- A_b main cross-sectional area of the ball
- d_{eq} equivalent damping coefficient
- C_s viscous damping coefficient of structure
- m_s effective mass at the fundamental mode of structure
- n number of dampers in the MTLCBD
- R ball-to-tube diameter ratio
- R_b radius of the ball

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C_t	equivalent damping coefficient of TLCBD	t	time variable
F_{ext}	external excitation (wind load)	x_b	displacement of the ball
F	amplitude of the external excitation	x_s	displacement of the main structure
g	gravitational acceleration	y	displacement of the surface of the liquid
h	undisturbed length of liquid in the vertical column	α	ratio of the horizontal length to total length of the liquid in the tube
J_c	mass moment of inertia of the ball about its center of mass	ρ	Density of the liquid
K_s	effective stiffness at the fundamental mode of structure	ω_{ext}	frequency of external excitation
L	total length of liquid within the tube	ω_s	natural frequency of the main structure
m_b	mass of the ball	ω_t	frequency of damper
m_f	total mass of liquid	μ	the ratio of total mass of liquid to the total mass of structure
		ξ_t	equivalent viscous damping ratio of damper

1. Introduction

High-rise building structures are typically vulnerable to the dynamic effects of wind loads. The wind-induced vibrations in such structures may result in accelerations that are beyond the residents, comfort limits and sometimes may cause damages to the building. This is particularly more pronounced when the structure is subjected to resonance or near resonance excitations. The structural response to wind loads may be attenuated with the aid of supplemental damping devices such as viscoelastic dampers or mass dampers (Samali and Kwok [1]). Tuned mass dampers (TMDs), and tuned liquid dampers (TLDs) have been successfully employed to control the wind-induced excitations of high-rise structures (e.g. Li et al. [2], Colwell and Basu [3]). The other means of control devices such as base isolation (Shahabi et al. [4]), or buckling restrained braces (Seyed Razzaghi and Hatami [5]), etc., are not as effective in the wind-response attenuation of structures.

The application of tuned liquid column dampers (TLCDs) as a means of wind-response attenuation in high-rise buildings has been investigated by several scholars (e.g., Sakai et al. [6], Liang et al. [7], and Diana et al. [8]). A TLCD consists of a U-shaped tube that is filled with water up to an appropriate height. These dampers are usually located at the highest level (typically the last story) of the structure. At the middle of the horizontal portion of the U-shaped tube, an internal orifice is installed to dissipate the kinetic energy of the sloshing water by making a barrier against liquid motions within the tube. This causes inherent damping for the device itself and limits the liquid sloshing. A semi-active version of TLCDs, with an adaptive frequency tuning capacity, may be employed to mitigate the buffeting response of long-span cable-stayed bridges during their various construction stages (Shum et al. [9]).

In a tuned liquid column ball damper (TLCBD), the orifice is removed, and instead, a rolling steel ball is located at the horizontal portion of the U-shaped tube to dissipate the energy of sloshing water. Al-Saif and Aldakan [10] developed the equations of motion, and experimentally evaluated the response characteristics of a model scale TLCBD. The performance of a TLCBD, as a passive vibration control device, is found to be more effective at low-frequency excitations (e.g. Al-Saif et al. [10]). The ball-to-tube diameter ratio is a critical design parameter that influences the performance of a TLCBD-system. An optimum value of 0.8 for this parameter has been recommended by Ref. (Al-Saif et al. [10]). Analytical studies suggest that the performance of TLCBDs is superior to that of TLCDs under the effects of random loads (e.g. Chatterjee and Chakraborty [11], Gur et al. [12]), as well as harmonic loads (Al-Saif et al. [10]).

Toopchi-Nezhad and Panahian [13] compared the response characteristics of TLCDs with those of TLCBDs via a comprehensive analytical study. Despite the superior performance of TLCBDs, their response is more sensitive to the excitation frequency as compared to TLCDs (Toopchi-Nezhad and Panahian [13]). Gupta and Kakulate [14] introduced the concept of a spring-loaded liquid column ball damper (SLLCBD), wherein two springs are attached at the opposite sides of the steel ball within the damper to improve its response attenuation capability. Mass dampers may be employed in vibration control of other structural systems in addition to tall buildings. Such structural systems include; wind turbine towers (Mensah and Osorio [15], Rahman and Ong [16]), suspended bridges (e.g. Gkoumas et al. [17]), and pedestrian bridges (Reiterer and Hochrainer [18]). Tuned mass dampers are more effective in mitigating the steady-state response of structures to wind excitations. However, the seismic response of these systems has been investigated by many scholars (e.g., Chang and Hsu [19], Yalla and Kareem [20], Wu et al. [21], and Gur et al. [12]). Pandey and Mishra [22] employed a combination of a TLCBD and a Circular Liquid Column Ball Damper (a TLCBD-CLCBD system) to control the torsional response of building structures under wind excitations.

The mass dampers may be employed as a group. The rationale behind employing multiple dampers instead of one includes response improvement, space, and/or budget considerations. Haroun and Pires [23] developed a hybrid-LCD system in which TLCDs were installed in parallel to TMDs. The performance of the hybrid system was found to be effective in displacement attenuation of high-rise structures (e.g. Haroun et al. [23]). Application of multiple-TLCDs has been reported in the literature (e.g. Gao et al. [24], Mohebbi et al. [25]). In an MTLCD-system, the size of each damper is significantly smaller than its equivalent single TLCD. As such, the dampers will occupy a relatively smaller space, and the total cost of the system may be lower than that of their equivalent single TLCD. The efficiency of an MTLCD-system is not necessarily increased with increasing the number of dampers within the system (Gao et al. [24]). The optimal design of MTLCDs for seismic vibration of multi-degree of freedom structures has been studied by Mohebbi et al.

[25]. Liquid dampers have been proposed for seismic response reduction of eccentric structures (Hue and Li [26]).

The literature is silent on the application of multiple tuned liquid column ball dampers (MTLCBDs) in response attenuation of structures. The rationale behind using a MTLCBD is the same as the reasons that justify the use of MTLCD. The main objectives of this research study are to develop the governing equations of motion for a MTLCBD system and to examine and compare the performance of a MTLCBD with its equivalent single TLCBD system. Also, the influence of many design parameters such as the mass ratio, length ratio, and the number of individual dampers in response mitigation efficiency of the system has been investigated.

2. Governing equations of motion for MTLCBD-systems

As shown in Figure 1 in a given single TLCBD system there are three degrees of freedom (independent unknowns), namely, x_s, y, and x_b.

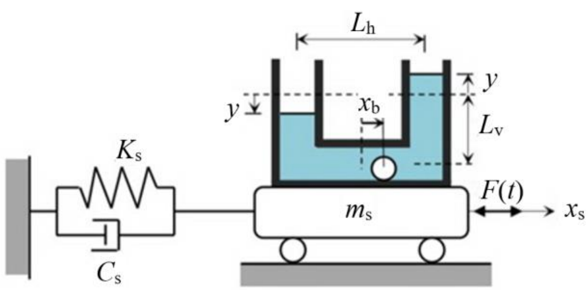

Fig. 1. 3 DOF-Model of a structure equipped with TLCBD.

Given the mass of the original structure, m_s, its horizontal stiffness, K_s, and damping of C_s, the natural frequency and the damping ratio of the structure will be obtained from the following equations:

$$\omega_s = \sqrt{\frac{K_s}{m_s}} \tag{1}$$

$$\xi = \frac{C_s}{2m_s\omega_s} \tag{2}$$

The governing equations for a structure equipped with a single TLCBD can be written as (Li et al. [2])

$$\left(m_s + m_f + \frac{J_c}{R_b^2}\right)\ddot{x}_s + C_s\dot{x}_s + K_s x_s + \alpha m_f \ddot{y} - \frac{J_c}{R_b^2}\ddot{x}_b = F_{ext}(t) \tag{3}$$

$$\alpha m_f \ddot{x}_s + m_f\ddot{y} + C_t\dot{y} + 2\rho g A y = 0 \tag{4}$$

$$-\frac{J_c}{R_b^2}\ddot{x}_s - d_{eq}\dot{y} - 2\rho g A_b y + \left(m_b + \frac{J_c}{R_b^2}\right)\ddot{x}_b + d_{eq}\dot{x}_b = 0 \tag{5}$$

The external loading, F_{ext}, in this research study has been taken as a harmonic loading with the following equation.

$$F_{ext}(t) = F_0 \sin(\omega_{ext} t) \tag{6}$$

Parameter J_c in Equations (3) and (5) shows the mass moment of inertia of the ball within the TLCBD. It is calculated based on the ball mass (m_b), and its radius (R_b) using the following equation.

$$J_c = \frac{2}{5} m_b R_b^2 \tag{7}$$

The equivalent damping coefficient, d_{eq}, is evaluated using the ball radius (R_b), and the coefficient of absolute viscosity of the liquid (ν) by the following equation (Al-Saif and Aldakan [10]).

$$d_{eq} = 6\pi R_b \nu \tag{8}$$

In a TLCBD that contains water, the liquid coefficient of absolute viscosity can be considered as $\nu = 10^{-3} Ns/m$ (Al-Saif et al.

[10]). In general, Equations (3) to (5) may be written in matrix form as follows:

$$[M^*]\{\ddot{u}\} + [C^*]\{\dot{u}\} + [K^*]\{u\} = \{F_{ext}\} \quad (9)$$

where, the displacement vector $\{u\}$ includes the three degrees of freedom of the system, namely, x_s, y, and x_b. The matrices $[M^*]$, $[C^*]$, and $[K^*]$, represent generalized mass, damping, and stiffness of a structure equipped with a single TLCBD system. These matrices can be easily evaluated from Eqs. (3) to (5).

In a multiple tuned liquid column ball damper (MTLCBD), two or more TLCBDs are employed in the system (see Figure 2). In such a system each TLCBD acts independently. Equation (9) with a new definition for its generalized mass, damping, and stiffness matrices may be employed in a MTLCBD system. The displacement vector includes all of the degrees of freedom of the system.

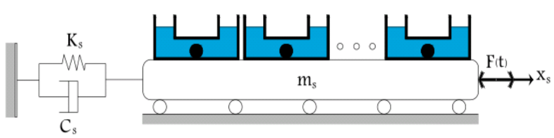

Fig. 2. A structure equipped with MTLCBD.

The definition of the mass matrix of a MTLCBD ($[M^*]_{(2n+1)*(2n+1)}$) can be found in the appendix of this paper.

Also, the elements of matrix $[M^*]$ are introduced in the appendix of the paper. The acceleration vector, $\{\ddot{u}\}$, in Equation (9) in the case of a MTLCBD system that comprises n individual dampers is defined as

$$\{\ddot{\boldsymbol{u}}\} = \{\ddot{\boldsymbol{x}}_{\boldsymbol{s}} \quad \ddot{\boldsymbol{y}}_{\boldsymbol{1}} \quad \dots \quad \ddot{\boldsymbol{y}}_{\boldsymbol{n}} \quad \ddot{\boldsymbol{x}}_{\boldsymbol{b1}} \quad \dots \quad \ddot{\boldsymbol{x}}_{\boldsymbol{bn}}\}^{\boldsymbol{T}}$$
(10)

The damping matrix $[C^*]$ of a MTLCBD is defined as follows.

$$[C^*] = \begin{bmatrix} C_s & [0]_{1\times n} & [0]_{1\times n} \\ \{0\}_{n\times 1} & [C_{ti}]_{n\times n} & [0]_{n\times n} \\ \{0\}_{n\times 1} & [-d_{eqi}]_{n\times n} & [d_{eqi}]_{n\times n} \end{bmatrix}$$
(11)

The description of the elements of matrix $[C^*]$ can be found in the appendix. The velocity vector $\{\dot{u}\}$ in Equation (9) for a MTLCBD of n individual dampers is defined as follows.

$$\{\dot{\boldsymbol{u}}\} = \{\dot{\boldsymbol{x}}_{\boldsymbol{s}} \quad \dot{\boldsymbol{y}}_{\boldsymbol{1}} \quad \dots \quad \dot{\boldsymbol{y}}_{\boldsymbol{n}} \quad \dot{\boldsymbol{x}}_{\boldsymbol{b1}} \quad \dots \quad \dot{\boldsymbol{x}}_{\boldsymbol{bn}}\}^{\boldsymbol{T}}$$
(12)

The stiffness matrix $[K^*]$of a MTLCBD can be written as

$$[\boldsymbol{K}^*] = \begin{bmatrix} \boldsymbol{K_s} & [\boldsymbol{0}]_{\boldsymbol{1\times n}} & [\boldsymbol{0}]_{\boldsymbol{1\times n}} \\ \{\boldsymbol{0}\}_{\boldsymbol{n\times 1}} & [\boldsymbol{2\rho g A_i}]_{\boldsymbol{n\times n}} & [\boldsymbol{0}]_{\boldsymbol{n\times n}} \\ \{\boldsymbol{0}\}_{\boldsymbol{n\times 1}} & [\boldsymbol{-2\rho g A_{bi}}]_{\boldsymbol{n\times n}} & [\boldsymbol{0}]_{\boldsymbol{n\times n}} \end{bmatrix}$$
(13)

The elements of $[K^*]$ are introduced in the appendix. The displacement vector $\{u\}$ is given by the following equation.

$$\{\boldsymbol{u}\} = \{\boldsymbol{x_s} \quad \boldsymbol{y_1} \quad \dots \quad \boldsymbol{y_n} \quad \boldsymbol{x_{b1}} \quad \dots \quad \boldsymbol{x_{bn}}\}^{\boldsymbol{T}}$$
(14)

The external load that represents the wind harmonic loading imposed on the structure is defined as follows.

$$\{\boldsymbol{F_{ext}}\} = \{\boldsymbol{F_0}\, \boldsymbol{sin}(\boldsymbol{\omega_{ext}}(\boldsymbol{t})) \quad \boldsymbol{0} \quad \dots \quad \boldsymbol{0}\}^{\boldsymbol{T}}$$
(16)

As mentioned earlier, the definition of the elements of matrices $[M^*]$, $[C^*]$, and $[K^*]$ can be found in the appendix of this paper.

3. Problem statement

In this section, the wind response of a 75-story skyscraper building that is equipped with a MTLCBD system is evaluated and compared to that of the same building equipped with a compatible single TLCBD. The 75-story building is modeled using an equivalent SDOF system with the mass of $m_s = 4.61\times10^7$ N.s^2/m, damping coefficient of $C_s = 1.04\times10^6$ N.s/m, and stiffness of $K_s = 5.83\times10^7$ N/m. These properties represent the first mode of the building as reported by Chang and Hsu [19] and Wu et al. [27]. The circular frequency of the structure is calculated to be $\omega_s = 1.12$ rad/s.

The wind is a stochastic natural phenomenon. In the absence of real wind data, there are different simulation methods for the generation of stochastic processes. The most commonly used method in wind engineering is based on the sum of harmonic signals with random phases. In this method, a spectral density matrix decomposition is used to calculate the weights of the harmonic signals (Chaghakaboodi, S., & Toopchi-Nezhad [28]). Although the stochastic wind loading methods are effective, they demand significant computational efforts. Given the scope of the current study, the wind loading is simulated as a harmonic excitation at resonant frequencies. This is a common technique that produces statistically reliable results (Wu et al. [27]). The wind excitation is assumed to be simulated using a harmonic function of $F_0 \sin \omega_{ext}(t)$, with $F_0 = 5\times10^5$ N, and an excitation frequency of $\omega_{ext} = 1.11$ rad/s. The frequency ratio, defined as $= \frac{\omega_{ext}}{\omega_s}$, is 0.99. This indicates a near resonance excitation for the SDOF-structure.

In this research study the performance of a single TLCBD system has been compared with a large set of different MTLCBD systems of the following properties: n = 1, 2, 3, and 4 (where, n represents the number of individual TLCBDs in the system); α_i =0.5, 0.6, 0.7, 0.8, and 0.9 (where, $\alpha_i$indicates the ratio of the length of the horizontal portion of the damper to its total length); and μ_i =0.0025, 0.005, 0.0075, 0.01, and 0.0125 (where, $\mu_i = m_{f,i}/m_s$ represents the mass ratio of each damper). The total length of the liquid columns for each damper forming the MTLCBD system is taken $L = 15.8$ m. The ratio of ball diameter to the tube diameter is selected to be $R = 0.8$ based on the results of a previous research study (Al-Saif et al. [10]). Additionally, the equivalent viscous damping ratio of each damper is assumed to be ξ_t= 0.046.

In addition to damper properties, the influence of various inherent structural damping ratios, namely, ξ = 0.01, 0.02, and 0.03 has also been investigated.

A MATLAB code was developed to solve the governing differential equations of the system with the aid of a 4th order Rung-Kutta (Chapra and Canale [29]) approach. The size of time steps was considered to be $\Delta t = 0.01$ s that is significantly smaller than the fundamental natural period of the structure. The analysis results are presented in the following section.

4. Analysis results and discussion

Figure 3 shows the displacement time history of the original structure having an effective inherent structural damping ratio of ξ = 0.01 to the harmonic wind excitation. As seen in this figure, the response amplitude is increased by time and reaches a plateau at a very slow rate due to the small inherent damping of the structure. The peak response of the structure can be decreased by 122% using a single TLCBD of 0.01, α = 0.5, and 0.99 (where ω_t represents the frequency of the damper). Figure 3 shows the displacement history of the structure equipped with such a single TLCBD. It should be noted that the displacements, x, shown in Figure 3 are normalized to the peak displacement, $x_{s,max}$, of the original structure (with no mass damper). As an alternative, the structural response may be mitigated using an equivalent dual TLCBD system in which each damper comprises a mass ratio of (0.01/2), and the same length ratio of α = 0.5 (see Figure 4). As seen in Figure 4 the response history of the two equivalent TLCBD systems is identical. Additionally, the use of four parallel TLCBDs each having a mass ratio of (0.01/4) will result in the same response history as the previous cases. Results of this study suggest that for any given constant total length, L, and length ratio, α, the use of n-TLCBDs of mass ratios of μ_i will result in the same response mitigation that is achieved by an equivalent single TLCBD of mass ratio. It should be noted that in a MTLCBD system, the individual dampers are smaller in size and contain a lower volume of liquid as compared to their equivalent single TLCBD system. The cross-section area of the damper tube and its ball diameter is decreased proportionally with decreasing the mass ratio of the damper. Space and budget limitations may justify the use of multiple small dampers instead of a single equivalent huge damper. Additionally, there is a potential for a MTLCBD system to mitigate the higher mode excitations of the original structure if some of the individual mass dampers within the MTLCBD system are tuned for the higher vibration modes of the structure.

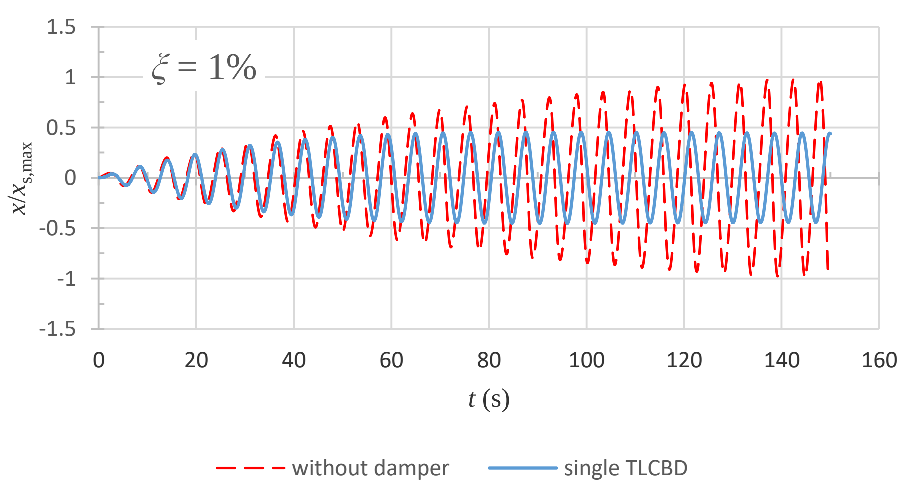

Fig. 3. Normalized displacement time history of the structure to a harmonic wind excitation of β = 0.99, a comparison between the responses of the original structure (without damper) and the structure equipped with a single TLCBD of μ = 1%, and α = 0.5.

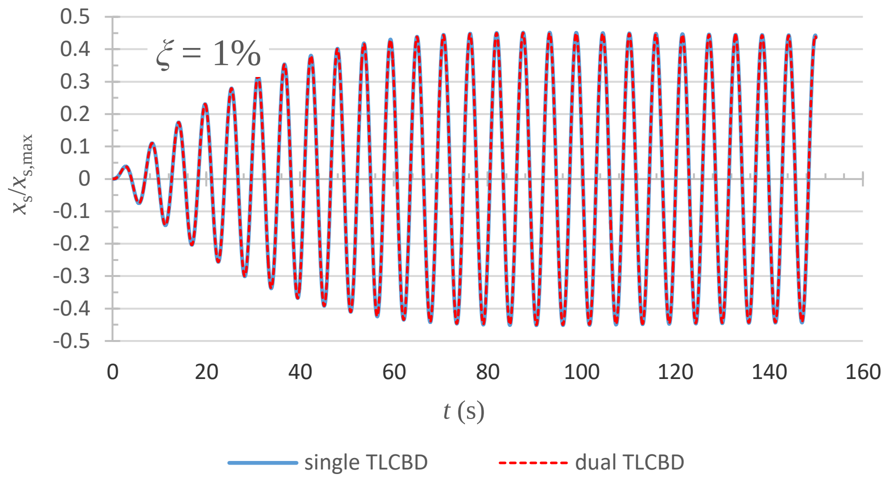

Fig. 4. Time history of normalized displacements in the structure equipped with a single TLCBD of μ = 0.01 and an equivalent dual TLCBD of μ_i = 0.005 (α = 0.5 for all dampers).

To examine the influence of the total mass ratio, μ, of the individual dampers on the performance of a MTLCBD system, a large set of dampers having various mass and geometrical properties were studied in this paper. Figure 5 shows the variation of the normalized peak displacement of the structure-TLCBD system to the total mass ratio μ (where, $\mu = \sum_{i=1}^{n} \mu_i$) of the MTLCBD system. The figure is plotted for a dual TLCBD system in which both dampers share the same values of mass and length ratio. As seen in Figure 5, for any given length ratio, α, the response mitigation capability of the damping system is increased by increasing the total mass ratio μ. This trend of behavior is consistent with that observed in single TLCBDs (Toopchi-Nezhad and Panahian [13]) and can be expected in any general MTLCBD system, regardless of the number of dampers that are employed in the group.

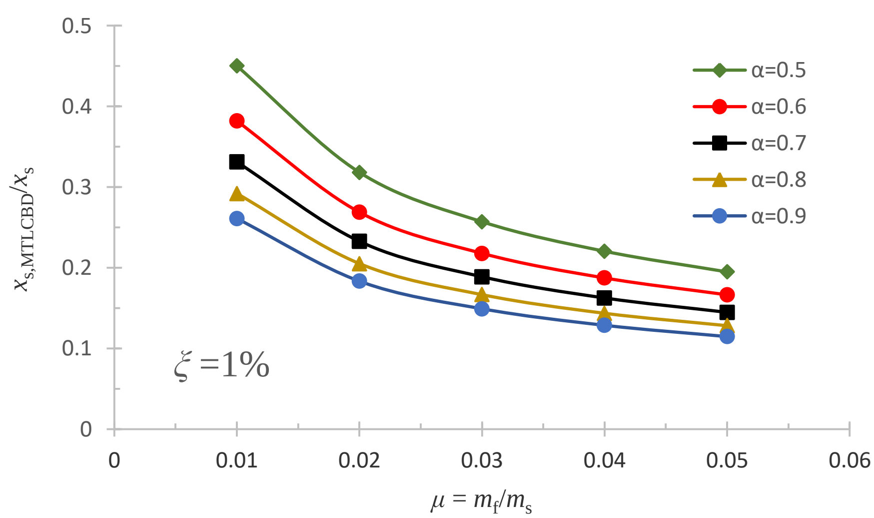

Fig. 5. The influence of total mass ratios, $\mu = \sum_{i=1}^{2} \mu_i$, on response mitigation of a dual TLCBD system with various length ratios (α).

Figure 6 shows the variation of the normalized peak displacements to the length ratio of individual dampers within a dual-TLCBD system. Both dampers of the dual system are assumed to be identical in terms of mass ratio and length ratio. As seen in this figure, for any given mass ratio, the response mitigation efficiency of the system is decreased with increasing length ratio. Results of this study indicate that this observation can be generalized for any given MTLCDB system. A recent study (Toopchi-Nezhad and Panahian [13]) confirms the applicability of this trend of behavior for single TLCBDs.

Figure 7 shows the variation of normalized peak displacement with the total mass ratio ($\mu = \sum_{i=1}^{n} \mu_i$) of a MTLCBD that comprises four individual dampers. All of the dampers within the group are of equal mass ratio (i.e., $\mu_1 = \mu_2 = \mu_3 = \mu_4$), and the same length ratio (i.e., $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.7$). The inherent damping ratio of the original structure varies between $\xi = 0.01$ to $\xi = 0.03$. The peak displacements are shown in Figure 7 are normalized with respect to the peak displacement of the original structure without supplemental dampers. As seen in Figure 7, regardless of the magnitude of the inherent structural damping, an increase in the total mass ratio of the MTLCBD improves the response mitigation of the system. Results of this study indicate that in general, the trend of behavior seen in Fig. 7 is independent of the number of dampers within the group.

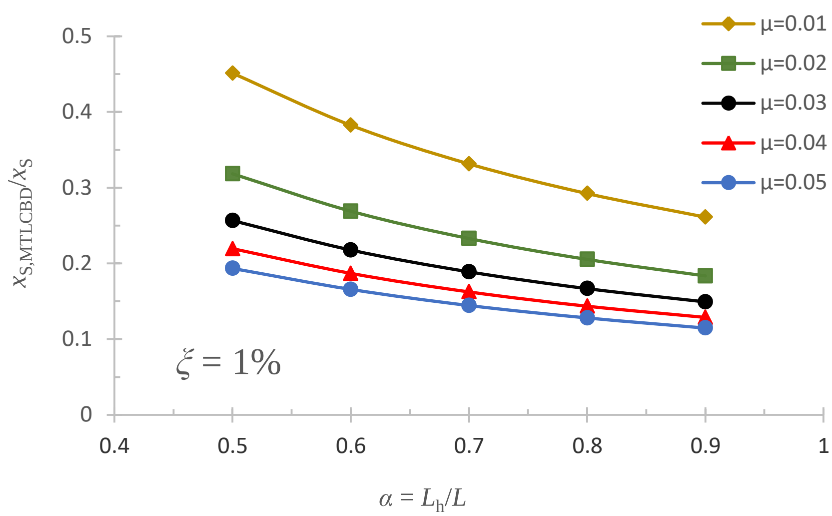

Fig.6. The influence of the length ratio of individual dampers, α, on the response mitigation of a dual TLCBD system with various mass ratios *(μ)*.

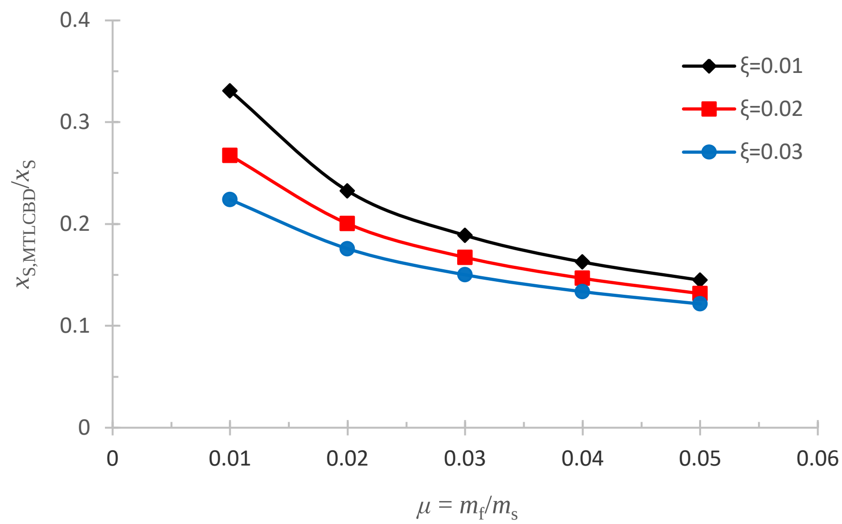

Fig. 7. The influence of total mass ratio ($\mu = \sum_{i=1}^{n} \mu_i$) in response mitigation of a foursome TLCBD in which the individual dampers share the same mass ratio, and have a length ratio of $\alpha = 0.7$.

In Figs. 8 and 9 comparisons are made between the performance of a single TLCBD with that of a dual, triad, and foursome TLCBDs. The mass ratio of individual dampers is assumed to be identical. As with the previous analysis runs the inherent damping ratio of the original structure is assumed to be 1%. In Figure 8, the length ratio of the individual dampers is $\alpha_i = 0.5$, and in Figure 9, a larger length ratio of $\alpha_i = 0.9$ has been taken into accounts. As seen in Figs. 8 and 9, for any given mass ratio, the response mitigation is improved by increasing the number of individual dampers within an MTLCBD. Moreover, the performance is boosted by increasing the length ratio. The evaluation of the optimal values of the design parameters could be the subject matter of a future study.

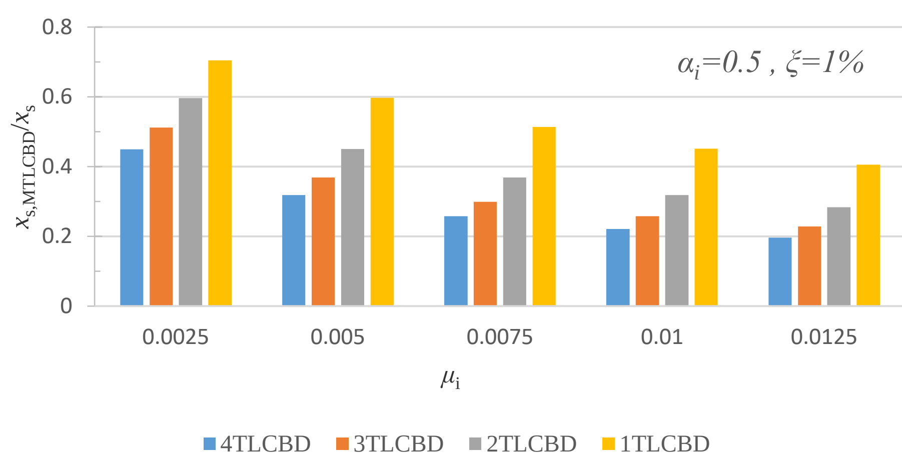

Fig. 8. The influence of the number of individual dampers (n) on the response mitigation of a MTLCBD with length ratio of $\alpha_i = 0.5$, in a structure with an inherent damping ratio of $\xi = 1\%$.

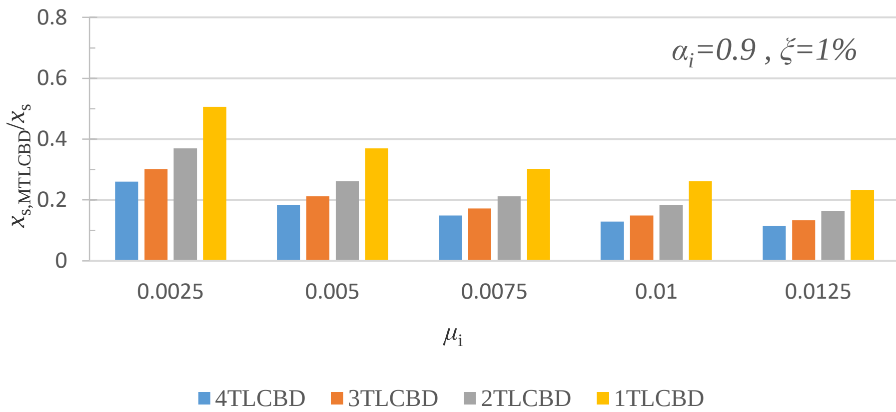

Fig. 9. The influence of the number of individual dampers (n) on the response mitigation of a MTLCBD with a length ratio of α_i = 0.9, in a structure with an inherent damping ratio of ξ =1%.

5. Conclusions

In this paper, the governing equations of motion for a multiple tuned liquid column ball damper (MTLCBD) were developed and a parametric study was conducted to examine the influence of damper design parameters, including mass ratio and length ratio on the response attenuation of the MTLCBD-system. The main results of this research study can be stated in brief as follows:

- The performance of an MTLCBD-system comprising *n* parallel individual dampers of equal mass ratios $\mu_i = \mu$, and constant length ratios of $\alpha_i = \alpha$, will be similar to that of a single TLCBD with α and $\mu = n\mu_i$.
- The response attenuation efficiency of an MTLCBD-system may be improved using either of the following strategies: i) increasing the number of individual dampers within the group; ii) increasing the mass ratio of individual dampers within the group; or iii) increasing the length ratio of dampers within the group.
- Application of an MTLCBD, instead of a single TLCBD, may be an attractive solution when space or cost limitations impose the use of a group of dampers of relatively smaller size at a more affordable cost. Additionally, the use of a MTLCBD system may be helpful to mitigate the contribution of higher vibration modes in the dynamic response of a structure to wind excitations. These may be investigated in a separate future study.

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Appendix: Supplemental equations

$$[M^*] = \begin{bmatrix} m_s + m_{f1} + m_{f2} + \ldots + m_{fn} + \frac{J_{c1}}{R_{b1}^2} + \ldots + \frac{J_{cn}}{R_{bn}^2} & [\alpha_i m_{fi}]_{1\times n} & \left[-\frac{J_{ci}}{R_{bi}^2}\right]_{1\times n} \\ \{\alpha_i m_{fi}\}_{n\times 1} & [m_{fi}]_{n\times n} & [0]_{n\times n} \\ \left\{-\frac{J_{ci}}{R_{bi}^2}\right\}_{n\times 1} & [0]_{n\times n} & \left[m_{bi} + \frac{J_{ci}}{R_{bi}^2}\right]_{n\times n} \end{bmatrix} \quad (17)$$

Elements of matrix [M*] in Equation (17):

$$[\alpha_i m_{fi}]_{1\times n} = [\alpha_1 m_{f1} \quad \alpha_2 m_{f2} \quad \cdots\cdots\cdots \quad \alpha_n m_{fn}] \quad (18)$$

$$\left[-\frac{J_{ci}}{R_{bi}^2}\right]_{1\times n} = \left[-\frac{J_{c1}}{R_{b1}^2} \quad -\frac{J_{c2}}{R_{b2}^2} \quad \cdots\cdots\cdots \quad -\frac{J_{cn}}{R_{bn}^2}\right] \quad (19)$$

$$\{\alpha_i m_{fi}\}_{n\times 1} = \{\alpha_1 m_{f1} \quad \alpha_2 m_{f2} \quad . \quad . \quad . \quad . \quad . \quad \alpha_n m_{fn}\}^T \quad (20)$$

$$[0]_{n\times n} = \begin{bmatrix} 0 & . & . & . & 0 \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ 0 & . & . & . & 0 \end{bmatrix}_{n\times n} \quad (21)$$

$$[m_{fi}]_{n\times n} = \begin{bmatrix} m_{f1} & 0 & . & . & . & . & . & 0 \\ 0 & m_{f2} & 0 & . & . & . & . & 0 \\ 0 & 0 & m_{f3} & 0 & . & . & . & 0 \\ . & & & . & & & & . \\ . & & & & . & & & . \\ . & & & & & . & & . \\ . & & & & & & . & 0 \\ 0 & . & . & . & . & . & 0 & m_{fn} \end{bmatrix} \quad (22)$$

$$\left\{-\frac{J_{ci}}{R_{bi}^2}\right\}_{n\times 1} = \left\{-\frac{J_{c1}}{R_{b1}^2} \quad -\frac{J_{c2}}{R_{b2}^2} \quad . \quad . \quad . \quad . \quad . \quad -\frac{J_{cn}}{R_{bn}^2}\right\}^T \quad (23)$$

$$\left[m_{bi} + \frac{J_{ci}}{R_{bi}^2}\right]_{n\times n} = \begin{bmatrix} m_{b1} + \frac{J_{c1}}{R_{b1}^2} & 0 & . & . & . & . & . & 0 \\ 0 & m_{b2} + \frac{J_{c2}}{R_{b2}^2} & 0 & . & . & . & . & 0 \\ 0 & 0 & m_{b3} + \frac{J_{c3}}{R_{b3}^2} & 0 & . & . & . & . \\ . & & & . & & & & . \\ . & & & & . & & & . \\ . & & & & & . & & . \\ . & & & & & & . & 0 \\ 0 & . & . & . & . & . & 0 & m_{bn} + \frac{J_{cn}}{R_{bn}^2} \end{bmatrix} \quad (24)$$

Elements of matrix [C*] in Equation (11):

$$[0]_{1\times n} = [0 \quad \ldots\ldots\ldots \quad 0]_{1\times n} \tag{25}$$

$$\{0\}_{n\times 1} = \begin{Bmatrix} 0 \\ . \\ . \\ . \\ 0 \end{Bmatrix}_{n\times 1} \tag{26}$$

$$[C_{ti}]_{n\times n} = \begin{bmatrix} C_{t1} & 0 & . & . & . & . & . & 0 \\ 0 & C_{t2} & 0 & . & . & . & . & 0 \\ 0 & 0 & C_{t3} & 0 & . & . & . & 0 \\ . & & & . & & & & . \\ . & & & & . & & & . \\ . & & & & & . & & . \\ . & & & & & & . & 0 \\ 0 & . & . & . & . & . & 0 & C_{tn} \end{bmatrix} \tag{27}$$

$$\left[-d_{eqi}\right]_{n\times n} = \begin{bmatrix} -d_{eq1} & 0 & . & . & . & . & . & 0 \\ 0 & -d_{eq2} & 0 & . & . & . & . & 0 \\ 0 & 0 & -d_{eq3} & 0 & . & . & . & 0 \\ . & & & . & & & & . \\ . & & & & . & & & . \\ . & & & & & . & & . \\ . & & & & & & . & 0 \\ 0 & . & . & . & . & . & 0 & -d_{eqn} \end{bmatrix} \tag{28}$$

$$\left[d_{eqi}\right]_{n\times n} = \begin{bmatrix} d_{eq1} & 0 & . & . & . & . & . & 0 \\ 0 & d_{eq2} & 0 & . & . & . & . & 0 \\ 0 & 0 & d_{eq3} & 0 & . & . & . & 0 \\ . & & & . & & & & . \\ . & & & & . & & & . \\ . & & & & & . & & . \\ . & & & & & & . & . \\ 0 & . & . & . & . & . & 0 & d_{eqn} \end{bmatrix} \tag{29}$$

Elements of matrix [K*] in Equation (13):

$$[2\rho g A_i]_{n\times n} = \begin{bmatrix} 2\rho g A_1 & 0 & . & . & . & . & . & 0 \\ 0 & 2\rho g A_2 & 0 & . & . & . & . & 0 \\ 0 & 0 & 2\rho g A_3 & 0 & . & . & . & 0 \\ . & & & . & & & & . \\ . & & & & . & & & . \\ . & & & & & . & & . \\ . & & & & & & . & 0 \\ 0 & . & . & . & . & . & 0 & 2\rho g A_n \end{bmatrix} \tag{30}$$

$$[-2\rho g A_{bi}]_{n\times n} = \begin{bmatrix} -2\rho g A_{bi} & 0 & . & . & . & . & . & 0 \\ 0 & -2\rho g A_{b2} & 0 & . & . & . & . & 0 \\ 0 & 0 & -2\rho g A_{b3} & 0 & . & . & . & 0 \\ . & & & . & & & & . \\ . & & & & . & & & . \\ . & & & & & . & & . \\ . & & & & & & . & . \\ 0 & . & . & . & . & . & 0 & -2\rho g A_{bn} \end{bmatrix} \quad (31)$$