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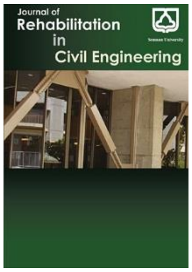

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Probabilistic Active Control of Structures Using a Probabilistic Fuzzy Logic Controller

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ABSTRACT

Because uncertainty is inherent in engineering structures, it is essential to improve the procedures of structural control. The present study focuses on applying a probabilistic fuzzy logic system (PFLS) in active tendons for the covariance response control of buildings. In contrast to an ordinary fuzzy logic system, PFLS integrates the probabilistic theory into a fuzzy logic system that can handle the linguistic and stochastic uncertainties existing in the process. To investigate the proficiency of the proposed controller, a single degree of freedom (SDOF) system and a three-story multiple degree of freedom (MDOF) system with different arrangements of tendons on the floors are considered. The structures are subjected to a random dynamic load modeled using Gaussian white noise, and the modeling parameters such as damping, stiffness, and mass are considered to be random Gaussian samples with a dispersion coefficient of 10%. The results of the proposed intelligent control scheme are compared with those of an uncontrolled structural model and a linear quadratic regulator (LQR) controller model. The numerical results reveal that the probabilistic fuzzy logic controller (PFLC) is more efficient than the LQR controller in decreasing the structural covariance responses. Moreover, the maximum and minimum reductions in displacement responses for the MDOF structures are, respectively, about 36% and 12.5%compared to the LQR controller. It is also showed that the PFLC is more accurate because it includes stochastic uncertainty.

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1. Introduction

The concept of likelihood has become a preferred approach to deal with unreliability and unpredictability. This has been brought into question with the development of the fuzzy logic systems [1]. A significant advantage of conventional fuzzy logic systems (FLSs) is their ability to map uncertain information to a linguistic domain. However, fuzzy logic systems cannot handle uncertainty in practical applications [2, 3]. Uncertainty results from lack of information. The aspects of uncertainty are inherently different and should be addressed by process modeling. One aspect is linguistic or non-stochastic uncertainty and the other one is statistical uncertainty, which has been represented using the possibility theory. Fuzzy logic is a mechanism to handle and manipulate linguistic uncertainty. This feature enables the use of connoisseur science in the form of rules to transform unpredicted membership functions (MFs) for input and/or output segments of rules; however, the knowledge itself is uncertain. Stochastic uncertainty affects the appearance of an event in the future. This kind of uncertainty gives the likelihood of the results that may or may not happen.

In the past decades, the implementation of intelligent control schemes such as fuzzy logic has been improved. Because of the capacity of fuzzy logic to manage linguistic uncertainties, it has been widely used. In 1965, Zadeh proposed the fuzzy logic of type-1 [4]. It is evident that the information available for the creation of fuzzy rules contains unreliable data, but this unreliability is not assumed in the fuzzy logic methods of type-1. Because the result of a fuzzy logic system of type-1 is a single number, it requires some measures of dispersion to increase the understanding about its uncertainties [5]. To overcome this shortcoming, Zadeh presented the fuzzy logic system of type-2 in 1975 [6] to provide a measure of dispersion. This is now considered to be essential for the design of structures that contain linguistic uncertainty. Membership function of a type-1 fuzzy set is two-dimensional, so the membership value is a crisp quantity within [0, 1]. In contrast, type-2 fuzzy sets have three-dimensional membership functions, i.e., there is a secondary membership value for each primary element of a fuzzy set within [0, 1]. This secondary membership value should be a constant value or a function. A major advantage of type-2 fuzzy sets is their capability to manage linguistic unreliability. Mendel et al. [5, 7-10] improved the basic concepts related to type-2 fuzzy sets. To decrease the computational complexity of a type-2 FLS, an interval type-2 fuzzy logic system (IT2FLS) was produced [11-13]. For simplicity, IT2FLS assumes that the secondary membership is one. To define the unreliability boundaries in IT2FLS, upper and lower membership functions are considered in interval type-2 fuzzy sets [14].

The application of fuzzy control algorithms has increased for civil engineering structures. Al-Dawod et al. [15-19] used an active tuned mass damper (ATMD) on the top floor of buildings with 5 and 76 stories. These structural models have been studied under various loading types, including earthquakes and wind loads. Pourzeynali et al. [20] analyzed an 11-story realistic shear building for various types of earthquakes. They stated that FLS is very beneficial in minimizing the structural response in comparison with the linear quadratic regulator (LQR) and linear quadratic Gaussian (LQG) methods. Liang and Mendel [11] proposed an approach for estimating input and antecedent operations inIT2FLS by focusing on the upper and lower membership functions. Golnargesi et al. [21, 22] demonstrated that IT2FLS is efficient in reducing the structural responses. They used IT2FLSs to specify an active control force. Previous works show that the current fuzzy control algorithms have not considered stochastic uncertainty. Zabihi and Ghanooni-Bagha [23] introduced a semi-active controller using the combination of thermal exchange and intelligent fuzzy logic controller.

For addressing the insufficiencies of probability theory [1, 24], it is useful to integrate FLS with probabilistic features to process uncertainties that include both fuzzy and probabilistic aspects. Loginov [25] considered a connection between fuzzy and probabilistic sets. He recommended that the membership function can be described as a dependent likelihood. This integration has been studied by others [26-28]; however, they only studied the relation between randomness and fuzziness, which cannot be directly applied to engineering applications. A probabilistic fuzzy logic system (PFLS) was proposed by Meghdadi et al. [27]. In this system, a true value has a specified quantity in the interval [0, 1] that is called “degree of truth” with a “likelihood of truth” that is identified by a likelihood value or likelihood distribution function. Probabilistic fuzzy logic utilizes a probabilistic three-dimensional membership function to show probabilistic uncertainties. The probabilistic membership function contains three segments: the input signal, the fuzzy degree, and the related likelihood. The likelihood segment of the probabilistic membership function can be used to describe the probabilistic unreliability [29]. To present stochastic uncertainties in an FLS, the primary membership function should be a set of fuzzy numbers in [0, 1] and the secondary membership function is related to the probability density function (PDF). Several studies have examined the theoretical concepts of PFLS. The probabilistic fuzzy set was suggested by Liu and Li [2], who introduced probability theory to the conventional fuzzy field to describe the random property of membership degree. The fuzzy degrees in a probabilistic fuzzy set become the probabilistic parameters, which allow it to obtain both probabilistic and fuzzy unreliability. Initially, probabilistic fuzzy sets were used to approximate functions and control problems. Liu et al. [30] and Li et al. [31] introduced a concise review and easy tutorial on the improvement of PFLS when there exist both probabilistic and fuzzy unreliability data. They integrated PFLS into a neural network to develop its efficiency under time-varying conditions. For example, the stochastic nature of wind makes the estimating of wind speed a complex problem. Zhang et al. [32] designed a practical wind speed forecasting pattern utilizing probabilistic fuzzy theory. Simulations using real collected data for wind speed showed that their wind speed estimation model performs better than the conventional fuzzy types, interval type-2 fuzzy method, and neural networks. Huang et.al [33] introduced a novel procedure to modify PFLS characteristics based on general probabilistic fuzzy sets (GPFS). Shaheen et al. [34] proposed an adaptive probabilistic TSK fuzzy proportional-integral-derivative (APTSKF-PID) controller to deal with linguistic and stochastic complexities of nonlinear dynamic operations. The results of two uncertain systems showed that the proposed scheme can handle both uncertainties better than the adaptive TSK fuzzy PID (ATSKF-PID) controller. Nguyen [35] introduced a fuzzy logic system along with stochastic uncertainty. In the proposed fuzzy logic system, the consequent part of the rules considered all feasible assumptions with different likelihoods.

To the best of the authors' knowledge, no study has been dedicated to probabilistic active control in which a probabilistic fuzzy logic controller (PFLC) as a novel technique has been applied to civil engineering structures. The present study has aimed to use a PFLC, which is a combination of fuzzy and stochastic theories, for controlling the response of structures subject to a dynamic random load modeled by Gaussian white noise. The mass, stiffness, and damping variables of structures are assumed to be random Gaussian variables. The dispersion coefficient of random parameters is assumed to be 10%. To verify the efficiency of the proposed control approach, active tendon control is applied to two different structural models. One of these models has a single degree of freedom (SDOF), and has been empirically investigated by Chung et al. [36]. The other one has a three-story multiple degree of freedom (MDOF) system, which has also been investigated by Chung et al. [37]. The results of the proposed PFLC technique are compared with those of an uncontrolled structural model and the LQR controller. It is observed that the proposed PFLC has better efficiency for minimizing the structural covariance responses. The advantages of the intelligent controller over a classic controller are highlighted in the present study.

2. Research significance

In the last decades, the fuzzy logic control method has been considered as an intelligent controller in engineering systems. Despite the significant efficiency of FLS to overcome linguistic uncertainties, it must be noted that FLS is not effective in situations where the controlled systems are subjected to stochastic uncertainties. It is more beneficial when the probability theory can be combined with the fuzzy theory. Several uncertainties result from structural characteristics, mathematical model insufficiency, dynamic characteristic of earthquake excitation, and lack of information in civil engineering structures. These uncertainties reduce the efficiency of the control systems. Thus, it is essential to apply PFLS in controlled structures. This research utilizes a probabilistic fuzzy logic system to reduce the covariance responses of controlled structures. The findings of this study show that PFLS is more beneficial in reducing structural covariance responses. The probabilistic fuzzy logic controller can reduce the structural covariance responses by applying linguistic and stochastic uncertainties which resulted in increasing of system reliability.

3. Problem formulation

The uncertainty in the movement equation of a building can be considered as a random variable Δ. This variable is a q-dimensional vector with the mean μ_Δ, covariance σ_Δ, and a joint likelihood distribution. Movement equations for an n-degree of freedom building can be represented by the framework of state-space as [38]:

$$\dot{z} = A(\Delta)z + B(\Delta)u + E(\Delta)w \tag{1}$$

and the measurement equation is [38]:

$$y = C(\Delta)z + D(\Delta)u + F(\Delta)v \tag{2}$$

where z is the $2n$-dimensionalvector of velocity and displacement, A denotes the $2n \times 2n$system plant matrix, u is the r-dimension input vector, B is a $2n \times 2n$ matrix describing the position of the applied control forces, w denotes the l-dimensional vector of excitation, E denotes a $2n \times l$ matrix defining the excitation effects on the building, y is an m-dimension measuring vector, C is the$m \times 2n$ output matrix of the combination of measured states, D is the $m \times r$ feed-through matrix, F is an $m \times m$ matrix that affects the measurement noise, and v is the$m$-dimensional vector of noise measuring. $[w' \; v']'$ is the white noise vector of zero mean and autocorrelation function as [38]:

$$E\begin{bmatrix} w \\ v \end{bmatrix} = 0 \tag{3}$$

$$E\left[\begin{Bmatrix} w(t) \\ v(t) \end{Bmatrix}\{w'(t+\tau) \quad v(t+\tau)\}\right] = 2\pi S\delta(\tau) \quad (4)$$

where $E[\cdot]$denotes the mathematical expectation, S is the matrix of uniform spectral density, and δ is the Dirac function. Non-white noise is integrated into the equation by reinforcing the movement formulation with an effective perturbation filter [39].

4. Covariance control fundamentals

Studying the improvement of the covariance control theory emerged in the 1980s [40]. The main concept of covariance control is to obtain a state covariance by solving the closed-loop feedback methodology. Thus, it is needed to describe the system specifications in the form of covariance, variance, or root mean square (RMS).The essential reason for improving the theory of covariance control is the expression of engineering systems in terms of variance [41]. The basic quantities that are the product of unpredictable inputs and the primary conditions exerted one at a time to the mechanism are presented in Eqs. (1) and (2) [41]. Let the system be driven only by u, so an impulsive input is inserted into the i^{th} input source, and other data are assumed to be zero, and the results are added as shown in Eq. (5):

$$u_i = \mu_i \delta(t) \quad (5)$$

where μ_i is the intensity of the strike and $\delta(t)$ denotes the Dirac function. The linear matrix in Eq. (6) describes the state covariance matrix X [41].

$$0 = XA^T + AX + BUB^T \quad (6)$$

where $U = \begin{bmatrix} \mu_i^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mu_{n_u}^2 \end{bmatrix}$ is the square of the matrix of input impulsive disturbance magnitudes (intensity). The state covariance matrix $X_w$is produced due to exerting impulses one at a time to each of the disturbance sources. Moreover, ($w_i = \omega_i \delta(t)$) [41] satisfies Eq. (7) as:

$$0 = X_w A^T + AX_w + DWD^T \quad (7)$$

where $W = \begin{bmatrix} \omega_i^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_{n_w}^2 \end{bmatrix}$ is the square of the matrix of disturbance magnitudes (intensity). To complete the possibilities, if the preliminary conditions are exerted one by one, state covariance matrix X_x satisfies Eq. (8) as [41]:

$$0 = X_x A^T + AX_x + X_0 \quad (8)$$

where $X_0 = \begin{bmatrix} x_{o_1}^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & x_{o_{n_x}}^2 \end{bmatrix}$ is the square of the matrix of the initial condition intensities. Eventually, Eq. (9) represents the full results of all excitations exerted one at a time [41].

$$0 = X_{uwx} A^T + AX_{uwx} + BUB^T + DWD^T + X_0 \quad (9)$$

where $X_{uwx} = X + X_w + X_x$. This describes the sum effect of excitation from initial conditions and impulsive inputs in $u(t)$ and $w(t)$ that have been applied one at a time. Matrix X_w includes data about the excitation of the system due to impulsive disturbance and X includes data about the excitation of the system due to impulsive inputs in $u(t)$. These basic concepts are the foundation for the improvement in the theory of covariance control [42]. For a wide class of control techniques, as in Eq. (1), the closed-loop state is shown as follows [38]:

$$\dot{\tilde{z}} = A_{cl}(\Delta)\tilde{z} + E_{cl}\begin{Bmatrix} w \\ v \end{Bmatrix} \quad (10)$$

where $\tilde{z}$ denotes the state vector, $A_{cl}(\Delta)$ is the plant matrix of the closed-loop state, and matrix E_{cl} denotes the effect of estimated noise on the closed-loop technique. By assuming that the structural state parameters are completely assessable, the state-space matrix of the closed-loop feedback controller is given as follows [38]:

$$A_{cl}(\Delta) = A(\Delta) - B(\Delta)K \quad (11)$$

where K denotes the feedback gain matrix. The Lyapunov mathematical problem is showed in Eq. (12). For the linear dynamic mechanism in Eq. (10), the covariance matrix of the response can be written as the outcome of the Lyapunov mathematical problem [38].

$$\dot{\Sigma}_{\tilde{z}} = A_{cl}\Sigma_{\tilde{z}} + \Sigma_{\tilde{z}}A'_{cl} + 2\pi E_{cl}SE'_{cl} \quad (12)$$

With the initial conditions $\Sigma_{\tilde{z}}(0) = \Sigma_0$. The stationary covariance matrix can be obtained by solving Eq. (12) using $\dot{\Sigma}_{\tilde{z}} = 0$ [38].

5. Structural model

5.1. Structural model of single degree of freedom

An SDOF structural model with active tendons is depicted in Fig. 1. The prestress force of each tendon during a static state is denoted by R.

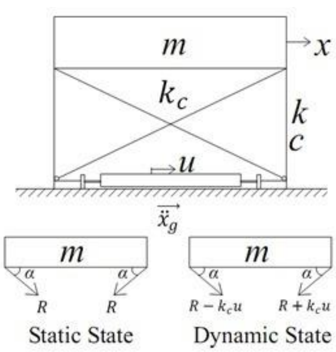

Fig. 1. SDOF model with active tendons [36].

The movement equation for uncontrolled and controlled SDOF buildings with active tendons is presented in Eqs. (13) and (14), respectively, as:

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_g \quad (13)$$

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_g - 4k_c u\cos\alpha \quad (14)$$

where x denotes the horizontal relative displacement, and u denotes the activator situation; c, k, and m denote, respectively, the damping, stiffness, and mass of the building and $\ddot{x}_g$ is the ground acceleration.

Control force is generated by pulling a collection of active tendons and liberating the others [38].Then, the state-space description of the movement equations is:

$$\dot{z} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & -\dfrac{c}{m} \end{bmatrix} z + \begin{bmatrix} 0 \\ -\dfrac{4k_c\cos\alpha}{m} \end{bmatrix} u + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \ddot{x}_g \quad (15)$$

where $z = [x \quad \dot{x}]'$. In this study, the ground acceleration is considered as Gaussian white noise. Table 1 lists the SDOF model parameters [38].

Table 1. Model parameters of SDOF structure [38].

	Mean (μ)	Standard deviation (σ)
c (lb-s/in)	9.02	0.902
k (lb/in)	7934	793.4
m (lb-s^2/in)	16.69	1.669
k_c(lb/in)	2124	0
α (degrees)	36	0

5.2. Multiple degrees of freedom

The MDOF systems with various tendon controller placements are used in more complex structures. Each system is a three-story structure with a single-bay exposed to one-dimensional earthquake excitation. Figure 2 shows the placement of three tendons. Case A features tendons only on the first floor. Cases B and C have tendons on all floors. In case C, activator devices are situated on the bottom floor [43]. Figure 3 shows the dynamic tendon forces for prestress forces (denoted by *R*). Equation (16) presents the mass, stiffness, and damping matrices for a simple shear frame model. Dynamic equations of the movement are written in a state-space form as Eqs. (17), (18), and (19) for cases A, B, and C, respectively.

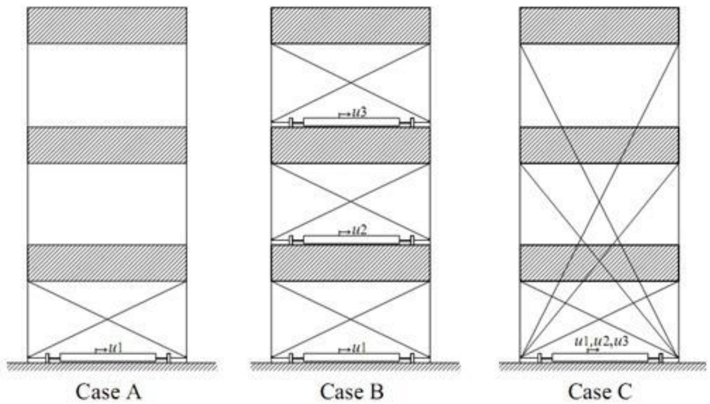

Fig. 2. Three MDOF systems with active tendons [37].

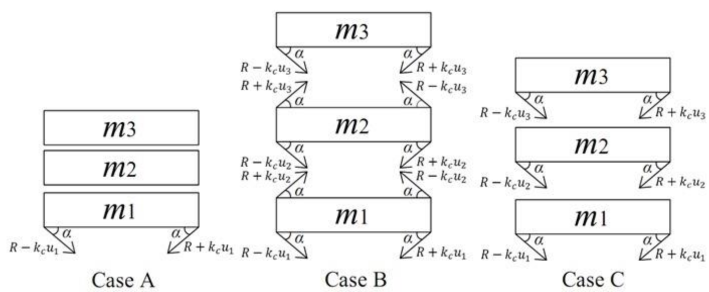

Fig. 3. Tendon forces in dynamic state for all cases [43].

$$[M]=\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, [C]=\begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix}, [K]=\begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$$
(16)

$$[M]\ddot{X}+[C]\dot{X}+[K]X=-[M][1]\ddot{X}_g-4K_c\cos\alpha\begin{bmatrix} u_1 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

$$[M]\ddot{X}+[C]\dot{X}+[K]X=-[M][1]\ddot{X}_g+4K_c\cos\alpha\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{18}$$

$$[M]\ddot{X}+[C]\dot{X}+[K]X=-[M][1]\ddot{X}_g-4K_c\begin{bmatrix} u_1\cos\alpha \\ u_2\cos\beta \\ u_3\cos\theta \end{bmatrix} \tag{19}$$

In the above equations c_i, k_i, and m_i are damping, stiffness, and mass, respectively, related to the *ith* floor of the structure. The situation of the activator is denoted by u. Equations (17)- (19) can be formulated in a matrix framework as:

$$M_s\ddot{x} + C_s\dot{x} + K_s x = B_s u - M_s\Gamma_s\ddot{x}_g \qquad (20)$$

If the state vector is defined as $z = \left[x' \quad \dot{x}'\right]'$, Eq. (1) can be expressed in the framework of the state-space matrices:

$$A = \begin{bmatrix} 0 & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix},\ B = \begin{bmatrix} 0 \\ M_s^{-1}B_s \end{bmatrix},\ E = \begin{bmatrix} 0 \\ -\Gamma_s \end{bmatrix} \qquad (21)$$

In this study, the simulation parameters, i.e., damping, stiffness, and mass, are Gaussian random variables. A dispersion coefficient of 10% is considered for the random variables, and the controller is assumed to be definite. Table 2 lists the model parameters as reported by Chung et al. [37].

Table 2. Three-story structure model parameters [37]

	Mean (μ)	Standard deviation (σ)
c_1 (lb-s/in)	2.6	0.26
c_2 (lb-s/in)	6.3	0.63
c_3 (lb-s/in)	0.35	0.035
k_1 (lb/in)	5034	503.4
k_2 (lb/in)	10965	1096.5
k_3 (lb/in)	6135	613.5
m_1 (lb-s^2/in)	5.6	0.56
m_2 (lb-s^2/in)	5.6	0.56
m_3 (lb-s^2/in)	5.6	0.56
k_c (lb/in)	2124	0
θ (degrees)	65	0
β (degrees)	55	0
α (degrees)	36	0

6. Controller method

6.1. LQR controller

A state feedback LQR controller is introduced by modeling the ground acceleration as Gaussian white noise and reducing the efficiency index in both the SDOF and MDOF systems. The quadratic performance indices for the SDOF and MDOF systems are shown in Eqs. (22) and (23), respectively.

$$J = \lim_{T\to\infty}\frac{1}{T}E\left[\int_0^T (kx^2 + \gamma k_c u^2)dt\right] \qquad (22)$$

$$J = \lim_{T\to\infty}\frac{1}{T}E\left[\int_0^T (x'K_s x + \gamma k_c u^2)dt\right] \qquad (23)$$

where γ denotes the control scheme factor. With increasing γ, more weight is given to incoming energy, while as γ decreases, more weight is imposed on the strain energy. An infinite value for γ denotes the uncontrolled case [37].

6.2. Probabilistic fuzzy logic controller

As described for an ordinary FLS, a PFLS contains four significant segments: probabilistic fuzzification, fuzzy rules, probabilistic fuzzy inference engine, and probabilistic defuzzification (Fig. 4).

The rule base develops from the expert knowledge to specify a relationship between input domain$X_1 \times X_2 \times \cdots \times X_n \in R^n$ and output domain$Y \in R$. These rules are in the form of an IF-THEN expression as follows[3]:

Rule i: If x_1 is $\tilde{A}_{1,i}$ and x_2 is $\tilde{A}_{2,i}$ $\cdots$and x_n is $\tilde{A}_{n,i}$, then y is $\tilde{B}_i$ (24)

where $\tilde{A}_{j,i}$ (j = 1, 2,… ,n) (i= 1, 2,…, J) is a priori in the view of the j^{th} input variable x_j, in the i^{th} rule, and $\tilde{B}_i$ is a subsequent section associated with the output parameter y [3]. In contrast to traditional fuzzy sets, antecedent parts $\tilde{A}_{j,i}$ and consequent parts $\tilde{B}_i$ are probabilistic fuzzy sets (PFSs) in PFLS.

6.2.1. Probabilistic fuzzification

One of the major differences between usual fuzzy logic and probabilistic fuzzy logic is that the fuzzification and defuzzification methods in a probabilistic fuzzy logic system focus on PFS. Thus, the significance of a PFS is introduced as follows:

- Description 1(PFS): The probabilistic fuzzy set $\tilde{A}$ can be presented by a likelihood space $(U_x, \wp, p)$, where U_x is the collection of all probable occurrences, $U_x = [0,1]$, and $\wp$ is a σ-field. The input parameter is $x \in X$ and the fuzzy degree is $\mu \in [0,1]$.

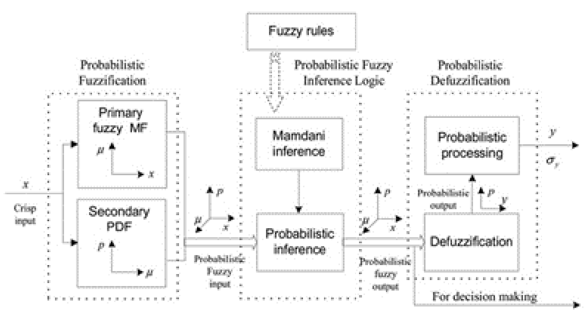

Fig. 4. Structure of PFLS [3].

For any μ in Ux, the PDF can be defined by $\wp$ as [3]:

$$p(\mu) \geq 0, \quad \int_0^1 p(\mu) d\mu = 1 \tag{25}$$

PFS can be demonstrated as Eq. (26) as shown in Fig. 5.

$$\tilde{A} = \bigcup_{x \in X} (U_x, \wp, p) \tag{26}$$

An important idea in the probabilistic fuzzy approach is that a PFS is a combination of primary MF and secondary PDF [31]. For an input x, its fuzzy membership degree $\mu(x)$ is a statistical parameter with the secondary PDF.

The primary MF of PFSs $\tilde{A}_{j,i}$ and $\tilde{B}_i$ can be employed as [32]:

$$\mu(x_{j,i}) = \exp\left(-\frac{(x_j - c_{j,i})^2}{2\xi_{j,i}^2} \right) \tag{27}$$

where $\mu(x_{j,i})$ represents the primary fuzzy membership degree, and $\xi_{j,i}$, and $c_{j,i}$ are, respectively, the width and center of PFS. To consider the ability to handle stochastic uncertainty, the secondary PDF is presented by the randomization of the variables in the primary membership function (Fig. 5).In this paper, the center $c_{j,i}$ has been randomized to follow a Gaussian distribution. The secondary PDF can be written as [32]:

$$p_{\tilde{A}_{j,i}}(\mu_{j,i}, x_j) = \frac{1}{2\sqrt{2\pi}\mu_{j,i}\sigma_{j,i}} \sqrt{\frac{-2\xi_{j,i}^2}{\ln \mu_{j,i}}} \times \left(\exp\left(-\frac{\left(\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}} + x_j - u_j\right)^2}{2\sigma_{j,i}^2} \right) + \exp\left(-\frac{\left(-\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}} + x_j - u_j\right)^2}{2\sigma_{j,i}^2} \right) \right) \tag{28}$$

where $\mu_{j,i} \in [0,1]$ is the primary fuzzy degree parameter, $p_{\tilde{A}_{j,i}}(\mu_{j,i}, x_j)$ denotes the PDF, and $\sigma_{j,i}$ and $u_{j,i}$ are, respectively, the standard deviation and mean of the Gaussian distribution in the view of $c_{j,i}$. The full details of Eq. (28) are presented in Appendix A [32].

6.2.2. Operation techniques of PFS

- Description 2 (union operation of PFS): probabilistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ can be written as [3]:

$$\tilde{A} = \bigcup_{x\in X}(U_{\tilde{A}}, \wp_{\tilde{A}}, p_{\tilde{A}}), \quad \tilde{B} = \bigcup_{x\in X}(U_{\tilde{B}}, \wp_{\tilde{B}}, p_{\tilde{B}}) \tag{29}$$

The union operation of $\tilde{A}$ and $\tilde{B}$ can be expressed as:

$$\tilde{A}\cup\tilde{B} = \bigcup_{x\in X}(U_{\tilde{A}\vee\tilde{B}}, \wp_{\tilde{A}\vee\tilde{B}}, p_{\tilde{A}\vee\tilde{B}}) \tag{30}$$

with

$$p(\mu_{\tilde{A}\vee\tilde{B}}) \geq 0$$

$$p(\mu_{\tilde{A}\vee\tilde{B}}) = p(\mu_{\tilde{A}} \vee \mu_{\tilde{B}}) = p(\mu_{\tilde{A}})P(\mu_{\tilde{B}}) + P(\mu_{\tilde{A}})p(\mu_{\tilde{B}}) \tag{31}$$

$$\int_0^1 p(\mu_{\tilde{A}\vee\tilde{B}}) = 1 \tag{32}$$

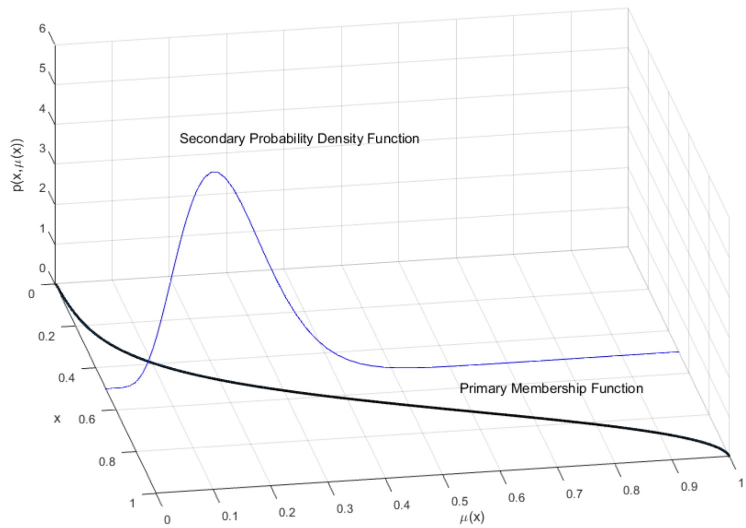

Fig. 5. Probabilistic fuzzy set.

The cumulative distribution functions of fuzzy degrees $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ are $P(\mu_{\tilde{A}})$ and $P(\mu_{\tilde{B}})$, respectively.

- Description 3 (intersection of PFS): The intersection ofprobabilistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ can be written as [3]:

$$\tilde{A}\cap\tilde{B} = \bigcup_{x\in X}(U_{\tilde{A}\wedge\tilde{B}}, \wp_{\tilde{A}\wedge\tilde{B}}, p_{\tilde{A}\wedge\tilde{B}}) \tag{33}$$

where

$$p(\mu_{\tilde{A}\wedge\tilde{B}}) \geq 0$$

$$p(\mu_{\tilde{A}\wedge\tilde{B}}) = p(\mu_{\tilde{A}} \wedge \mu_{\tilde{B}}) = p(\mu_{\tilde{A}})(1-P(\mu_{\tilde{B}})) - P(\mu_{\tilde{A}})p(\mu_{\tilde{B}}) + p(\mu_{\tilde{B}}) \tag{34}$$

$$\int_0^1 p(\mu_{\tilde{A}\wedge\tilde{B}}) = 1 \tag{35}$$

6.3. Probabilistic fuzzy inference engine

As previously described, the PFS consists of continuous PDF, which is identified as Eq. (28). Therefore, the inference engine of PFLS can be obtained under a probabilistic framework. The probabilistic fuzzy inference engine is a nonlinear representation in the input domain $X_1\times X_2\times\cdots\times X_n$ and output domain Y as follows [3, 32]:

$$R_{\tilde{A}_{1,i}\times\cdots\times\tilde{A}_{n,i}\to\tilde{B}_i}(x, y) \tag{36}$$

where $\tilde{A}_{1,i}\times\cdots\times\tilde{A}_{n,i}$ represent the Cartesian product of $\tilde{A}_{1,i},\cdots,\tilde{A}_{n,i}$. For an input x =

$(x_1,\ldots,x_n)$ and associated membership function $\mu_X(x)$, the fuzzy relation set R_i in Y can be computed as [32]:

$$\mu_{R_i}(y) = \mu_{A_{1,i}} \circ \mu_{A_{2,i}} \circ \ldots \circ \mu_{A_{M,i}} \circ \mu_{B_i} \tag{37}$$

where $\mu_{A_{j,i}}$ and μ_{B_i} define the fuzzy membership. The symbol "$\circ$" indicates a t-norm functioning [32]. The minimum functioning has been exerted in this paper. Equation (38) denotes the probabilistic fuzzy inference in the i^{th} rule[32].

$$\mu_{R_i}(y) \sim p_{R_i} = p(\mathop{T}_{j=1}^{n} \mu_{A_{j,i}}(y_i) * \mu_{B_i}(y)) = p(\min(\mu_{A_{1,i}}(x_1),\ldots,\mu_{A_{M,i}}(x_M)) * \mu_{B_i}(y)) \tag{38}$$

where p_{R_i} denotes the PDF of $\mu_{R_i}(y)$, and T and * indicate the minimum functioning [32]. In Description 3, the PDF of the input firing level can be presented as [3, 32]:

$$\begin{aligned} p(\mu_{A_i}(x)) &= p(\mathop{T}_{j=1}^{n} \mu_{\tilde{A}_{j,i}}(x_j)) \\ &= p(\min(\mu_{\tilde{A}_{1,i}}(x_1),\ldots,\mu_{\tilde{A}_{n,i}}(x_n))) \\ &= \sum_{t}^{n-1} p(\mu_{\tilde{A}_{t,i}}(x_t)) \prod_{j=t+1}^{n} (1 - P(\mu_{\tilde{A}_{j,i}}(x_j))) \\ &+ \sum_{t=1}^{n-1} p(\mu_{\tilde{A}_{t,i}}(x_t)) \left[- \sum_{k=t+1}^{n} p(\mu_{\tilde{A}_{k,i}}(x_k)) \times \prod_{\substack{j=t+1 \\ j\neq k}}^{n} (1 - P(\mu_{\tilde{A}_{j,i}}(x_j)) \right] \\ &+ p(\mu_{\tilde{A}_{n,i}}(x_n)) \end{aligned} \tag{39}$$

The firing level of the input variable in the i^{th} rule is $\mu_{A^i}(x)$. $P(\mu_{\tilde{A}_{j,i}}(x_j))$ is the cumulative distribution function (CDF) of $\mu_{\tilde{A}_{j,i}}$. The PDF of inference PFS $p(\mu_{R_i}(y))$ can be written as [3, 32]:

$$\begin{aligned} p(\mu_{R_i}(y)) &= p(\min(\mu_{A^i}(x), \mu_{\tilde{B}_i}(y))) \\ &= p(\mu_{A^i}(x))(1 - P(\mu_{\tilde{B}_i}(y))) \\ &- P(\mu_{A^i}(x)) p(\mu_{\tilde{B}_i}(y)) + p(\mu_{\tilde{B}_i}(y)) \end{aligned} \tag{40}$$

where $P(\mu_{\tilde{B}_i}(y))$ and $P(\mu_{A^i}(x))$ are the CDF of $\mu_{\tilde{B}_i}(y)$ and $\mu_{A^i}(x)$, respectively.

6.4. Probabilistic defuzzification

The defuzzification process is associated with fuzzy sets. Since the inference engine is based on the probabilistic fuzzy set, a probabilistic defuzzification is proposed in this paper. The mathematical expectation of a stochastic output produces the ultimate output by the concept of stochastic defuzzification. In this case, the center-of-set probabilistic defuzzification method is proposed. The probabilistic output of the probabilistic fuzzy logic system can be determined as [3]:

$$y_{PFLS} = \frac{\sum_{i=1}^{J} y_i \mu_{A^i}}{\sum_{i=1}^{J} \mu_{A^i}} \tag{41}$$

where y_i is the center of probabilistic fuzzy set $\tilde{B}_i$ in rule i, J is the number of rules, and μ_{A^i} is the firing level in rule i; y_i and μ_{A^i} are stochastic variables. One of the limitations of the probabilistic theory is that it is challenging to obtain the likelihood distribution of the product of two parameters [32]. To overcome this drawback, Zhang et al. [32] suggested replacing y_i by the mathematical expectation of y_i as follows:

$$y_{PFLS} = \frac{\sum_{i=1}^{J} E(y_i) \mu_{A^i}}{\sum_{i=1}^{J} \mu_{A^i}} \tag{42}$$

Then, a discretization method is needed to obtain y_{PFLS} as Eq. (43) [32]. In this process, firing level $\mu_A^{\ i}$ ($p(\mu_A^{\ i}) > 0$) should be discretized into Q regions $[\underline{\mu_{A^{i,t_i}}}, \overline{\mu_{A^{i,t_i}}}]$, which are centered at $\mu_{A^{i,1}}, \mu_{A^{i,2}}, ..., \mu_{A^{i,Q}}$ and the associated probability $P(\mu_{A^{i,1}}), P(\mu_{A^{i,2}}), ..., P(\mu_{A^{i,Q}})$ can be estimated as Eq. (44) [32].

$$y_{PFLS} = \left\{ \frac{\sum_{i=1}^{J} E(y_i)\mu_{A^{i,t_i}}}{\sum_{i=1}^{J} \mu_{A^{i,t_i}}} \right\}, \ t_i \in \{1,...,Q\} \quad (43)$$

$$P(\mu_{A^{i,t_i}}) = \int_{\underline{\mu_{A^{i,t_i}}}}^{\overline{\mu_{A^{i,t_i}}}} p(\mu_{A^i}) d(\mu_{A^i}) \quad (44)$$

Finally, every possible combination of $\{\mu_{A^{1,t_1}}, \mu_{A^{2,t_2}}, ..., \mu_{A^{J,t_J}}\}$ $(t_i = 1,\ldots,Q, i = 1,\ldots,J)$ and the concerned probabilities are investigated to find all y_{PFLS} and their associated $P(y_{PFLS})$ as follows [32]:

$$P(y_{PFLS}) = \prod_{i=1}^{J} P(\mu_{A^{i,t_i}}). \quad (45)$$

As mentioned, the crisp output y can be determined using the expected value of y_{PFLS} as [32]:

$$y = E(y_{PFLS}) = \sum y_{PFLS} \cdot P(y_{PFLS}) \quad (46)$$

The probabilistic fuzzy logic controller scheme is shown in Fig. 6.

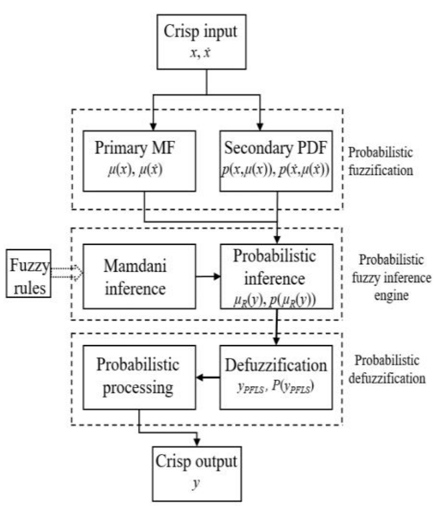

Fig. 6. Flowchart of the utilized PFLC.

7. Probabilistic fuzzy logic design

A probabilistic fuzzy logic controller uses uncertain information directly obtained from the building model. This information is defined as PFS. In this research, PFLC uses two input variables, each with three primary membership functions, and one output variable with seven primary membership functions. The velocity and displacement of the structures are the inputs of PFLC and the output parameter is the active control force. The reason for using two input variables is to illustrate the efficiency of the PFLS strategy in the control problem. The primary membership functions for the input and output parameters are Gaussian, as denoted in Eq. (27), and are introduced for the common interval [-1,1]. Previous studies have used triangular linear membership functions. However, Gaussian functions are more capable of estimating and improving the results. In this paper, PFLC is used as a probabilistic active controller. Thus, the membership functions are transformed from a simple mathematical model into probabilistic parameters. It is assumed that the integration of Gaussian and probability within membership functions can produce better responses. Gaussian functions determine uncertainties more effectively than other functions. In this study, PFS has been constructed by randomly selecting the center of the Gaussian fuzzy set; thus, the membership function becomes a random parameter that can be introduced by the secondary PDF function. As presented in Eq. (28), the standard deviation and mean of the fuzzy sets are two secondary PDF features of each primary membership function. The proposed primary membership functions for input and output parameters are presented in Figs. 7 and 8, respectively. The same standard deviation for the center of the fuzzy sets is considered for both the inputs and output. The mean centers of the primary membership functions for the input variables are -1, 0, 1 and for the output variable are -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75.Table 3 shows the fuzzy parameters to define the fuzzy domain [21]. Table 4 presents the inference rules, which have been generated from expert's experiences.

Table 3. Fuzzy variables.

Membership function	Variable	Definition
Input	*P*	positive
	Z	zero
	N	negative
Output	*PB*	positive big
	PM	positive medium
	PS	positive small
	Z	zero
	NS	negative small
	NM	negative medium
	NB	negative big

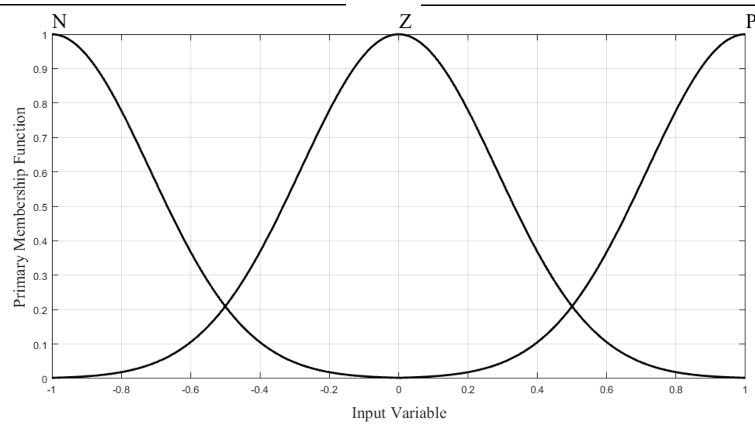

Fig. 7. Primary membership functions of input parameters.

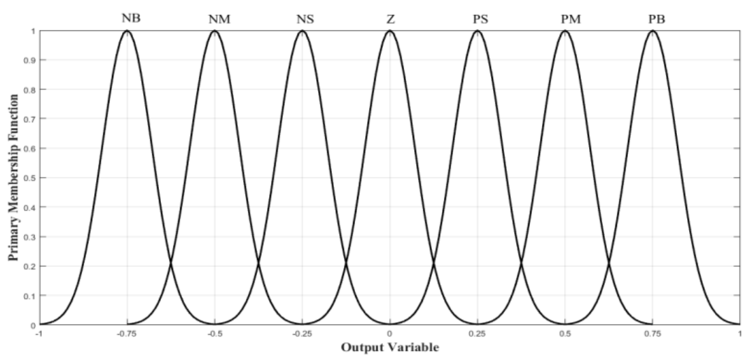

Fig. 8. Primary membership functions of output parameters.

Table 4. Inference rules for PFLS

Displacement	Velocity		
	N	*Z*	*P*
N	*PB*	*PM*	*PS*
Z	*PS*	*Z*	*NS*
P	*NS*	*NM*	*NB*

8. Results and discussion

To study the efficiency of the proposed probabilistic active control strategies for decreasing the building responses under Gaussian white noise excitation, an SDOF and three MDOF structural systems are chosen as sample problems. The results of these building responses, which are controlled by the PFLC method, are compared with those controlled by the active LQR control method. In the configuration of the LQR controller, a full state feedback closed-loop mechanism is utilized. For the LQR controller, the control design parameter $\gamma = 1$ is chosen for the performance index in Eqs. (22) and (23). Spencer et al. [38] showed that, with the use of a smaller value of γ, more weight can be imposed on the strain energy, so smaller covariance matrix

(RMS) responses are obtained for displacement. In the optimal control theory, finding the control gain matrix is the main problem, which can be solved using the Riccati algorithm. The gain matrices are presented in Tables 5 and 6 for SDOF and MDOF systems, respectively.

Table 5. Transfer function of the LQR controller for SDOF.

	Control gain matrix (*G*)	
SDOF	-1.0969	-0.0717

Table 6. Transfer function of the LQR controller for MDOF.

	Control gain matrix (*G*)					
Case A	-2.6315	1.6872	-0.0281	-0.0646	-0.0273	-0.0099
Case B	-1.1264	0.1119	0.0453	-0.0365	-0.0158	-0.0132
	1.0777	-1.1220	-0.0533	0.0207	-0.0308	-0.0137
	-0.1181	1.1065	-1.-258	0.0026	0.0197	-0.0363
Case C	-1.3385	0.4691	0.1108	-0.0453	0.0021	0.0028
	-0.1124	-0.8408	0.3168	0.0015	-0.0425	0.0033
	0.0585	-0.0819	-0.5044	0.0015	0.0024	-0.0393

The simulation analyses of the single and three-story benchmark models with tendon systems are conducted using unit intensity white noise. The covariance matrix responses are compared with the LQR controller and PFLC under the same simulation conditions. Tables 7, 8, 9, and 10 show the simulation results of the covariance matrix of displacement and velocity of the uncontrolled and controlled models for the top story of the SDOF structure, and for each story of cases A, B, and C. The calculation of the response reduction percentage is introduced as follows:

Response reduction (%) = {(Uncontrolled covariance response-Controlled covariance response)/(Uncontrolled covariance response)} × *100*. (47)

Table 7. RMS displacement and velocity responses of SDOF model with LQR controller and PFLC.

Covariance response					
σ_x(in)			$\sigma_{\dot{x}}$(in/s)		
No control	LQR	PFLC	No control	LQR	PFLC
0.1688	0.0162	0.0099	3.6862	0.4940	0.2442

Table 8. RMS displacement and velocity responses of case A using LQR controller and PFLC

Floor number	Covariance response (Case A)					
	σ_x(in)			$\sigma_{\dot{x}}$(in/s)		
	No control	LQR	PFLC	No control	LQR	PFLC
1	0.3986	0.0306	0.0254	6.0211	0.5409	0.4342
2	0.5319	0.0445	0.0338	7.9492	0.8104	0.7379
3	0.6648	0.0624	0.0546	9.9477	1.1646	0.9598

Table 9. RMS displacement and velocity responses of case B using LQR controller and PFLC.

Floor number	Covariance response (Case B)					
	σ_x(in)			$\sigma_{\dot{x}}$(in/s)		
	No control	LQR	PFLC	No control	LQR	PFLC
1	0.3796	0.0183	0.0146	5.8932	0.4113	0.3760
2	0.5320	0.0255	0.0175	8.1934	0.5703	0.4745
3	0.6133	0.0310	0.0198	9.4697	0.6576	0.5929

Table 10. RMS displacement and velocity responses of case C using LQR controller and PFLC.

Floor number	Covariance response (Case C)					
	σ_x(in)			$\sigma_{\dot{x}}$(in/s)		
	No control	LQR	PFLC	No control	LQR	PFLC
1	0.3596	0.0133	0.0116	5.5017	0.3711	0.2591
2	0.4863	0.0183	0.0149	7.3076	0.4690	0.3844
3	0.6582	0.0236	0.0189	9.9450	0.5766	0.4887

The results in Tables 7, 8, 9, and 10 reveal that the efficiency of the probabilistic fuzzy controller is higher than the case of the LQR controller. As shown in Table 7 for the SDOF model, the controlled covariance response decreases in both LQR controller and PFLC methods. Tables 8, 9, and 10 reveal that the proposed intelligent control system also reduces the controlled covariance responses of displacement and velocity for each floor of cases A, B, and C, respectively.

Table 11. Displacement covariance response reduction using PFLC and LQR control systems.

Floor number	Covariance response reduction of displacement (%)							
	SDOF		Case A		Case B		Case C	
	LQR	PFLC	LQR	PFLC	LQR	PFLC	LQR	PFLC
1	90.4	94.1	92.3	93.6	95.2	96.2	96.3	96.8
2	-	-	91.6	93.7	95.2	96.7	96.2	97.0
3	-	-	90.7	91.8	94.9	96.8	96.4	97.1

Table 12. Velocity covariance response reduction using PFLC and LQR control systems.

Floor number	Covariance response reduction of velocity (%)							
	SDOF		Case A		Case B		Case C	
	LQR	PFLC	LQR	PFLC	LQR	PFLC	LQR	PFLC
1	86.6	93.4	91.0	92.8	93.0	93.6	93.2	95.3
2	-	-	89.8	90.7	93.0	94.2	93.5	94.7
3	-	-	88.3	90.4	93.1	93.7	94.2	95.1

Tables 11 and 12 show the covariance response reduction for displacement and velocity, respectively. The LQR controller reduces the covariance responses of displacement and velocity for the SDOF system by 90.4% and 86.6%, respectively. The corresponding reductions for PFLC are 94.1% and 93.4%, respectively. As shown in Tables 11 and 12, simulation results for case A indicate that PFLC can decrease the covariance responses of displacement and velocity of the top floor by 91.8% and 90.4%, respectively. However, the associated reductions are, respectively, 90.7% and 88.3% for the LQR controller. In case B, in which there are tendons on all floors, PFLC reduces the covariance responses of velocity and displacement of the roof level by 93.7% and 96.8%, respectively. Further, the reductions corresponding to the LQR controller are93.1% and 94.9%, respectively (Tables 11 and 12). It is evident in Tables 11 and 12 that, for case C, PFLC reduces the covariance responses of displacement and velocity, respectively, by about 97% and 95% compared to the uncontrolled responses for the top floor. Tables 11 and 12 demonstrate that the response reductions in displacement and velocity for the LQR controller are 96.4% and 94.2%, respectively, for the top floor. Table 13 shows the reduction in covariance response of PFLCascompared to the LQR controller. The results indicate that PFLC reduces the displacement and velocity responses of the SDOF system by 38.6% and 50.6%, respectively, compared to the LQR controller. As shown, the displacement covariance response reduction compared to the LQR controller for the top floor of cases A, B, and C are 12.5%, 36.1%, and 20%, respectively. Besides, these results for the velocity covariance response are 17.6%, 9.83%, and 15.2%, respectively.

Table 13. Reduction in covariance response in PFLC as compared to LQR controller.

Floor number	Covariance response reduction (%)							
	SDOF		Case A		Case B		Case C	
	Displacement	Velocity	Displacement	Velocity	Displacement	Velocity	Displacement	Velocity
1	38.6	50.6	16.8	19.7	20.2	8.58	12.5	30.2
2	-	-	24.1	8.95	31.4	16.8	18.1	18.0
3	-	-	12.5	17.6	36.1	9.83	20.0	15.2

The RMS control force (σ_u) for SDOF and MDOF are presented in Tables 14 and 15. The results show that the RMS for the control force in the SDOF structure in PFLC is about 130.70 lb, while that in LQR controller is about 91.27 lb. As can be observed from the results, the control force of PFLC for case A increases by about 30% compared to the LQR controller. In case B, the control force of PFLC for the first and second floors decreases by about 65% and 16%, respectively, compared to the LQR controller. However, in the third floor, it increases by about 99%. The results of Table 15 for case C indicate that the control force of PFLC increases in all floors. The maximum and minimum values of increase for the second and first floors are about 176% and 47%, respectively. As a result, in PFLC, long tendons of structure in case C produce greater control forces compared to other cases. The active tendons on the top floor provide a side effect so that the tendons experience reaction forces in the opposite direction to the major control forces. One of the important reasons for the small top floor RMS displacement response in case C compared to case B is that the resistance control forces in case C are protected by the land surface. Because the tendons are very long in case C, the utilization of this structure is not reasonable, so case B is more preferable. As observed from the analytical results, the active control force that is needed to reduce the structural responses of displacement and velocity is greater in PFLC than in the LQR controller. Therefore, greater response reduction in PFLC compared to that in the LQR controller causes a significant reduction in the member size of structures.

By studying and analyzing the results, it can be understood that the LQR controller is not efficient in considering the uncertainty in structural control. However, the ability of the PFLC approach to handle the stochastic uncertainties in fuzzy rules results in a reduction in the structural responses in all floors. These reductions are greater in PFLC than in the LQR approach. Therefore, the PFLC provides more reliable results than the classic LQR system.

It is worth mentioning that the reported results correspond to the analysis of a specific case and a wider range of analyses seems necessary.

Table 14. RMS horizontal control force of SDOF for LQR controller and PFLC.

Floor number	LQR	PFLC
	σ_u(lb)	σ_u(lb)
top floor	91.27	130.70

Table 15. RMS horizontal control force of MDOF for LQR controller and PFLC.

Floor number	Case A		Case B		Case C	
	LQR	PFLC	LQR	PFLC	LQR	PFLC
	σ_u (lb)	σ_u (lb)	σ_u (lb)	σ_u (lb)	σ_u (lb)	σ_u (lb)
1	130.14	169.23	35.07	12.20	35.81	52.70
2	-	-	19.38	16.24	32.70	90.33
3 (top)	-	-	29.87	59.60	30.20	60.80

9. Conclusion

This study presented a probabilistic active control using a probabilistic fuzzy logic system for civil engineering structures with uncertain characteristics subject to dynamic random loads, which were modeled as Gaussian white noise. The mass, stiffness, and damping variables of structures were considered to be random Gaussian parameters. The dispersion coefficient of random parameters was assumed to be 10%. Fuzzy and stochastic theories were integrated using a PFLC for response control of structures. For numerical evaluation, the active tendon system was implemented in two types of the structural models, one using a SDOF system and the other one using a three-story MDOF system. The results of PFLC were compared with those of an uncontrolled structure and an LQR controller. The following conclusions can be drawn from this study:

(1)The results of this study demonstrated that PFLC is quite efficient in decreasing the structural covariance responses compared to the LQR controller. The results of comparison of controlled covariance responses of building floors in cases A, B, and C showed that PFLC reduces the responses of the floor smore effectively than the LQR controller.

(2) The SDOF system with PFLC produced better covariance response values than with the LQR controller.

(3) Case C of the MDOF system with PFLC showed the greatest reduction in the covariance response.

(4) Because the tendons are very long in case C, and there are six activators on the land surface, the use of case C is not feasible and case B is more feasible for application.

(5) The results also showed that the control force required to reduce the covariance response in PFLC was greater than that in the LQR controller.

It is worth noting that these results were obtained by an assumption that earthquake excitation is a Gaussian white noise. It is strongly recommended that this research be continued using far-field and near-field earthquakes.

Disclosure statement

The corresponding author notes on behalf of all the authors that there is no conflict of interest.

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Appendix A

Derivation of probability distribution function (PDF) of fuzzy set

Using central limit theory, the center $c_{j,i}$ in Eq. (27) is a normally distributed parameter as:

$$c_{j,i} \approx N\left(u_{j,i}, \sigma_{j,i}{}^2\right) \quad \text{(A.1)}$$

The probability distribution function is:

$$p(c_{j,i}) = \frac{1}{2\sqrt{2\pi}\sigma_{j,i}}\left(\exp\left(-\frac{\left(c_{j,i}-u_{j,i}\right)^2}{2\sigma_{j,i}{}^2}\right)\right) \quad \text{(A.2)}$$

Because $\mu_{j,i} \in (0,1)$ increments in $(-\infty, x_j)$ and reduces in $(x_j, +\infty)$, its PDFis obtained when $\mu_{j,i} \leq 0$, $F_\mu(\mu_{j,i}) = P(\mu < \mu_{j,i}) =$ 0and when $0 < \mu_{j,i} \leq 1$as described below.

$$F_\mu(\mu_{j,i}) = P(\mu < \mu_{j,i}) = P\left(exp\left(-\frac{(x_j-c_{j,i})^2}{2\xi_{j,i}^2}\right) < \mu_{j,i}\right) = P\left(c_{j,i} < x_j - \sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}\, or c_{j,i} > x_j + \sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}\right) = 1 - \int_{x_j-\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}}^{x_j+\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}} P(c_{j,i})\, dc_{j,i}. \quad \text{(A.3)}$$

Therefore,

$$F_\mu(\mu_{j,i}) = \begin{cases} 1 - \int_{x_j-\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}}^{x_j+\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}} P(c_{j,i})\, dc_{j,i}, 0 < \mu_{j,i} < 1 \\ 0, otherwise \end{cases} \quad \text{(A.4)}$$

$$p_{\tilde{A}_{j,i}}(\mu_{j,i}) = (F_\mu(\mu_{j,i}))' = \frac{1}{2\sqrt{2\pi}\mu_{j,i}\sigma_{j,i}}\sqrt{\frac{-2\xi_{j,i}^2}{\ln \mu_{j,i}}} \times \left(exp\left(-\frac{\left(\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}+x_j-u_{j,i}\right)^2}{2\sigma_{j,i}{}^2}\right) + exp\left(-\frac{\left(-\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}+x_j-u_{j,i}\right)^2}{2\sigma_{j,i}{}^2}\right)\right) \quad \text{(A.5)}$$

Therefore, the PDF of the PFS can be summarized as [3]:

$$p_{\tilde{A}_{j,i}}(\mu_{j,i}, x_j) = \begin{cases} \frac{1}{2\sqrt{2\pi}\mu_{j,i}\sigma_{j,i}}\sqrt{\frac{-2\xi_{j,i}^2}{\ln \mu_{j,i}}} \times \left(\exp\left(-\frac{\left(\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}+x_j-u_j\right)^2}{2\sigma_{j,i}{}^2}\right) + \exp\left(-\frac{\left(-\sqrt{-2\xi_{j,i}^2 \ln \mu_{j,i}}+x_j-u_j\right)^2}{2\sigma_{j,i}{}^2}\right)\right), 0 < \mu_{j,i} < 1 \\ 0, otherwise \end{cases}$$

(A.6)