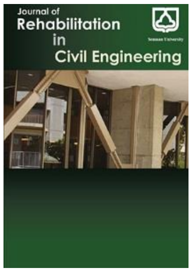
Journal of Rehabilitation in Civil Engineering 11-3 (2023) 158-179



Journal homepage: http://civiljournal.semnan.ac.ir/

A Review on Soil-Foundation-Interaction Models

Gaurav Pandey[1*], Dhirendra Patel[1], Vishal Kumar Mourya[1], Rajesh Kumar[2], Suresh Kumar[3]

1. Research Scholar, Department of Civil Engineering, IIT (BHU) Varanasi, India.
2. Professor, Department of Civil Engineering, IIT (BHU) Varanasi, India.
3. Assistant Professor, Department of Civil Engineering, IIT (BHU) Varanasi, India.

Corresponding author: *gauravp.rs.civ16@iitbhu.ac.in*

ARTICLE INFO

Article history:
Received: 22 December 2021
Revised: 26 July 2022
Accepted: 21 September 2022

Keywords:
Improved Winkler models;
Continuum models;
Improved continuum models;
Modulus of subgrade reaction;
Raft foundation.

ABSTRACT

The process in which response of soil influences the motion of foundation and vice-versa, is known as Soil-Foundation Interaction (SFI). This paper deals with the different types of soil-foundation interaction models useful for structural as well as geotechnical engineers and researchers for safe and economical raft design. The study offers a gist of all the models in literature and their applications. Several approaches for analysis which include analytical methods and numerical methods have been described here. Analytical models based on Winklerian and Continuum approach have been discussed. Moreover, modified forms of such approaches have also been discussed. In general, all the models make use of a parameter known as Modulus of Subgrade Reaction to model soil interaction. This parameter can be calculated either through experiments or empirical formulas. Different numerical methods have also been presented along with a few literatures. Further, some of the studies on raft foundation are also included.

1. Introduction

The topic on plates on elastic foundation finds its importance in several practice areas of engineering including structural, aerospace, mechanical, geotechnical etc. Common examples involve foundation slabs, RCC pavements, airport runways etc. Raft foundation made up of concrete carrying a structure behaves like a plate supported on an elastic foundation which resists the load of the structural components. For every civil engineering structure, analyzing and designing a good foundation is the primary

How to cite this article:
Pandey, G., Patel, D., Mourya, V., Kumar, R., & Kumar, S. (2023). A Review on Soil-Foundation-Interaction Models. Journal of Rehabilitation in Civil Engineering, 11(3), 158-179.
https://doi.org/10.22075/JRCE.2022.25247.1570

concern. The structural loads which include self-weight and applied loads are transferred over the earth through the foundation. The soil settlement causes additional deformation and stresses in the superstructures. Most of the time, superstructure analysis is done based on the fixed base idealization. However, for realistic and more accurate modeling, the soil-structure interaction effect is desirable. The most common types of foundations that we come across are isolated footings. However, depending upon soil properties, the structural load, and the plan area, we choose a better alternative for designing such as combined footings, raft foundations, piles, caissons etc. When the foundation area of any structure covers more than half of the total built-up area, raft foundations are generally preferred in that case [1]. The raft/ mat foundations are also used either when the supporting soil is loose, or column loads are heavy or if there is the possibility of differential settlement. Heavy and multi-storied buildings, silos, storage tanks on soft soil are few examples that require raft foundation.

Safe and economic analysis is the foremost criteria in almost every field of engineering. An analytical solution is the most precise method for the analysis, but it has limited applications. As soon as the boundary conditions and the structural geometry start getting complex, calculations involved in the analytical methods become cumbersome. Therefore, we require switching on to such an alternative which can deal with the material and the geometric non-linearity. Finite element methods, finite difference methods, finite strip methods & boundary element methods are some of the numerical methods that are widely used for such problems.

IS:2950-1981 [2] suggests different types of mat foundations depending upon column spacing and the bearing load. For design purpose, several parameters such as size, shape, load eccentricity, soil properties and rigidity of foundation as well as that of the superstructure are usually taken into consideration. The conventional method is considering raft as rigid compared to the soil. The contact pressure variation is assumed linear in this case. The relative stiffness factor in such a case is greater than 0.5. The rigid analysis assumptions are commonly justified for soft soils such as peat and muck. For the flexible foundation analysis, Winkler's model as discussed in the next section is usually used. However, several other models have been given till date in order to capture the actual behavior of raft over soil. Following sections briefly illustrates such models which have proposed over a period.

2. Modelling

2.1. Analytical Modelling

The approach for solving soil-foundation-interaction (SFI) problem can be broadly categorized into Winklerian Approach and Elastic Continuum Approach. The basic tactic is either to initiate with the Winkler Model and introduce some interaction element; or start with the Continuum model and provide a few simplifying assumptions related to stresses and/or displacements. Based on these approaches, several models have come into existence till date in attempt to include the realistic behaviour of soil-foundation-interaction. Initially elastic models have been discussed which include Winkler model, Winklerian based models, Continuum models and improved continuum models. Lastly, advanced models which considers rheology and non-linearity of soils have also been discussed. A brief introduction about these models has been provided in the subsequent sections.

2.1.1. Winkler Model

The first known, as well as the simplest mechanical model, was proposed by Winkler [3] which states that the contact pressure, p at every point varies linearly with the soil deformation at that point. Mathematically, it is represented as

$$p = kw \quad (1)$$

where w is the soil deformation and k is the modulus of subgrade reaction.

The governing differential equation for a plate supported over an elastic foundation is thus given as

$$D\nabla^2\nabla^2 w + kw = q \quad (2)$$

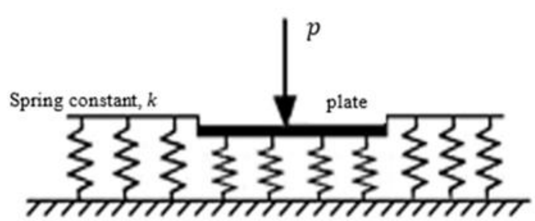


Fig 1. Winkler Model.

The model idealizes the soil as several springs having spring constant, k which are assumed as closely spaced, independent, distinct, and linearly elastic as well. This model suggests the deformation restricted only to the loaded area and zero deformation at the points outside it. The constant of proportionality in the above equation, k, which is also called as modulus of subgrade reaction is the parameter of prime concern for accurately defining the nature of the soil. Several methods by which we can determine the subgrade modulus are tried to be dealt with in the subsequent sections.

Liou and Lai [4] proposed a model using Winkler spring with constant spring constant over the entire mat footing area. Yield Line Theory was used to join the springs to the raft with grid beams used as stiffeners. Finally, the proposed model was compared with the FE model using ANSYS software to check its efficiency.

An interesting thing to be noted is that this model is analogous to the Archimedes principle in the case of floating structures [5]. Despite its limitations, the model is used for its wide applications and the simplicity to be used in a computer program. Winkler model was first used for railroad tracks analysis but since then, it has been extended to several engineering problems at present such as grillage structures, shrink-fit problems, and plates [6].

Modulus of Subgrade Reaction

The subgrade modulus is defined as the ratio between the contact pressure, p at any given point and the corresponding deformation, w at that point.

$$k = \frac{p}{w} \quad (3)$$

Due to the non-linear p-w relation, the value of k is not constant and varies with the type of soil. In addition to this, the magnitude of subgrade modulus also depends upon the foundation's shape, position, and dimensions. Also, since the soil is generally stratified with varying thickness of each layer, the complexity is much more in its determination. Different approaches for determination of subgrade modulus can be broadly classified as: field and laboratory tests, empirical formulas, tables, and graphs.

Field and laboratory tests: Plate Load Test is the most common field test for determination of subgrade modulus. For more of the lab and field tests such as consolidation test, California Bearing Ratio test and triaxial test, the reader may go through the references presented in the review article by Dutta and Roy [7]. Bearing capacity of the soil is the simplest parameter which is used to design a small foundation and assumes the foundation to be rigid. However, when the flexible analysis is to be done as in case of large foundations with several footings, modulus of subgrade reaction is taken into account.

Empirical methods: Several empirical formulas have been there till date for different applications. More extended representation of the empirical formulations of subgrade modulus given by Lee and Jeong [8] has been provided in Table 1.

Apart from the equations provided in Table 1, Sall et al. [13] presented several recent works of literature which shows the equations in the form of $k = a \frac{E_s}{1-\mu^2}\left(\frac{E_s B^4}{EI}\right)^{\gamma}$,

where the constants a and γ vary conferring to different authors. Horvath [14] discussed the advantages and shortcomings for several approaches used in determining the subgrade modulus value and proposed an elastic theory-based model known as Reissner Simplified Continuum (RSC) in which modulus of elasticity is kept constant and stresses σ_x, σ_y, τ_{xy} as zero. Body forces were also not considered.

Kirsch [15] observed more than 100% difference in variation in the determination of the subgrade modulus while comparing several existing empirical approaches. When compared with 2D and 3D FE results using an example, the analytical settlement was found to be in good agreement for that example. However, inconsistent results were observed between 2D and 3D model for pile as well as piled-raft foundation.

Graphs and tables: Chandra et al. [16] presented two separate columns of values for linear and non-linear subgrade modulus applicable for his non-linear model. Bowles [17] portrayed a table that presents a range for 'k' values for different types of soil. Daloglu and Vallabhan [18] presented graphs to determine the subgrade modulus for given properties and geometry of the entire system. Comparative study for Winkler and modified Vlasov model had also been done.

Table 1. Modulus of Subgrade Reaction.

S. No.	Proposer	Year	k_s	Application
1	Biot [9]	1937	$k_s = \frac{0.95E_s}{B(1-\mu_s^2)}\left\{\frac{E_sB^4}{EI(1-\mu_s^2)}\right\}^{0.108}$	Infinite beam on elastic soil continuum
2	Timoshenko & Goodier	1951	$k_s = \frac{1.13E_s}{(1-\mu_s^2)}\frac{1}{\sqrt{A}}$	Rigid plate on a semi-infinite elastic soil medium subjected to point load
3	Terzaghi [10]	1955	$k_s = k_p\left(\frac{B+0.3}{2B}\right)^2$ (sands) $k_s = k_p\frac{1}{B}$ (clays)	Rigid plate on a horizontal subgrade surface
4	Vesic [11]	1961	$k_s = \frac{0.65E_s}{B(1-\mu_s^2)}\sqrt[12]{\frac{E_sB^4}{EI}}$	Beams on elastic half-space
5	Meyerhof & Baikie	1963	$k_s = \frac{E_s}{12(1-\mu_s^2)}$	Buried circular conduits
6	Vlassov	1966	$k_s = \frac{E_s(1-\mu_s)}{(1+\mu_s)(1-2\mu_s)} \times \frac{\mu_s}{2B}$	Beams & plates on elastic half space
7	Kloppel & Glock	1979	$k_s = \frac{2E_s}{B(1+\mu_s)}$	Buried circular conduits
8	Selvadurai	1985	$k_s = \frac{0.65E_s}{12(1-\mu_s^2)}$	Buried circular conduits
9	AASHTO [12]	1993	$k_s = \frac{M_R}{19.4}$ (M_R is resilient modulus)	Flexible pavement
10	Lee & Jeong	2016	$k_j = j_f\frac{E_s}{B(1-\mu_s^2)}$ j_f is the joint reduction factor	Jointed Rock Mass

2.1.2. Improved Winkler Models

Several other improved mechanical models are proposed since then to provide continuity among the discrete springs and hence providing more realistic and precise modeling of the soil. This continuity is provided in each of these models through some structural element applied to the conventional Winkler model. Some of these models are often called two-parameter models as these are defined by two independent elastic constants.

A. Filonenko-Borodich Model

Filonenko-Borodich [19] suggested that the required interaction to some extent is acquired by bridging the top of the springs with a stretched elastic skin subjected to constant tension, *T*. Taking the equilibrium of the membrane-spring system into account, the mathematical equation is modified as

$$p = kw - T\nabla^2 w \tag{4}$$

On the contrary, the response of plate on tensionless Winkler foundation was analyzed by Celep [20] using Galerkin's method. He concluded that the partial contact shows non-linear behavior whereas nature is linear in case of full contact. In addition, he showed that the dependence of the ratio of loadings on the contact region is visible only in the case of multiple loads.

B. Hetenyi Model [21]

The interaction in this model has been achieved by providing an elastic plate or an elastic beam experiencing flexural deformation, instead of an elastic layer as in the previous case. The governing equation, in this case, is given by

$$p = kw + D\nabla^2\nabla^2 w \tag{5}$$

Here, D is the flexural rigidity.

$D = E_b I_b$ for beam & $D = \frac{E_p}{12(1-\mu^2)}$ for plate

C. Pasternak Model [22]

Ends of the discrete springs in this model were assumed to be connected to a beam or a plate undergoing the shear deformation only. The response is thus characterized by the shear layer consideration as

$$p = kw - G\nabla^2 w \tag{6}$$

where G is the shear modulus of the layer.

All the above equations [(4) to (6)] reduce to the Winkler model as the limiting case when the parameters T, D and G become zero.

Axi-symmetric non-linear response of orthotropic circular plate on Pasternak foundation was analysed by Dumir [23]. Point collocation method was used in the analysis and geometric non-linearity was considered. Shear modulus used in this model as interacting element was found to have considerable influence in the study.

D. Generalized Foundation Model

Besides Winkler's hypothesis, this model proposed the contact moment to be relatable to the angle of rotation [24]. Thus, mathematically,

pressure $p = kw$

and moment in n-direction is given by

$$m_n = k_1 \frac{dw}{dn} \tag{7}$$

Here, k and $k_1$are proportionality factors and n is the direction in the foundation plane.

E. Kerr Model

This two-layered model proposed by Kerr [25] is, in fact, an extension of the Pasternak model. It assumes a shear-layer sandwiched between two spring layers of different spring constants to offer the coupling among them. The pressure-deflection relationship is thus modified into the following equation

$$\left(1 + \frac{k_2}{k_1}\right) p = \frac{G}{k_1}\nabla^2 p + k_2 w - G\nabla^2 w \tag{8}$$

F. Beam-Column Analogy Model

This model has been established by Horvath [26] in which the Winkler hypothesis is combined with the Pasternak model for the beams on elastic foundation. Hence, the resulting differential equation is represented as

$$(EI\nabla^4 - c_2\nabla^2 + c_1)w(x) = q(x)$$

□□□□□□□□□□

C_1 & C_2 being constants.

The aforementioned-equation can be considered equivalent to a beam-column supported on Winkler subgrade of stiffness C_{p2} and subjected to a tensile force, C_{p1}. Ti et al. [27] suggested the determination of C_{p1} & C_{p2} by Pasternak-type simplified continuum (PSTC) as the derivation is directly in terms of the elastic parameters and subgrade thickness.

The parameter values determined by PSTC derivation in case of homogeneous & isotropic layer lying beneath a rigid base (Horvath 1979) is given as

$$c_1 = \frac{E_s}{H} \,;\, c_2 = \frac{G_s H}{2} \tag{10}$$

The additional parameter in all the above models enhances the replication accuracy but on the other hand, it increases the complexity due to difficulty in determining the parameter.

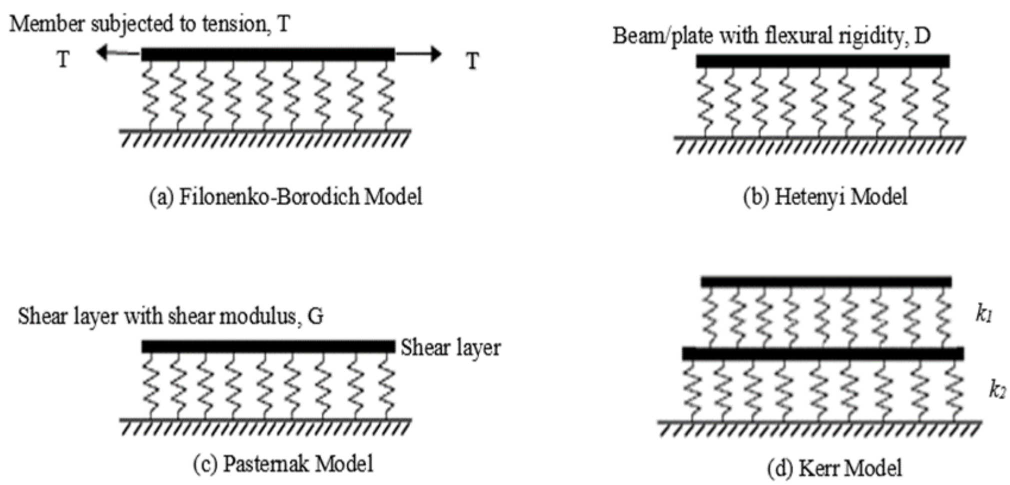


Fig 2. Improved Winkler Model.

Table 2. Winklerian Models.

Model	**Proposer**	**Year**	**Expression**
Winkler Model	Winkler	1867	$p = kw$
Filonenko Borodich Model	Filonenko Borodich	1940	$p = kw - T\nabla^2 w$
Hetenyi Model	Hetenyi	1946	$p = kw + D\nabla^2\nabla^2 w$
Generalized Foundation Model	Sokolov	1952	$p = kw \,;\, m_n = k_1 \frac{dw}{dn}$
Pasternak Model	Pasternak	1954	$p = kw - G\nabla^2 w$
Kerr Model	Kerr	1965	$\left(1 + \frac{k_2}{k_1}\right) p = \frac{G}{k_1}\nabla^2 p + k_2 w - G\nabla^2 w$
Beam Column Analogy Model	Horvath	1993	$\left(E_b I_b \nabla^4 - C_{p2}\nabla^2 + C_{p1}\right) w = q$

G. New continuous Winkler Model

Kurian and Manojkumar [28] presented a different continuous model for SSI using case studies on different structural elements in which intermeshed springs were used. This model nicely considers the interaction with the nearby soil using other springs that are not directly connected to the foundation plate.

2.1.3. Continuum Models

Another approach is to idealize the soil as elastic, homogeneous, isotropic and semi-infinite continuum. Since it is well known that deflections under loaded region are not restricted to area immediately under it but also in the nearby regions, an alternative to overcome this shortcoming of Winkler model is to consider soil as a semi-infinite continuum. The response at any point in the soil continuum subjected to a point load can be found out by the concept of Boussinesq equation [29]. This theory has been widely used to develop several Continuum models in order to capture the soil-foundation interaction. Most basic and notable among them are the Reissner and the Vlasov Model.

A. Reissner Model

A model using elastic continuum approach was proposed by Reissner [30] in which constraints to stress and displacement were made.

Assumptions: (a) Horizontal displacements $u = v = 0$; (b) in-plane stresses $\sigma_{xx} = 0$; $\sigma_{yy} = 0$; $\tau_{xy} = 0$.

Response function:

$$c_1 w - c_2 \nabla^2 w = p - \frac{c_2}{4c_1} \nabla^2 p \quad (11)$$

where $c_1 = \frac{E}{H}$, $c_2 = \frac{HG}{3}$

B. Vlasov Model [31]

Vlasov proposed a two-parameter model by simplifying the elastic isotropic continuum approach. Using variational method, Vlasov imposed certain constraints on probable displacements in an elastic layer and obtained a soil response function. Several practical problems have also been discussed in this text.

Assumptions: (a) Horizontal displacement $u(x,z) = 0$ everywhere in the soil; (b) Vertical displacement $w(x,z) = \overline{w}(x).\phi(z)$ such that $\phi(0) = 1$ and $\phi(H) = 0$; $w(x,0) = \overline{w}(x)$. The function $\phi(z)$ portrays the variation in vertical displacement and can be determined by principle of minimum potential energy. $\phi(z) = 1 - (z/H)$ for foundation of finite thickness H and $\phi(z) = \frac{\sin\gamma\left(1-\frac{z}{H}\right)}{\sinh\gamma}$ for thick foundation where γ is a parameter that presents stress distribution within foundation.

Response function:

$$p = kw - 2t \frac{d^2 w}{dx^2} \quad (12)$$

where $k = \frac{E_0}{1-{\mu_0}^2} \int_0^H \left(\frac{d\phi}{dz}\right)^2 dz$;

$t = \frac{E_0}{4(1+\mu_0)} \int_0^H \phi^2 dz$; $E_0 = \frac{E}{1-\mu^2}$;

$\mu_0 = \mu(1-\mu)$

Advantage of this model is that it includes all the advantages of a continuum model and maintains the simplicity of spring model too. It can also be reduced to Kerr and Reissner model with proper consideration of vertical deformation profile.

2.1.4. Improved Continuum Models

A. Modified Reissner Model

Horvath [14] neglected body forces too in addition to the in-plane stress components as in Reissner Model. The advantage of this model was that only geometry and modulus E were needed to find the parameters and finite difference methods could be simply applied to analyse the problem. Apart from solving the Reissner model where E was assumed constant, he also varied the elastic modulus linearly and with square root of depth. He found the governing equation to be in similar form in all the cases as follows

$$c_1 w - c_2 \nabla^2 w = p - c_3 \nabla^2 p \quad (13)$$

where expressions for $c_1, c_2 \,\&\, c_3$ vary according to the case.

B. Modified Vlasov Model

Jones & Xenophontos [32] used a different variational formulation for Vlasov's model. The advantage of this model over Vlasov's is that it can also be used to determine the vertical deformation profile. They presented a relationship between γ and displacement characteristics. They also investigated rigid beams on elastic foundation experimentally and found these results to be in good agreement with the theoretical results.

Vallabhan & Das [33] used an iterative approach to find the exact value of γ, which was recommended to be between 1 and 2 by Vlasov till then. Investigations proved $\left(\frac{H}{l}\right), \left(\frac{E_s}{E_b}\right), \left(\frac{l}{d^3}\right)$ as the non-dimensional parameters that influence values of γ and k. For the case of beams subjected to uniformly distributed loading, they found the parameter γ to be dependent on (H/l) ratio but independent of E_s/E_b and $(l/d)^3$.

$$\left(\frac{\gamma}{H}\right)^2 = \frac{1-2\mu}{2(1-\mu)} \frac{\int_{-\infty}^{\infty} \left(\frac{d\overline{w}}{dx}\right)^2}{\int_{-\infty}^{\infty} \overline{w}^2 dx}$$

An important point is that this γ is not the same as used by Vlasov and is dimensionless.

Vallabhan & Das [34] further used finite difference method to solve the fourth-order governing equations and compared them with FE results. Three loading cases were discussed in the analysis and the parameter γ was found to vary with loading conditions. The study also suggested to carry out convergence study while using FEM to get fairer results. Vallabhan & Daloglu [35] analysed plates on layered soil-medium using consistent FE Vlasov model. Assuming soil modulus varying linearly from E_1 at top to E_2 at bottom, modified values for parameters k and $2t$ were presented. Teodoru & Musat [36] used the modified Vlasov foundation model for analysis of beam supported on elastic foundation and validated the results with 2D-plain strain FE solution in MATLAB. The model was observed to give conservative results when compared to FE results.

2.1.5. Generalized Continuum Model

Worku [37] presented a new continuum approach without neglecting any stress, strain or deformation. Several variants of the above model can be produced by different combinations of the depth functions. A few such variants based on Winkler, Pasternak and Kerr were also discussed. Most of the continuum models can be shown as special cases of the presented model. The generalized model is given by

$$p - \frac{G}{E}\frac{1}{K_I}\left(L_{gI} - \frac{K_{gI}L_g}{K_g}\right)\nabla^2 p = \frac{E}{K_g} w - \frac{GL_{gI}}{K_g K_I}\nabla^2 p \quad (14)$$

where $K_I, K_g, K_{gI}, L_g, L_{gI}$ are constants as given in Worku (2010). It is also referred to as 'the generalized Kerr-type continuum model' due to its similarity with Kerr's model.

Table 3. Continuum Models.

Model	Proposer	Year	Expression
Vlasov Model	Vlasov	1960	$p = kw - 2t\nabla^2 w$
Reissner Model	Reissner	1958	$p - \frac{c_2}{4c_1}\nabla^2 p = c_1 w - c_2 \nabla^2 w$
Modified Vlasov Model	Jones & Xenophontos	1977	$p = kw - 2t\nabla^2 w + D\nabla^4 w$
Modified Reissner Model	Horvath	1983	$p - c_3\nabla^2 p = c_1 w - c_2 \nabla^2 w$
Generalized Continuum Model	Worku	2010	$p - \frac{G}{E}\frac{1}{K_I}\left(L_{gI} - \frac{K_{gI}L_g}{K_g}\right)\nabla^2 p = \frac{E}{K_g} w - \frac{GL_{gI}}{K_g K_I}\nabla^2 p$

Further, the application of this continuum model was presented for beams over elastic foundations by Worku & Degu [38]. FE software PLAXIS 3D foundation was used for numerical analysis of finite & infinite beams and was compared to proposed Kerr-variant models. Kerr-type II model was found to provide results closer to the FE results.

Formulas for determination of foundation model parameters were obtained by Worku [39]. The work demonstrated calibration factor for beams and plates. These factors based on Winkler and Kerr models were determined and its direct application in software was recommended. However, differential equations of Kerr model intended to have some amount of complexity in solving.

Since two-parameter models are easier to analyse and are much familiar as compared to three-parameter Kerr-type model, Worku [40] developed calibrated Pasternak-type model by neglecting the lateral deformations. To overcome the problem of overestimated stiffness while neglecting lateral deformations in basic continuum models, thickness was eliminated by calibrating the Kerr-type model. This calibrated Pasternak-type model gave better results when compared with other simplified-continuum models.

2.1.6. Advanced Models

A. Elasto-plastic Models

The non-linearity of soil can be better described by an elastoplastic model. The linear portion is modeled using a Hookean spring element while the curved portion as a Coulomb unit. St. Venant element is that when both these elements are connected in series. Several parallel St. Venant elements epitomize a better elasto-plastic model [41].

Like in the case of elastic models, basic elastoplastic models can be established either by combining plasticity effects in Winkler or Winkler-based models, or by continuum idealization modified for considering plastic flow effects.

Selvadurai [42] analysed a rigid circular plate supported on elastoplastic Pasternak foundation. The plasticity effect was introduced through the shear interaction layer only and load was monotonically increasing. This model considered punching failure instead of bearing capacity failure for compressible soils and the results showed the yielding initiating at plate edges. It was also noticed that after incorporation of plastic effects, contact stresses were altered significantly. However, no experimental verification was done.

Papadopoulos and Taylor [43] analysed Reissner-Mindlin plate based on elasto-plastic model. It was generalized plane stress model considering von Mises yield criterion where elasto-plastic correction was done after the basic elastic step.

Performance of such model was then compared with thin plate solutions using a FEA program. Several yield criteria for soils such as Mohr-Coulomb, Drucker-Prager, Naylor-Zienkiewicz etc. were used for elasto-plastic analysis by Noorzaei et al [44]. The analyses included determination of collapse load, settlements, contact pressures, bending moments etc. Interaction between plane frame-footing-soil system was investigated through elastic-perfectly plastic analysis. Lastly, this interactive analysis was compared with the analysis of structure with fixed base.

Liang et al. [45] proposed an elastoplastic model for clay considering non-orthogonality. Plastic strain magnitude is determined by consistency condition, hardening parameter and yield function while the plastic flow direction by slope of plastic potential surface. The model captures behaviour of clay with variable stiffnesses. All the parameters used in this model can be obtained by tri-axial tests. Considering the fractional order as unity, this non-orthogonal model reduces to orthogonal one which is also known as Modified Cam-Clay (MCC) model. Again, elastoplastic modelling of sand with dilatancy considering non-orthogonality was presented by Liang et al. [46]. Hardening parameter was improved for capturing dilatancy and additional two parameters have been used in this model. The performances of both these models were validated with various experimental data from previous literature.

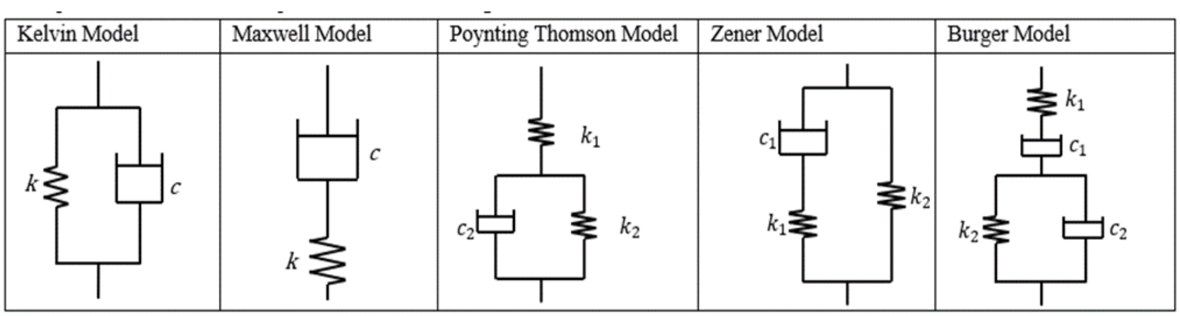


Fig 3. Viscoelastic Models.

B. Visco-elastic Models

The rheological properties of soil induce important time-dependent phenomena in interaction problems of plates on foundation. Rheology is the study of flow and deformation of matter. The rheology of soil thus can be considered into model using different arrangements of elastic and viscous element. Spring and dashpots are mostly used for such modelling. A few such arrangements have been shown in Table which considers the viscoelastic behaviour of soil that have been used by the researchers.

Moreover, the most common rheological models are Kelvin-Voigt Model and Maxwell model in which spring and dashpot are connected in parallel and series respectively. However, only two-parameters in these models were found to predict poorer results which encouraged researchers to introduce additional viscoelastic elements in different arrangements.

Therefore, an arrangement where Winkler spring is in parallel combination with Maxwell model is known as Zener Model. Similarly, different other combinations of three spring/dashpot elements form Poynting-Thomson Model.

Again, a four-element model in different combinations can be called as Burger Model. A series combination of Maxwell and Kelvin model is an example of Burger model [47].

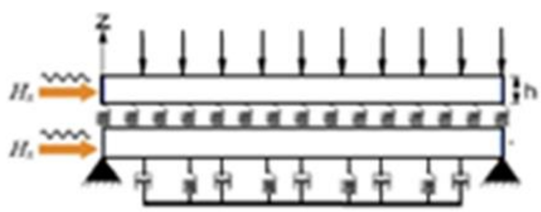


Fig 4. Visco-Pasternak Model.

In addition to this, various other models based on hyper-elasticity, hypo-elasticity, visco-plasticity and elasto-viscoplasticity of the soil behavior has been briefly discussed by Ti et al [27]. Double-layered graphene sheets supported on visco-Pasternak foundation has been analyzed in the presence of the magnetic field in the literature by Arani and Jalaei [48]. The static and dynamic response was analytically determined using Laplace inversion method and Navier's approach and the effect of several parameters such as structural damping, aspect ratio, van der wall interaction etc. on the response has been analyzed.

Cone models are also a substitute for soil modeling. These models have been presented for translational as well as rotational response. In these models, cones are considered equivalent to the inter-linking of masses, springs and dashpots. Discrete element model suggested by Veletsos & Meek [49] for rotation was improved as a monkey tail model and spring-damper model. The raft foundation supported over homogeneous soil has been analyzed dynamically by Meek & Wolf [50]. Anvarsamarin et al. [51] used modified cone models in analysis of moment-resisting structure considering SSI.

Table 4. Expressions of Viscoelastic Models.

Models	Governing Equation
Kelvin-Voigt	$p = kw + c\frac{\partial w}{\partial t}$
Maxwell	$\frac{\partial w}{\partial t} = \frac{1}{k}\frac{\partial p}{\partial t} + \frac{1}{c}p$
Zener	$\frac{\partial w}{\partial t} = \frac{\frac{\partial p}{\partial t} + \frac{k_1}{c_1}p - k_1 k_2 w}{k_1 + k_2}$
Poynting-Thomson	$k_1\frac{\partial w}{\partial t} + \frac{k_1 k_2}{c_2}w = \frac{\partial p}{\partial t} + \frac{k_1 + k_2}{c_2}p$
Burger	$\left[\frac{\partial^2}{\partial t^2} + \left(\frac{k_1}{c_1} + \frac{k_1}{c_2} + \frac{k_2}{c_2}\right)\frac{\partial}{\partial t} + \frac{k_1 k_2}{c_1 c_2}\right]p = \left[\frac{k_1 + k_2}{c_2}\frac{\partial}{\partial t} + k_1\frac{\partial^2}{\partial t^2}\right]w$

2.1.7. Other Models

Horvath & Colasanti [52] developed a modified Kerr-Reissner model using ANSYS software, which is an improvement to Winkler's model for soil-structure interaction studies. This model also uses inherent spring coupling and claims to be used for problems in practice.

Non-linearity in interaction can also be included using Beam on Nonlinear-Winkler-Foundation (BNWF). Such an interaction uses zero-length element between foundation and soil continuum. However, it is limited to 2-dimensional problems. Ganjavi & Rezagholilou [53] studied the seismic effect over moment frame using the BNWF model and compared the same with the one with fixed base

When a fractional derivative element is substituted in place of the dashpot element, the model is known as Fractional Merchant Model (FMM). This concept was firstly introduced by Gemant [54] in which fractional-order differential operators were used in place of integral-order operators. Due to its complex nature it has not been used widely in plate-foundation interaction problem. However, a few literatures can be studied in references by Zhang et al [55]. Zhang et al. also proposed a four-parameter FMM for plate-foundation interaction problem. to analyze the long-term performance of plates on the foundation. Results show that for sandy soil foundation, the fractional differential order is lesser as compared to that for clayey soil foundation. The parameters are spring stiffnesses k_0 and k_1, viscosity coefficient η and fractional differential order α.Validation was done by comparing foundation reactions, deflections and bending moments of this model with the results of Standard Merchant Model and elastic models. The stress-strain relationship and the governing equation are given by

$$E_0(D_{RL}^{\alpha} + 1/\tau_1^{\alpha})\varepsilon(t) = (D_{RL}^{\alpha} + 1/t_1^{\alpha})\sigma(t)$$

and

$$k_0(D_{RL}^{\alpha} + 1/\tau_1^{\alpha})w(x,y,t) = (D_{RL}^{\alpha} + 1/t_1^{\alpha})R(x,y,t) \quad (15)$$

where D_{RL}^{α} is the Riemann-Liouville fractional differentiation of order α, $\tau_1 = \eta/E_1$, τ is the creep time, R is the foundation reaction and $t_1 = \tau_1/\sqrt[\alpha]{1 + E_0/E_1}$. FMM reduces to SMM when $\alpha = 1$.

2.2. Numerical Modelling

The most common methods nowadays are the numerical methods as they can be applied in cases of complex boundary conditions and complex loadings. Apart from linear and non-linear problems, they can be commonly used for considering the elasto-plastic and viscoelastic behavior of geotechnical media. A basic introduction of different numerical methods is presented below.

A. Finite Difference Method (FDM): FDM is one of the most general numerical methods to be used for analysis in structural mechanics. It is also used to solve a variety of dynamic and elastic stability problems of plates. In FDM, the governing plate equation is transformed into a set of simultaneous algebraic equations by means of the corresponding boundary conditions. The solution is then achieved by means of computers or advanced calculators. Some of the improved finite difference methods have been dealt with in the text by Szilard [56]. In earlier works, Straughan [57] used FDM to analyse the plates based on modified Vlasov model. Since it considered rectangular geometry, it was easy to use FDM but for complex geometries such as plate with hole, use of FDM becomes difficult.

B. Finite Grid Method (FGM): Another numerical method is the finite grid method which considers plate as various interconnected strips which together forms a continuous surface. This method is advantageous in case of complex boundary conditions and loadings. Karasin et al. [58] analyzed circular plates on two-parameter elastic foundation considering the circular plates as radially and tangentially interconnected beam elements. They recommended this method to be used in mat foundations and other plate problems since it can also take care of plates with varying thickness and foundation properties. This method was also used for studying the case studies for circular and annular plates resting on elastic foundation and it was found that increasing divisions in tangential or radial directions does not indicate a better convergence, instead there is a limitation to the number of divisions.

C. Finite Element Method (FEM): Due to its versatility and flexibility, FEM is the most widely used numerical technique nowadays. Almost all software packages which are used to analyze rafts are based on FEM. The representation of the overall system including boundary conditions and loading conditions is close enough in case of FEM [56]. The application has now been extended from structural mechanics problem to several other engineering applications such as heat conduction & fluid dynamics problems. FEM can be used for most of the complexities in geometries, loading conditions as well as boundary conditions [59,60].

Cheung & Zinkiewicz [61] used FEM to analyze the tanks and plates on elastic foundation. Foundation both as Winkler's model as well as on elastic continuum was analyzed. They stated the irregularities in plates to be handled easily and the calculations involved to be solved rapidly using FEM. Further, Cheung & Nag [38] observed the significant horizontal deformations along with the vertical ones affecting the stresses that should be taken care of while analyzing.

D. Boundary Element Method (BEM): In recent years, BEM has gained a lot of popularity in several engineering areas. There are several researchers who had worked on the thin plates resting either on Winkler foundation [63–35] or on continuous subgrade [66].

Shear deformation is also considered in addition to simple bending while modeling a raft as a thick plate [30]. Pieces of literature which deal with the thick plates on elastic foundation using BEM can be found [67,68]. Fundamental solutions for BEM have also been provided. Most of the times, plates on the elastic foundation are analyzed on the basis of thin plate theory but the inclusion of the shear-deformation effect tends to give a more precise result. When shear deformation comes into account, the plate is considered thick, and it has three degrees of freedom.

Depending on the parameters of the plate and the foundation, Balas et al. [69] suggested three cases of the fundamental solutions in which they could analyze one case while other two cases were solved by Rashed et al. [68]. A few software based on BEM have also been established for modeling tunneling, crack mechanics, plates on elastic foundation etc.

E. FEM-BEM Coupling Approach: Mendonca & Paiva [70] observed the behavior of the raft and piled-raft foundations using FEM-BEM coupled approach. The raft modeling was done on FEM defining it as linearly elastic and the soil was considered as an elastic half-space in BEM. Responses of raft foundation were observed to be close to the other numerical and analytical solutions. However, in the case of rigid piled raft foundations, a significant change was observed in bending moment values probably due to higher stress concentrations compared to flexible foundations.

Padron et al. [71] considered viscoelastic half-space under earthquake waves to support the structure over piles. A dynamic formulation of piles in the frequency field was done through coupled 3D FEM-BEM. The simulation of piles as Bernoulli beams made use of FE analysis while the BEM was used to model the geotechnical media. Using this direct approach, the effects of a few more parameters related to the seismic effect has also been considered for proper investigation of soil interaction on shear structures.

Vasilev et al. [72] proposed a hybrid approach based on FEM-BEM coupling to observe the seismic response of an engineering structure considering a few geophysical and geological properties. They used BEM for modeling the soil media and FEM for handling the dynamic behavior of the structure and the surrounding soil media.

Schepers [73] suggested a quick method for soil-structure interaction problems using coupled FEM-BEM approach in which discretization of the structure has been done using FEM while BEM is employed for the interface discretization. The equation of motion in the frequency domain was derived for this dynamic problem and ANSYS v14 was used for FEM execution of the responses. Green's functions as a result of Thin Layer Method were used for solving the BEM integral equations.

Analysis of building foundation plates are done by Rashed & Aliabadi [74] considering quadratic isoparametric boundary integrals and the results are compared with the other methods discussed above. In the formulation, only the discretization of plate boundary is done rather than that of the entire problem.

3. Raft Foundation

A comparative study has been done by Chilton and Wekezer [75] for Timoshenko solution, Winkler based model and ANSYS solution. The Convergence test based on displacements for the cases of point as well as uniform loading showed that plate without the elastic foundation yields an upper bound solution whereas plate supported on springs shows lower bound convergence.

The swelling and shrinkage in expansive soils cause performance problems in the supported raft foundation. This variation in water-content can be described generally in terms of soil-suction. El-Garhy et al. [76] established a 2D soil-structure interaction model using the finite element analysis to determine the deformations, bending moments and shear forces for a raft on expansive soil. This model needs only the initial soil suction conditions along with the boundary conditions. Seo et al. [77] offered design charts for assessment of bearing capacity of clay over which the raft foundation is rested and then numerically analyzed elasto-plastic models to design piled raft foundation. The anisotropic property of the clay has also been considered in a few works [78,79]. Flexible circular foundation on cross-anisotropic elastic soil was solved by a variational method in the literature [80]. Minimum potential energy method with a six-degree polynomial has been used for the analysis under uniform and parabolically distributed load.

Rashed [66] analyzed the building raft as thick plate using a new BEM which can be used for non-homogeneous soil as well. The model was made more realistic by considering entire loads as column loads and free boundary conditions are provided. The non-linear behavior of layered soil below the plate due to vast variation in their moduli has been analyzed by Jayachandran et al. [81]. They have considered two terms of Maclaurin's series to derive non-linear stiffness matrices and parametric study has been carried out of the same.

A parametric analysis has been reported by Tabsh & El-Emam [82] using SAFE software in order to understand the behaviour of design variables. Linearly elastic 2D analysis showed that the effect of Young's modulus and Poisson's ratio of concrete on shear forces and bending moments is not significant, unlike its geometry. It was found that increase in raft thickness increases the raft rigidity, thus increasing the uniformity in soil bearing pressure. Lastly, the subgrade modulus tends to moderately effect the bending moments inside raft.

In thicker rafts, a large amount of cement is used which produces high heat of hydration. Resultant temperature stress can cause cracks in the foundation, particularly in the early stage. Jingliang and Dong [83] considered one practical problem and suggested that proper mix designing and using cement which has low hydration property can be used to reduce the crack occurrence. In addition to this, proper vibration and timely surface maintenance can reduce heat evolution.

Non-linear analysis of flexible raft foundation was done by El-Garhy et al. [84] using PLAXIS 3D where the supporting granular layer was displayed as a shear layer. Further, enhancement due to floating piles was also studied and reduction in displacements and bending moment was observed. An axisymmetric annular plate on Winkler foundation was analysed by Foyouzat & Mofid [85] for linearly varying subgrade modulus and subjected to linearly varying distributed load. This study used infinite power series method for the bending analysis and the results were compared using FE program. The study also suggested the presented approach to be applied to FG plates of variable thickness and even for other improved foundation models.

Ghalesari et al. [86] analysed raft foundation supported over mixture of coarse- and fine-grained soil. Drucker-Prager plasticity model was used for underlying soil and the foundation was subjected to eccentric vertical loading. Soil properties were determined by field & lab testing and then FE analyses were done to find out the response of eccentrically loaded foundation.

4. Summary

Designers conventionally treat raft as a rigid foundation and design it accordingly with guidance from some textbook or manual by following their procedures. This approach makes it safer but is not economical. Design basically means it should be safe and economical, hence considering raft as rigid foundation cannot always be a better option. This design makes thicker raft and asks for requirement of more reinforcement, making it expensive. So, a better exercise maybe to design it assuming flexible foundation which reduces the bending moment and hence the cost too. While designing a raft foundation, proper consultation of structural engineers as well as geotechnical engineers should always be taken and based on the problem, they should suggest the best model.

The most basic modelling starts with the Winkler model. Winkler model was proposed for beams on elastic foundation and later, extended to problems on plates. Since this model considers soil beneath the foundation as discrete springs, the only parameter to estimate was its stiffness, i.e., the modulus of subgrade modulus of soil. Hence different authors and researchers have suggested methods and empirical formulas to modify the subgrade modulus. From the literature, it was found that the modulus of subgrade reaction also depends upon depth of soil, stiffness of beam/plate and type of loading. Its value changes even for a certain type of soil in different conditions. However, models like those proposed by Kerr, Pasternak involved more than one parameter to encounter the effect of settlement towards surrounding soil by introducing some interaction element. Another approach is to consider soil as an elastic continuum and simplify it using certain assumptions related to stresses and deformations. In recent years, efforts have been made in generalizing this approach by using variants of Winkler-based models. While using FMM, fractional differential of low order is suitable for plate supported by sandy soil and that of higher order should be used for clayey soil foundation. By varying the differential order and viscosity coefficient, long-term performance of such problems can be replicated precisely. Lastly, a few advanced models such as elastic-plastic models and visco-elastic models which consider the non-linear behavior and rheology of soil have been discussed.

From the review, wide range of applications based on plates on elastic foundation can be seen. Bending analysis, buckling analysis as well as vibrational analysis have been in view of the researchers worldwide and the analyses have been carried out by analytical and numerical methods. Analyses based on soil properties, layering in soil, variation in loading, difference in shape of plate, method of analysis etc. are several areas touched by researchers. Due to development of several commercial software in recent years, numerical methods have been used widely due to its accountability to capture complex interaction models and the demand for quick calculations. However, effort in areas of analytical ones can also not be denied. FE softwares now-a-days have built-in models for capturing the elastic and plastic properties. They also have an option for modeling based on user-definition.

5. Conclusions

Based on review done in preceding sections, following conclusions regarding soil-foundation interaction can be drawn:

1. Effect of soil-foundation interaction must be considered in the analysis for static as well as dynamic cases.

2. For flexible foundation analysis, numerical methods could be preferred over the cumbersome analytical solutions. Numerical methods can be used for a wide range of applications which include the case of complex boundary and loading conditions.

3. A clear understanding of the modulus of subgrade reaction is always needed in case of a raft or piled raft foundation design. Other parameters included for interaction should also be carefully investigated.

4. Apart from the finite element analysis, boundary element method and FEM-BEM coupling approach are now widely used for the foundation analysis, especially in the dynamic cases.

5. Large numbers of commercial software such as ANSYS, ABAQUS, PLAXIS etc. are available nowadays to analyze linear & non-linear soil-structure interaction problems. The only problem is to deal with the non-uniformity and uncertainty in soil property throughout.

6. Uniform loading and geometry may be considered as a preliminary study. However, for actual detailed analyses, variation in the structural loads should also be taken into account which is actually possible in current era of modern computers.

7. It was also noticed that convergence studies have been done wherever required, being it the case of meshing or the selection of geometry. Since there is always a threshold value in such cases, the study will make the design efficient.

8. There is also a huge scope of research regarding the interface between soil and raft.

9. In order to consider the response in actual practice, proper selection of model and then its experimental validation should be done.

10. Several models have already been into existence, still, there could be scope of modified models that can replicate the problem more realistically.

References

[1] A.C. 336 (1993). "Suggested Analysis and Design Procedures for Combined Footings and Mats."

[2] Indian Standard Code of Practice for Design and Construction of Raft Foundations (1981).

[3] Winkler E (1867). Die Lehre von der Elasticitaet und Festigkeit: Dominicus,.

[4] Liou, G.S., Lai, S.C. (1996). “Structural analysis model for mat foundations.” Journal of Structural Engineering, Vol. 122, No. 9, pp. 1114–1117, DOI:10.1061/(asce)0733-9445(1996)122:9(1114).

[5] Wyman, M. (1950). “Deflections of an Infinite Plate.” Canadian Journal of Research, Vol. 28, No. A, pp. 292–295.

[6] Selvadurai, A.P.S. (1979). “Elastic Analysis of Soil-Foundation Interaction.” Elsevier Scientific Publishing Company.

[7] Dutta, S.C., Roy, R. (2002). “A critical review on idealization and modeling for interaction among soil – foundation – structure system.” Computer and Structures, Vol. 80, pp. 1579–1594.

[8] Lee, J., Jeong, S. (2016). “Experimental Study of Estimating the Subgrade Reaction Modulus on Jointed Rock Foundations” Rock Mechanics and Rock Engineering, Vol. 49, No. 6, pp. 2055–2064, DOI: 10.1007/s00603-015-0905-9.

[9] Biot, M.A. (1937). “Bending of an Infinite Beam on Elastic Foundation” Journal of Applied Mechanics, Vol. 59, pp. A1–A7.

[10] Terzaghi, K. (1955). “Evaluation of Coefficients of Subgrade Reaction” pp. 297–325.

[11] AB. Vesic (1961). “Bending of Beams Resting on Isotropic Solid.” J. Eng. Mech. Div. ASCE, Vol. 87, pp. 35-54.

[12] AASHTO (2004). “AASHTO Guide for Design of Pavement Structures.”

[13] Sall, O.A., Fall, M., Berthaud, Y., Ba, M. (2013). “Influence of the Elastic Modulus of the Soil and Concrete Foundation on the Displacements of a Mat Foundation.” Open Journal of Civil Engineering, Vol. 3, pp. 228–233.

[14] Horvath, J.S. (1983). “New Subgrade Model Applied to Mat Foundations.” Journal of Geotechnical Engineering, Vol. 109, No. 12, pp. 1567–1587, DOI: doi.org/10.1061/(ASCE)0733-9410(1983)109:12(1567).

[15] Kirsch, A. (2011). “Analytical and numerical investigation of the subgrade modulus for raft and pile-raft-foundations.” Calculation Methods in Geotechnics- Failure Mechanisms and Determination of Parameters,.

[16] Chandra S, Madhav M.R., Iyengar N.G.R. (1987). "A new model for nonlinear subgrades." Math. Model, Vol. 8, pp. 513–518.

[17] J. E. Bowles (1997). "Foundation Analysis and Design."

[18] Daloglu, A.T., Vallabhan, C.V.G. (2000). "Values of k for Slab on Winkler Foundation" Journal of Geotechnical and Geoenvironmental Engineering, Vol. 126, No. 5, pp. 463–471.

[19] Filonenko-Borodich MM. (1940). "Some approximate theories of elastic foundation." Uchenyie Zapiski Moskovskogo Gosudarstvennogo Universiteta [in Russian], Mekhanica, Vol. 46, pp. 3–18.

[20] Celep Z. (1989). "Rectangular Plates Resting on Tensionless Elastic Foundation." *Journal of Engineering Mechanics*, Vol. 114, No. 12, pp. 2083–2092.

[21] Hetenyi M., (1946). "Beams on elastic foundations, University of Michigan Press."

[22] Pasternak PL. (1954). "On a new method of analysis of an elastic foundation by means of two foundation constants." [in Russian]-Gosudarstvennoe Izdatelstvo Literaturi po Stroitelstvu I Arkhitekture, Moscow, USSR.

[23] Dumir, P.C. (1987). "Circular plates on Pasternak elastic foundations." *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 11, No. 1, pp. 51–60, DOI: 10.1002/nag.1610110105.

[24] Sokolov SN. (1952). "Circular plate on a generalized foundation.", [Russian]-Inzhinierny Sbornik, Academy of Sciences, USSR.

[25] Kerr, A.D. (1965). "A study of a new foundation model." *Acta Mechanica*, Vol. 1, No. 2, pp. 135–147, DOI: 10.1007/BF01174308.

[26] Horvath, J.S. (1993). "Beam-Column-Analogy Model for Soil-Structure Interaction Analysis." *Journal of Geotechnical Engineering*, Vol. 119, No. 2, pp. 358–364.

[27] Ti, K.S., Huat, B.B.K., Noorzaei, J., Jaafar, M.S., Sew, G.S. (2009). "A Review of Basic Soil Constitutive Models for Geotechnical Application." *Electronic Journal of Geotechnical Engineering*, Vol. 14, pp. 18.

[28] Kurian NP., Manojkumar NG.(2001). "A new continuous model for soil-structure interaction." J. Struc. Eng., Vol. 27, 269-276.

[29] Boussinesq M. J. (1885). "Application des potentiels a l'etude de l'equilibre et du mouvement des solides elastiques, principalement au calcul des deformations et des pressions que produisent, dans ces solides, des efforts quelconques exerces sur une petite partie de leur surface ou de leur interieur." Memoire suivi de notes etendues sur divers points de physique mathematique et d'analyse, GauthierVillars, Paris, 722.

[30] Reissner, E. (1947). "On Bending of Elastic Plates." *Quart Appl Math*, Vol. 5, pp. 55–68.

[31] Vlasov, V. Z., Leontiev, U. N. (1966). "Beams, plates and shells on elastic foundations." Israel Program for Sientific Translations, Jerusalem.

[32] Jones, R., Xenophontos, J. (1977). "The Vlasov Foundation Model." *International Journal of Mechanical Sciences*, Vol. 19, pp. 317–323.

[33] Vallabhan, C.V.G., Das, Y.C. (1989). "Parametric Study of Beams on Elastic Foundations." *Journal of Engineering Mathematics*, Vol. 114, No. 22996, pp. 2072–2082.

[34] Vallabhan, C.V.G., Das, Y.C. (1991). "Modified Vlasov Model for Beams on Elastic Foundations." *Journal of Geotechnical Engineering*, Vol. 117, No. 6, pp. 956–966.

[35] Vallabhan, C.V.G., Daloglu, A.T. (1999). "Consistent FEM-Vlasov Model for Plates on Layered Soil." *Journal of Structural Engineering*, Vol. 125, No. January, pp. 108–113.

[36] Teodoru, I., Musat, V. (2010). "The Modified Vlasov Foundation Model : An Attractive Approach for Beams Resting on Elastic Supports." *Electronic Journal of Geotechnical Engineering*, No. January, pp. 1–13.

[37] Worku, A. (2010). "Part I: A Generalized Formulation of Continuum Models for Elastic Foundations." *GeoFlorida 2010: Advanced in Analysis, Modeling & Design*, No. Gsp 199, pp. 2012–2021.

[38] Worku, A., Yimer, D. (2010). "Part II: Application of Newly Derived and Calibrated Continuum Subgrade Models in the Analysis of Beams on Elastic Foundations." *GeoFlorida 2010: Advanced in Analysis, Modeling & Design*, No. Gsp 199, pp. 1885–1894.

[39] Worku, A. (2013). "Calibrated analytical formulas for foundation model parameters." *International Journal of Geomechanics*, Vol. 13, No. 4, pp. 340–347, DOI: 10.1061/(ASCE)GM.1943-5622.0000214.

[40] Worku, A. (2014). "Development of a calibrated Pasternak foundation model for practical use." *International Journal of Geotechnical Engineering*, Vol. 8, No. 1, pp. 26–33, DOI: 10.1179/1938636213Z.00000000055.

[41] Smolira M. (1975). "Analysis of tall buildings by the force displacement method." UK: McGraw-Hill Book Company Limited.

[42] Selvadurai, A.P.S. (1976). "The response of a rigid circular plate resting on ana idealized elastic-plastic foundation." *International Journal of Mechanical Sciences*, Vol. 18, pp. 463–468.

[43] Papadopouios, P., Taylor, R.L. (1990). "Elasto-plastic analysis of Reissner-Mindlin plates." *Applied Mechanics Reviews, ASME*, Vol. 43, No. 5(2), pp. S40-50.

[44] Noorzaei, J., Viladkar, M.N., Godbole, P.N. (1995). "Elasto-plastic analysis for soil-structure interaction in framed structures." *Computers and Structures*, Vol. 55, No. 5, pp. 797–807, DOI: 10.1016/0045-7949(94)00432-3.

[45] Liang, J., Lu, D., Zhou, X., Du, X., Wu, W. (2019). "Non-orthogonal elastoplastic constitutive model with the critical state for clay." *Computers and Geotechnics*, Vol. 116, No. July, p. 103200, DOI: 10.1016/j.compgeo.2019.103200.

[46] Liang, J., Lu, D., Du, X., Wu, W., Ma, C. (2020). "Non-orthogonal elastoplastic constitutive model for sand with dilatancy." *Computers and Geotechnics*, Vol. 118, No. October 2019, p. 103329, DOI: 10.1016/j.compgeo.2019.103329.

[47] Younesian, D., Hosseinkhani, A., Askari, H., Esmailzadeh, E. (2019). "Elastic and viscoelastic foundations : a review on linear and nonlinear vibration modeling and applications." *Nonlinear Dynamics*, Vol. 97, No. 1, pp. 853–895, DOI: 10.1007/s11071-019-04977-9.

[48] Arani, A.G., Jalaei, M.H. (2016). "Nonlocal dynamic response of embedded single-layered graphene sheet via analytical approach." *Journal of Engineering Mathematics*, Vol. 98, No. 1, pp. 129–144, DOI: 10.1007/s10665-015-9814-x.

[49] Veletsos, A.S., Meek, J.W. (1974). "Dynamic behaviour of building-foundation systems." *Earthquake Engineering & Structural Dynamics*, Vol. 3, No. 2, pp. 121–138, DOI: 10.1002/eqe.4290030203.

[50] Meek, J.W., Wolf, J.P. (1992). "Cone Models for Homogeneous Soil." *Journal of Geotechnical Engineering*, Vol. 118, No. 5, pp. 667–685.

[51] Anvarsamarin, A., Rahimzadeh Rofooei, F., Nekooei, M. (2020). "Torsion effect on the RC structures using fragility curves considering with soil-structure interaction." Journal of Rehabilitation in Civil Engineering, Vol. 8-1, pp. 01-21.

[52] Horvath, J.S., Colasanti, R.J. (2011). "Practical Subgrade Model for Improved Soil-Structure Interaction Analysis : Model Development." No. February, pp. 59–64.

[53] Ganjavi, B., Rezagholilou, A. (2019). "Seismic evaluation of flexible-base low-

rise steel frames using Beam-on-Nonlinear-Winkler-Foundation" modeling of shallow footings." Journal of Rehabilitation in Civil Engineering, Vol. 7-4, pp. 57-71.

[54] Gemant, A. (1936). "A method of analyzing experimental results obtained from elasto-viscous bodies." *Journal of Applied Physics*, Vol. 7, No. 8, pp. 311–317, DOI: 10.1063/1.1745400.

[55] Zhang, C., Zhu, H., Shi, B., Liu, L. (2014). "Theoretical investigation of interaction between a rectangular plate and fractional viscoelastic foundation." *Journal of Rock Mechanics and Geotechnical Engineering*, Vol. 6, No. 4, pp. 373–379, DOI: 10.1016/j.jrmge.2014.04.007.

[56] Szilard, R. (2004). *Theories and Aplications of Plate Analysis*, John Wiley & Sons, Inc.

[57] Straughan, W.T. (1990). *Analysis of plates on elastic foundation*.

[58] Karasin, H., Gülkan, P., Aktas, G. (2014). "A Finite Grid Solution for Circular Plates on Elastic Foundations." *KSCE Journal of Civil Engineering*, Vol. 00, No. 0000, pp. 1–7, DOI: 10.1007/s12205-014-0713-x.

[59] Abdolrezayi A, Khayat N. Comparative Three-Dimensional Finite Element Analysis of Piled Raft Foundations. Comput Eng Phys Model 2021;4:19–36.

[60] Bunyamin S, Aghayan S. Settlement Modelling of Raft Footing Founded on Oferekpe/Abakaliki Shale in South East Region of Nigeria. Comput Eng Phys Model 2018;1:68–82.

[61] Cheung, Y.K., Zienkiewicz, O.C. (1965). "PLATES AND TANKS ON ELASTIC FOUNDATIONS-AN APPLICATION OF FINITE ELEMENT METHOD." *International Journal of Solids and Structures*, Vol. 1, pp. 451–461.

[62] Cheung, Y.K., Nag, D.K. (1968). "PLATES AND BEAMS ON ELASTIC FOUNDATIONS- LINEAR AND NON-LINEAR BEHAVIOUR." *Geotechnique*, Vol. 18, pp. 250–260.

[63] Katsikadelis, B.J.T., Armenakas, A. E., (1984). "Plates on elastic foundation by bie method." Vol. 110, No. 7, pp. 1086–1105, DOI: 10.1061/(ASCE)0733-9399(1984)110:7(1086).

[64] Costa, J. A., C. A. Brebbia (1985). "The boundary element method applied to plates on elastic foundations." *Engineering Analysis*, Vol. 2, No. 4, pp. 174–183, DOI: https://doi.org/10.1016/0264-682X(85)90029-2.

[65] Bezine, G. (1988). "A new boundary element method for bending of plates on elastic foundations." *International Journal of Solids and Structures*, Vol. 24, No. 6, pp. 557–565, DOI: 10.1016/0020-7683(88)90057-1.

[66] Rashed, Y. F. (2005). "A boundary domain element method for analysis of building raft foundations." *Engineering Analysis With Boundary Elements*, Vol. 29, pp. 859–877, DOI: 10.1016/j.enganabound.2005.04.007.

[67] Jianguo, W., Xiuxi, W., Maokuang, H. (1993). "A Boundary Integral Equation Formulation for Thick Plates on a Winkler Foundation." *Computer & Structures*, Vol. 49, No. I, pp. 179–185.

[68] Rashed, Y., Aliabadi, M. H., Brebbia, C. A. (1998). "The Boundary Element Method for Thick Plates on a Winkler Foundation." *International Journal for Numerical Methods in Engineering*, Vol. 1462, No. June 1997, pp. 1435–1462.

[69] Balaš, J., Sládek, V., Sládek, J. (1984). "The Boundary Integral Equation Method for Plates Resting on a Two-Parameter Foundation." *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 64, No. 3, pp. 137–146, DOI: 10.1002/zamm.19840640302.

[70] Mendonca, A.V., Paiva, J.B. (2003). "An elastostatic FEM / BEM analysis of vertically loaded raft and piled raft foundation." *Engineering Analysis With Boundary Elements*, Vol. 27, pp. 919–933, DOI: 10.1016/S0955-7997(03)00061-4.

[71] Padron, L.A., Aznarez, J.J., Maeso, O. (2009). "Dynamic structure-soil-structure interaction between nearby piled buildings under seismic excitation by BEM-FEM

model." *Soil Dynamics and Earthquake Engineering*, Vol. 29, pp. 1084–1096, DOI: 10.1016/j.soildyn.2009.01.001.

[72] Vasilev G., Parvanova S., Dineva P., Wuttke F. (2015)., "Soil-structure interaction using BEM – FEM coupling through ANSYS software package." Soil Dyn. Earthq. Eng., Vol. 70,pp. 104–117.

[73] Schepers, W. (2017). "Fast 3D FEM-BEM coupling for dynamic soil-structure interaction." *Procedia Engineering*, Vol. 199, pp. 391–396, DOI: 10.1016/j.proeng.2017.09.129.

[74] Rashed, Y. F., Aliabadi, M. H. (2000). "Boundary element analysis of foundation plates in buildings." *Engineering Analysis With Boundary Elements*, Vol. 24, pp. 201–206, DOI: 10.1007/s004660050503.

[75] Chilton, D.S., Wekezer, J.W. (1991). "Plates on Elastic Foundation." *Journal of Structural Engineering*, Vol. 116, No. 11, pp. 3236–3241.

[76] El-Garhy, B.M., Wray, W.K., Youssef, A.A. (2000). "Using soil diffusion to design raft foundation on expansive soils." *ASCE*, pp. 586–601.

[77] Seo, Y.-K., Choi, K.S., Jeong, S.-G. (2003). "Design Charts of Piled Raft Foundations on Soft Clay." *Proceedings of The Thirteenth (2003) International Offshore and Polar Engineering Conference*, Vol. 5, pp. 753–755.

[78] Borowicka H. (1936). "Influence of rigidity of a circular foundation slab on the distribution of pressure over the contact surface." In: Proc. First Int. conf. on Soil Mech. Found. Eng., Vol. 2, pp. 144–149.

[79] Hooper, J.A. (1974). "Analysis of a circular raft in adhesive contact with a thick elastic layer." *Geotechnique*, Vol. 24, No. 4, pp. 561–580.

[80] Gazetas, G. (1981). "Variational estimation of the settlement of a circular raft on anisotropic soil." *Japanese Society of Soil Mechanics and Foundation Engineering*, Vol. 21, No. 4.

[81] Jayachandran, S.A., Seetharaman, S., and Abraham, S. (2008). "Simple Formulation for the Flexure of Plates on Nonlinear Foundation." *Journal of Engineering Mechanics, ASCE*, Vol. 134, No. January, pp. 110–115, DOI: 10.1061/(ASCE)0733-9399(2008)134:1(110).

[82] Tabsh, S.W., El-Emam, M.M. (2014). "Finite element-based parametric analysis of mat foundations." *Numerical Methods in Geotechnical Engineering- Hicks, Brinkgreve & Rohe (Eds)*, No. June 2014, pp. 693–697, DOI: 10.1201/b17017-124.

[83] Jingliang, Y., Dong, Y. (2014). "Construction Technical Measures and Control of Concrete Temperature Cracking for a Mass Flat Plate Type Raft Foundation." *Applied Mechanics and Materials*, Vol. 484–485, pp. 717–721, DOI: 10.4028/www.scientific.net/AMM.484-485.717.

[84] El-garhy, B., Galil, A.A., Mari, M. (2017). "Analysis of flexible raft resting on soft soil improved by granular piles considering soil shear interaction." *Computers and Geotechnics*, No. September,DOI: 0.1016/j.compgeo.2017.09.007.

[85] Foyouzat, M.A., Mofid, M. (2019). "An analytical solution for bending of axisymmetric circular/annular plates resting on a variable elastic foundation." *European Journal of Mechanics, A/Solids*, Vol. 74, No. September 2018, pp. 462–470, DOI: 10.1016/j.euromechsol.2019.01.006.

[86] TaghaviGhalesari, A., Tabari, M.K., Choobbasti, A.J., EsmaeilpourShirvani, N. (2019). "Behavior of eccentrically loaded shallow foundations resting on composite soils." *Journal of Building Engineering*, Vol. 23, No. March 2018, pp. 220–230, DOI: 10.1016/j.jobe.2019.01.036.