

Journal of Rehabilitation in Civil Engineering

Journal homepage:<https://civiljournal.semnan.ac.ir/>

Evaluation and Retrofit of Circular Reinforced Concrete Silos Built in Lima, Peru, in the 1970s

Jorge L. Bazan 1,* ; Victor I. Fernandez-Davila ²

1. Top Consult Ingeniería, Lima, Peru

2. Associate Professor. Faculty of Civil Engineering, National University of Engineering, Lima, Peru

* Corresponding author: *<jlbazans@uni.pe>*

ARTICLE INFO ABSTRACT

Article history: Received: 16 October 2022 Revised: 15 March 2023 Accepted: 05 July 2023

Keywords: Circular silos; Reinforced concrete; Non-conventional structure; Evaluation; Seismic retrofit.

The assessment of two circular reinforced concrete (RC) silos built in the 1970s is presented. The methodology implemented includes data collection, testing, analysis requirements, structural modeling, evaluation results, and retrofit design. The RC silos were originally designed in the 1970s with very low seismic demand and insufficient engineering details. Evaluation of this specialized structure has reported both stability and strength, as well as severe issues that need to be addressed. National and international codes and guidelines were consulted to define the most appropriate seismic demand, gravitational, and incidental loads, as well as load combination criteria, evaluation and diagnostic requirements, structural design criteria, and effective retrofitting techniques. The main retrofit strategy focused on improving the stability of the silos against overturning, reducing normal stresses on the ground, and strengthening the RC walls to withstand combined axial and bending effects in line with current requirements. The retrofit design included foundation extension and RC jacketing of the silo walls to address these issues. A rational approach to evaluating and retrofitting this type of non-conventional RC structure was developed. Additionally, a cost assessment based on the comparison of concrete and steel reinforcement volumes between the existing silo and the strengthened components is presented.

E-ISSN: 2345-4423

© 2024 The Authors. Journal of Rehabilitation in Civil Engineering published by Semnan University Press. This is an open access article under the CC-BY 4.0 license. [\(https://creativecommons.org/licenses/by/4.0/\)](https://creativecommons.org/licenses/by/4.0/)

How to cite this article:

Bazan, J., & Fernandez-Davila, V. (2024). Evaluation and Retrofit of Circular Reinforced Concrete Silos Built in Lima, Peru, in the 1970s. Journal of Rehabilitation in Civil Engineering, 12(2), 1-13. <https://doi.org/10.22075/jrce.2023.28722.1731>

1. Introduction

The silos are designed to store various materials, such as grain, cement, coal, aggregates, and other granular substances, for multiple purposes. They are typically characterized by a slender geometry with circular cross-sections. Due to industrial needs, this type of non-conventional structure is often required, especially in the food and construction sectors. Additionally, the most common materials for silo construction are reinforced concrete (RC) and structural steel.

Because silos are very rigid structures, it might present an undesirable brittle failure. They are designed mainly to support gravitational loads (self-weight, and live loads), and the effects due to the content that generate the internal pressures, and the frictional loads in the walls due to the flow of the material during its process of filling and discharging [1]. In addition, eventual loads like ground motion movement due to the earthquake must be considered.

The failure modes of these types of structures are mainly associated with combined loading such as compression and tension due to bending at the base of the walls or columns (Fig. 1(a)), the overturning of the structure (Fig. 1(b)), the overstressing of the soil, or settlement of the foundation (Fig. $1(c)$) [2], [3], as well as bending failures in the walls due to the pressures of the material deposited inside.

Graham and Rodriguez [4] proposed a theoretical linear solution for the impulsive and convective pressures of a fluid contained in a moving rectangular container. Housner [5] developed simplified equations to determine the linear hydrodynamic pressures on an asymmetrical container with thin, massless walls. Kana [6] developed a nonlinear analytical model to represent fluid motion in a spherical container. Li and Wang [7] presented an exact solution for an equivalent model of oscillating fluid in a rectangular tank, which supplements the traditional solutions by Graham and Rodriguez, as well as Housner. Lopez and Fernandez-Davila [8] developed simplified expressions to evaluate the seismic response of circular RC silos, achieving results within acceptable error margins. Currently, there are documents such as ACI 313 [9] and Eurocode 1 and 8 [10], [11] for the design and construction of silos and storage structures. These standards have been based on depth and detailed analysis that has been achieved with reliable theory and experience. However, many silos have been designed with imprecise assumptions and built following less demanding and detailed guidelines [2], [3]; consequently, many of these structures have experienced severe damage or collapse (Fig. 1). The collapse of silos not only represents economic losses linked to reconstruction, discontinuation of economic activities, and loss or contamination of content but also poses a significant risk to the health and safety of the personnel working in these industrial spaces.

The purpose of this report is to present an important case study corresponding to the structural evaluation of an existing silo, which was designed and built in the 1970's decade. In this decade the seismic design philosophy was still developing thanks to the support of several eminent researchers, and the seismic codes were evolving slowly and did not contain the lessons learned from the great earthquakes that occurred [12]. The evaluation results confirmed the defects of the seismic designs of that decade.

The main structural deficiencies of the silo were fully identified in accordance with current standards and codes. Additionally, to ensure operational continuity, a rational approach was developed for evaluating and retrofitting this non-conventional RC structure.

Fig. 1. Some typical silo damage (a) at the base of a coal silo due to overloading [2]. (b) three silos were injured during the 1999 Turkey Earthquake [2]. (c) failure due to foundation overstresses [3].

2. Methodology

The ASCE-41 [13] guidelines were consulted to implement an efficient methodology using a three-tier approach. The first level involved gathering information on the existing structure, conducting in-situ tests on the RC walls, and assessing the foundation soil characteristics. At the second level, structural evaluation criteria were established, and an analysis of the existing structure was performed to identify the most critical deficiencies. Finally, at the third level, basic and detailed engineering plans for strengthening the silo were implemented.

2.1. First level of study

Description of the existing structure:

The structure is made up of two RC silos with circular sections, adjoining each other, with a uniform cross-section throughout the entire height and an outer diameter of $D = 6500$ mm (Fig. 2(a)).

The silo walls have an average thickness of *e*=270 mm, each section is erect from a circular combined foundation plate with a thickness h_p = 600 mm, and a diameter D_p = 8300 mm. (Fig. 2(b)). The reinforcement of the RC walls consisted of a double mesh of reinforcing steel distributed along the inner and outer faces.

The visual inspection of the silo indicated a good state of conservation of the RC. No cracks or signs of corrosion were observed in the reinforcing steel bars. Specific destructive and non-destructive tests were conducted to determine: (i) the quality of the existing concrete, (ii) the diameter and distribution of the reinforcement steel, and (iii) the condition of the steel bars. Overall, despite their considerable age, the steel bars were found to be in good condition.

Destructive and non-destructive studies:

Because the original design drawings were not available for this old structure, X-ray scans were performed on the silo walls (Fig. 2(c)). This procedure allowed for determining the diameter of the reinforcing steel bars, their distribution in each direction, and the thickness of the concrete cover. The existing RC silos were built with double mesh: vertical reinforcing steel bars \varnothing 1/2" (\varnothing) 300 mm, and horizontal steel bars α 1/2" @ 200 mm, with an average concrete cover of 90 mm at each face of the walls.

The concrete quality examination was implemented at twelve strategic points (six in each silo, six in the lower part, and six in the middle) through diamantine extractions and laboratory tests, according to ASTM C-42. Average resistance to concrete compression equal to 20.0 MPa and a standard deviation of \pm 2.3 MPa were obtained. Similarly, the carbonatation tests, according to UNE - EN 14630: 2007, showed that it had not touched the steel reinforcing bars, the results exhibited

a carbonation depth ranging from 15 to 50 mm.

Geotechnical studies:

The results of geotechnical studies showed that the foundation soil is stiff and dense, consisting mainly of gravel, classified as GW according to USCS. A static soil-bearing capacity (*σadm*) of 0.44 MPa was determined.

Fig. 2. Existing RC Silos. (a) General view. (b) Silos cross-section. (c) Steel reinforcement scan with X-Ray equipment.

2.2. Second level of study

The second level included: i) The selection of design parameters and criteria ii) The structural evaluation of the existing structure and iii) The seismic deficiencies identification. Also, a computational model was developed using finite element software representative of the RC silos.

Design load cases:

The following load cases, shown in Fig. 3, were considered:

a) Gravitational loads: dead loads (DL) from silos' self-weight; Live loads (LL) due to the live loads on the roof and the granular contents; also, granular pressure and friction loads (PLL).

b) Seismic loads (SL) are calculated from the silos' weight and a percentage of the weight corresponding to the impulsive masses $(M_0 = \sum m_i)$, and the interaction of the convective mass $(M₁)$ with the structure.

Granular loads (PLL):

Granular loads are classified as live loads, producing both static and dynamic effects on the structure during filling, emptying, and storage, as well as during extreme events such as seismic activity. These effects are described in the ACI-313-16 standard, and the principal loads are the following: the vertical pressure (*q*) of the stored material, horizontal pressure (*p*) of the material on the perimeter walls, and the friction force (V) , of the material on the perimeter of the silo (Fig. 3).

Fig. 3. (a) Gravity and (b) Seismic loads considered for this study.

These effects depend on the mechanical characteristics of the granular material such as its coefficient of friction (μ) , the angle of internal friction (Φ) , and its specific weight (*ϒ*). In addition, the type of material discharge was determined to be either mass flow or funnel flow [9], [10]. The case study presented a discharge of funnel flow, and the values of its corresponding effects were calculated with the expressions given by the ACI-313 [9] and EN-1991-4 [10] standards.

The vertical pressure *q* acting at depth *Y* was calculated with (1):

$$
qq = \frac{\gamma \cdot R_H}{\mu \cdot k} \left[1 - e^{-\mu \cdot k \cdot Y / R_H} \right] \tag{1}
$$

The horizontal filling pressure *p* acting at depth *Y* was calculated with (2):

$$
p = k \cdot q \tag{2}
$$

The horizontal pressure ratio *k* is calculated with (3) :

$$
k=1-sin\phi
$$
 (3)

The vertical friction load per unit length *V* on the perimeter wall acting at depth *Y* was calculated with (4):

$$
V = (\gamma \cdot Y - q) \cdot R_H \tag{4}
$$

The distribution of granular live loads along the height of the walls $(0 \le Y \le h)$ was

calculated, where the vertical *q* and horizontal *p* pressure components are the predominant loads.

Convective and impulsive masses:

The total stored mass of the silo subjected to the ground motion (*M*) can be decomposed into two types of masses, one that moves as a rigid body together with the structure, called impulsive mass, and another one that interacts with the structure called convective mass. Table 1 shows a summary of results by different theoretical hypotheses. The results of impulsive and convective masses and their respective center locations were normalized to the total stored mass (*M*) and its equivalent total height (*h*), respectively.

Seismic Parameters:

The seismic parameters for this type of nonconventional structure were selected according to the NTPE-030 [14] and ASCE-7 [15] standards, considering a 10% probability of exceedance in 50 years and an immediate occupancy performance level to enhance the silo's earthquake resistance. Table 2 presents

the seismic parameters required to develop the pseudo-acceleration design response spectrum (Sa vs. T), which is necessary for performing the elastic spectral modal analysis. Fig. 4 compares the design response spectrum obtained from both the local Peruvian NTPE-030 standard and the American ASCE-7 standard, adapted using local parameters.

Fig. 4 shows that the design response spectrum of the American standard is slightly more conservative than that of the Peruvian standard. Notably, in the short vibration period zone, the difference in spectral ordinates is more significant compared to the long vibration period zone. For this study, the design spectrum of the Peruvian standard was chosen because it accurately represents the seismicity of the area; however, the American standard could also have been used with a reasonable degree of precision and conservativeness.

Analysis Method:

For static loads, linear and elastic analysis methods compatible with the structural condition of the silos were considered. For seismic loads, a linear dynamic procedure (LDP) was implemented using modal spectral analysis with a model characterized by uncracked linear elastic stiffness and equivalent viscous damping. This method allowed us to determine the dynamic seismic shear forces (V_d) , the internal forces of each component of the silo, the base reactions, and the lateral elastic deformations caused by seismic loads (*Δ*). The seismic shear design forces and the lateral inelastic deformations were computed by multiplying the elastic response calculated using LDP with their

respective response modification factor according to the standard NTPE-030 [14].

The vertical component of the seismic load was not considered.

Load combinations:

The load combinations considered for evaluation using allowable stress design (S) [9], [15] were the following:

- \bullet S1: 1.0 DL + 1.0 LL
- S2: $1.0 \cdot DL + 0.75 \cdot LL \pm 0.75 \cdot SL_x$
- S3: $1.0 \cdot DL + 0.75 \cdot LL \pm 0.75 \cdot SL_v$

The following load combinations were considered for evaluation using strength design (U) [9], [15]:

- U1: 1.2DL+ 1.6LL
- \bullet U2: 1.2.DL+ 1.00.LL \pm SL_x
- \bullet U3: 1.2 \cdot DL+ 1.00 \cdot LL \pm SL_v
- \bullet U4: 0.9 DL SL_x
- \bullet U5: 0.9 DL SL_v
- U6: 1.2DL+ 1.6LL+ 1.45PLL
- U7: $1.2 \cdot DL + 1.00 \cdot LL + 1.00 \cdot PLL \pm SL_x$
- \bullet U8: 1.2 DL+1.00 LL+1.00 PLL \pm SL_v

Evaluation criteria (stability, stiffness, and resistance):

Table 3 presents the main evaluation criteria considered in this case study, related to the limit states of design by service (S) and strength (U).

ASPECT	CRITERIA	
The foundation's stress level (S)	Static: σ max $\leq \sigma$ adm Dynamic: σ max ≤ 1.3 σ adm	
Safety factor for overturning (S)	Full Silo: $SF > 1.33$ Empty Silo: $SF \geq 1.20$	
Stiffness under seismic load (U)	NTPE.030: Δ ht < 0.007 ASCE-7: Δ / ht \leq 0.010	
Strength design (U) \cdot	Φ ·Rn > Ru* Φ = 0.75 for shear in silo walls Φ = 0.70 for flexo-compression Φ = 0.90 for bending and flexo-tension	

Table 3. Aspects and criteria to evaluate the RC Silos.

*According to ACI 313 [9] and ACI 318 [16].

The foundation allowable bearing pressure was examined. Similarly, the seismic performance was reviewed according to the maximum lateral drift ratio (Δ/h_t) and the safety factors against overturning. Finally, the strength was evaluated for the principal RC elements, walls, and foundation slab.

Fig. 5. Views of retrofitted RC silos. (a) Elevation, and (b) Plan.

2.3. The third level of study

The third level involved the implementation of retrofitting strategies to meet the performance level selected. The strategies implemented were focused on improving the strength capacity of the RC foundations and the walls. In addition, a good safety factor against seismic overturning was considered.

Strengthening of the RC Foundation:

The capacity in the existing foundation was not feasible to be increased, especially due to the level of flexural demand. For this reason, it was decided to design a new foundation. This strategy is contemplated in Chapter 23 of FEMA 547 [17].

The size and expansion of the new foundation were crucial for improving the safety factor against overturning and reducing foundationbearing pressure. The design considered the condition of the existing facilities, which limited the proper expansion of the new foundation.

Strengthening of the RC walls:

The significant reinforcing steel deficit due to combined tension, compression, and bending demand in the walls was resolved by designing an increase on the existing walls' cross-section, supplementing new RC areas, and increasing the amount of reinforcing steel in the horizontal and vertical direction.

Fig. 5 illustrates the new foundation slab placed on the partially demolished existing foundation, along with the increased wall cross-section up to the required height. In both cases, reinforcing steel was added and positioned to meet the new demands of the retrofitted silo, ensuring compliance with the seismic performance and design criteria established in Table 3.

To ensure the proper behavior of the increase in the cross-section of the walls, a shear flow analysis by bending was carried out. In this way, the reinforcing steel connection dowels seen in the plan view of Fig. 5 (b) and the RC shear keys shown in the elevation of Fig. 5 (a) are arranged appropriately to transmit the shear flows between the wall of the existing silos and the new RC supplementary wall area.

Fig. 6. Finite element models. (a) Existing silos. (b) Retrofitted silos.

3. Discussion of the results

Two structural models were developed for this study, and the main results of the analysis are presented in this section. The first model represents the structure of the existing silos, and the second represents the one that integrates the retrofitting components discussed in the previous section. These computational models were developed using finite element software [18], specifically employing Shell-type elements from SAP2000, and considering a linear elastic dynamic procedure. Figs. 6(a) and 6(b) show

the models of the existing silos and the silos with retrofitting elements, respectively.

3.1. Existing silos results

The modal dynamic analysis revealed that the primary lateral vibration periods (*T*) for the existing silos, without the retrofitting components, are 0.43 sec in the short direction and 0.25 sec in the long direction (x and y directions, as shown in Fig. 5(b)).

Existing silos analysis exhibited a characteristic behavior of concentration of vertical and bending (around a horizontal axis) internal forces in the bottom of the walls, specifically due to high seismic demands, where have considerable insufficient vertical reinforcing steel. Similarly, the foundation has an excessive bearing pressure, particularly due to seismic loads.

The seismic drifts are within the allowable criteria; however, stability against overturning is inadequate, particularly in the short direction of the plan (the x-direction in Fig. 5(b)), due to the small dimensions of the existing foundation. The safety factors against overturning established in Table 3 are met with a maximum shear lateral force of no greater than $0.27 \cdot W$, significantly less than the specified lateral force demand, *Vs*, of 0.49·*W*.

3.2. Retrofitted silos results:

For the retrofitted silos, the modal dynamic analysis indicated that the predominant lateral vibration periods (*T*) are 0.33 sec in the short direction and 0.22 sec in the long direction.

Retrofitted silos analysis also exhibited a characteristic concentration of internal forces due to seismic demand in the bottom of the walls, where the steel reinforcement for the strengthening design has been concentrated (Fig. 7). The steel demand results showed significant concentrations in the foundation $(Z = -0.25 \text{ m})$, and in the hopper $(Z = 4.5 \text{ m})$; and smaller other at the end of the wall strengthening (*Z*= 24 m).

The vertical steel reinforcement area required for the retrofitted silo walls for each different load case combination (R_U) is shown in Fig. 7, where the reinforcement steel required has concentrations at the bottom of the wall (*Z* < 6.00 m), and a drastic reduction of this on the top are also observed $(Z > 22.00 \text{ m})$. The blue

line represents the existing steel, the red line represents the continuous new reinforcement steel, and the green lines represent additional new steel placed on the bottom of the walls. Similarly, the dashed blue line represents the existing steel in the section without strengthening.

Table 4. Results from the evaluation of the existing and strengthened RC Silo structure.

Results	Existing silo	Strengthened silo
Fundamental period (sec)	0.43	0.33
Seismic Weight W (kN)	18,994.12	27,060.59
Ratio V_s/W (%)	48.75%	48.75%
Static Seismic Force Vs (kN)	9,259.64	13,192.00
Dynamic Seismic Force Vd (kN)	7,418.32	10,599.71
Maximum drift (mm/mm)	0.0024	0.0023
Static ground bearing pressure (MPa)	0.21	0.26
Dynamic ground bearing pressure (MPa)	0.95	0.54
Safety factor SF to Overturning (full)	0.99	1.23
Safety factor SF to Overturning (empty)	0.94	1.35

Fig. 7. Vertical steel reinforcement (SR) area demand along the height (axis Z) for the different ultimate load combinations (RU). Existing SR in blue lines, and new SR for the retrofitted silos in red and green lines.

Table 4 presents a comparative summary of the key evaluation results for both the existing and the retrofitted silos. Notable findings include an increase in weight of 8,066 kN, which is attributed to the new foundation and the widening of the vertical walls. Additionally, there is a significant decrease in dynamic ground bearing pressure, along with an improvement in the safety factors for turning. The strengthening approach effectively met the design criteria requirements outlined in Table 3.

Regardless of the significant increase in shear seismic force demand, the maximum seismic drifts (Δ / h_t) for both the existing and retrofitted silos are adequate and comparable. Another notable result in Table 4 is the reduction in the fundamental vibration period due to the increased cross-section of the silo.

4. Cost assessment

This section presents a briefly cost assessment for the retrofit of the studied silo, which is proposed in practical terms through a comparison with the existing structure.

Table 4 presents the ratios of steel reinforcing per unit volume of concrete resulting from the reinforcement, and they are compared with the design ratios of the initial structure.

Table 4. Steel reinforcement ratio (kg/m3) for existing and strengthening components of the silo.

CAISUNG and Suchguiching components of the sho.						
	Retrofitting					
silo	components of					
	the silo					
120	200					
80	140					
	Existing					

A drastic change from the design steel ratios of the 1970s compared to the ratios of the reinforcing new components obtained with the currently selected design standards can be noticed. This critical change is due to the greater seismic demand requirements of current regulations (part of these are presented in Table 3), which can serve to raise consciousness about the need to evaluate and retrofit many existing structures designed with outdated standards.

Table 5 provides a detailed breakdown of the major construction activities for both the existing structure and the strengthened components involved in the retrofitting process. Additionally, it compares the quantity ratios of similar construction activities to offer a clearer understanding of the investment required for both retrofitting and safeguarding the existing structure.

	Table 5. Comparison of various quantities and ratios in sho construction activities.							
	Item	Unit	Existing components	Strengthening components	Strengthening / Existent silos ratio			
	Excavation	m^3	230.00	380.00	1.65			
	Localized demolitions	m ³		45.00				
	Formwork	m ²	1,570.00	1,100.00	0.70			
	Concrete	m ³	470.00	435.00	0.93			
	Reinforcing steel	Кg	37,000.00	76,000.00	2.05			
	Connection Dowell	Und	$\overline{}$	5,600.00				

Table 5. Comparison of various quantities and ratios in silo construction activities.

(-) No value.

5. Conclusions

This study presents a rational approach to evaluating and retrofitting reinforced concrete (RC) silos constructed in Lima, Peru, during the 1970s. The evaluation focused on identifying the main structural deficiencies of these non-conventional structures according to current regulations. Based on this assessment, a retrofitting procedure was proposed to meet the specified resistance and service criteria

outlined in Table 3. This proposal considers a seismic hazard level of 10% over 50 years and aims for immediate occupancy performance. The key conclusions drawn from this case study are as follows:

- a) The existing silos, designed and built in the 1970s, adequately meet the gravitational demands resulting from their self-weight and granular loads. However, when assessed for seismic loads, several critical deficiencies were identified regarding their service and resistance behavior. These deficiencies include: i) an inadequate safety factor (SF) against overturning; ii) excessive pressure on the foundation due to dynamic conditions; and iii) insufficient flexural strength in both the foundations and the upper third of the RC walls.
- b) The extension of the new foundations was essential for improving the safety factor (SF) against overturning, primarily due to the additional stabilizing weight. This extension also contributed to a reduction in dynamic ground-bearing pressure.
- c) New steel reinforcement was designed for the foundation to provide the required flexural strength. Additionally, new and complementary steel reinforcement was designed and placed in the wallstrengthening components to enhance flexural strength.
- d) The silo studied required nearly the same volume of existing concrete and twice the weight of the existing steel reinforcement to adequately satisfy the performance criteria established for continuous operation.
- e) It is highly advisable to conduct evaluations of similar structures designed and built in the 1970s, as the integrity of industrial personnel and future investments in the facilities could be at serious risk.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or non-profit sectors.

Conflicts of interest

The authors declare no conflict of interest.

Authors contribution statement

Conceptualization, methodology, FEM modeling, analysis, and writing of original draft preparation by Jorge Bazan. Supervision, writing – review & editing by Victor I. Fernandez-Davila.

Acknowledgment

The support of the TOP CONSULT INGENIERIA SAC company is gratefully acknowledged.

References

- [1] N. Kuczyńska, M. Wójcik, and J. Tejchman, "Effect of bulk solid on strength of cylindrical corrugated silos during filling," J. Constr. Steel Res., vol. 115, pp. 1–17, Dec. 2015, doi: 10.1016/J.JCSR.2015.08.002.
- [2] D. Adem, K. Zeki, D. Ahmet, and S. Halil, "Cause of Damage and Failures in Silo Structures," J. Perform. Constr. Facil., vol. 23, no. 2, pp. 65–71, Apr. 2009, doi: 10.1061/(ASCE)0887-3828(2009)23:2(65).
- [3] M. Bozozuk, "Tower Silo Foundations," Canada Build. Dig., 1976.
- [4] E. W. Graham and A. M. Rodriguez, "The Characteristics of Fuel Motion Which Affect Airplane Dynamics," J. Appl. Mech., vol. 19, no. 3, pp. 381–388, Apr. 1952, doi: 10.1115/1.4010515.
- [5] G. W. Housner, "Dynamic pressures on accelerated fluid containers," Bull. Seismol. Soc. Am., vol. 47, no. 1, pp. 15–35, Jan. 1957, doi: 10.1785/BSSA0470010015.
- [6] D. D. Kana, "Validated spherical pendulum model for rotary liquid slosh," J. Spacecr. Rockets, vol. 26, no. 3, pp. 188–195, 1989, doi: 10.2514/3.26052.
- [7] Y. Li and J. Wang, "A supplementary, exact solution of an equivalent mechanical model for a sloshing fluid in a rectangular tank," J.

Fluids Struct., vol. 31, pp. 147–151, May 2012, doi: 10.1016/J.JFLUIDSTRUCTS.2012.02.012.

- [8] A. Lopez and V. Fernández-Dávila, "SIMPLIFIED METHOD FOR EVALUATING SEISMIC RESPONSE OF CIRCULAR REINFORCED CONCRETE SILOS," 2017.
- [9] ACI 313-16, Design Specification for Concrete Silos and Stacking Tubes for Storing Granular Materials. 2016.
- [10] C.-E. C. for Standardization, "Eurocode 1: Action on structures – Part 4: Silos and Tanks, EN-1991-4," vol. 1, no. 2005, 2006.
- [11] C.-E. C. for Standardization, "Eurocode 8 Design of Structures for Earthquake Resistance – Part 4: Silo, Tanks and Pipelines, EN-1998-4," vol. 1, no. 2005, 2006.
- [12] S. Otani, "Earthquake resistant design of reinforced concrete buildings past and future," J. Adv. Concr. Technol., vol. 2, no. 1, pp. 3–24, 2004, doi: 10.3151/jact.2.3.
- [13] ASCE/SEI 41-17, Seismic Evaluation and Retrofit of Existing Buildings. American Society of Civil Engineers, 2017. doi: 10.1061/9780784414859.
- [14] Ministerio de Vivienda Construcción y Saneamiento, Norma Técnica E.030 Diseño Sismorresistente. El Peruano, 2018.
- [15] ASCE/SEI 7-10, Minimum Design Loads and Associated Criteria for Buildings and Other Structures. 2010.
- [16] ACI 318-19, Building Code Requirements for Structural Concrete (ACI 318-19). American Concrete Institute, 2019.
- [17] FEMA, FEMA-547: Techniques for the Seismic Rehabilitation of Existing Buildings. U.S. Department of Homeland Security, FEMA ; U.S. Department of the Interior ; GSA, 2006.
- [18] Computer & Structures INC, "SAP2000: Integrated Solution for Structural Analysis and Design." 2020.