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Analytical Solution of the Closed-Form Equations Governing the Hybrid Performance of a Tuned Liquid Column Gas Damper Equipped With a Variable Orifice

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ABSTRACT

Structural vibrations are one of the main concerns of engineering in recent decades. The tendency towards a flexible structure such as tall structures or structures with long spans has caused in more intense movements of the structure under service loading. Limiting acceleration in tall and slender buildings, as well as controlling vibrations, is a complex design issue. In this research, the new hybrid damper; "tuned liquid column gas damper equipped with variable orifice (H-O-TLCGD)" is introduced. The dominant mechanism for confronting with vibration in this damper is based on liquid movement, and the vibration energy is dissipated by the effects of fluid turbulence and friction caused by the local pressure drop of the orifice opening. In order to achieve the actual performance behavior of the system, the equations governing the dynamic response of the structure equipped with this damper are obtained along with damping modification and removal of some uncertainties, which cause nonlinear equations. Also, in this research, according to the advantages of low energy demand, permanent stability of the system and economic efficiency of using semi-active control systems, the combination of semi-active and passive dampers and the increase of stiffness in the systems equipped with them by gas springs, with the aim of improving the performance of system are considered so that the performance level of structures equipped with this new control system improve at an acceptable level. This research presents the differential equations governing the axial performance of the liquid column damper, accounting for energy dissipation due to changes in flow cross-section and gas spring stiffness, and demonstrates how to combine these effects using hydrodynamics and structural control principles. Also, the closed-form analytical solution of these nonlinear equations is presented so that researchers can achieve their research goals in a shorter time. To facilitate practical applications, this research provides a methodology for designing systems equipped with this damper and optimizing its performance using semi-active control effects, which will be useful for researchers and construction engineers.

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1. Introduction

Passive control methods are limited by their dependence on the frequency content of external forces and environmental conditions, while active control methods, although effective, require significant external power, such as electricity. Furthermore, during an earthquake, this power source may not be available. Additionally, high maintenance costs made this system not very practical. In response to these challenges, two structural and semi-active control systems have been proposed.

Semi-active control systems are essentially passive systems that can adapt and adjust the mechanical characteristics of the system, hence they are often referred to as passive controllable devices. The mechanical properties of these systems are adjusted based on feedback measurements from the structural response. In a semi-active control scheme, a controller system (a computer) measures the feedbacks and based on the predetermined control algorithm, sends a suitable signal for the operation of the semi-active devices. Control forces, as a result of the movement of the structure itself and proper adjustment of the mechanical properties of the semi-active control system, are generated. In addition, due to the fact that the control forces in most semi-active control systems act against the direction of the structure's movement, therefore they cause the overall stability of the structure [1,2].

In semi-active systems combine the approaches of active and passive systems, allowing for simultaneous use of both. These systems are essentially passive, yet capable of adapting and changing their mechanical characteristics.

Shinozuka and Feng [3] proposed the use of a variable orifice damper as a power driver, which was later developed by Shinozuka et al. [4]. Kurata et al. [5] utilized a three-story structural frame, while Sack and Patten installed a controllable orifice hydraulic actuator in an Oklahoma highway bridge [6]. The force required to adjust the position of the valve is small, and therefore energy demand will be low. Also some passive dampers like yielding [7], viscous [8], friction [9] and new lateral earthquake systems [10–12] could tolerate seismic actions very effectively.

Korta's tests showed that only 30 W are needed for the operation of the valve. Variable orifice dampers exhibit a high capacity for resistant forces (up to 1-2 MN) and a dynamic range (with $c_{min}/c_{max} = 200$). Tuned Liquid Column Damper (TLCD): This device was first patented by Fram (1910) [13], which consisted of U-shaped tubes that were filled with a Newtonian fluid and used to reduce ship vibrations. It was proposed. But the first research and patent in the field of using liquid column damper adjusted in civil engineering was proposed by Sakai et al [14] for passive control of vibrations. This damper is a specialized type of liquid damper. The practical cases of using the adjusted liquid column damper include the Sofitel (former Kusima) hotel building in Tokyo (Japan), Hyatt hotel and Ichida building in Osaka (Japan), Comcast Center in Philadelphia (United States), Random House Tower in New York (United States), One Rincon Hill building in San Francisco (United States) [15]. Most of these buildings range from 30 to 60 stories in height, with the Comcast Center being the tallest at approximately 300 m.

The 48-story Wall One Center building in Vancouver, with a lean ratio of 1:7, features two tuned liquid column dampers holding 60,000 gallons (330 tons) of water to mitigate wind-induced vibrations. The dampers had the dominant frequency of the structure [16].

Jung-Cheng Wu et al. (2009) [17] investigated the optimal design parameters for a TLCD with a non-uniform and variable U-tube cross-section (including the orifice) in a one-degree-of-freedom

structure subjected to sinusoidal horizontal loads and also obtained white noise. In this method, calculations are facilitated by presenting and proposing a closed-form solution, which is an analytical solution that does not repeat the response, and increases the speed of the process. Analytical and numerical studies have been conducted, revealing the following new findings:

1. The maximum height drop is inversely proportional to the excitation amplitude, whereas the coefficient of the maximum frequency is independent of the maximum excitation amplitude
2. The structure's movement range spans all possible frequencies when the two resonance peaks in the structural response are equal, applicable to both normal and damped structures
3. A uniform TLCD consistently provides the best damping performance for structures, given the same mass coefficient and horizontal length coefficient conditions
4. Optimal performance is inversely related to pipe cross-section coefficients.

In the continuation of the research, the desired effective parameters are presented in the tables and the desired results are obtained for the cross section coefficient of 1.2 and 2 for non-uniform pipes (with orifices) and uniform pipes [17].

Guntur et al (2019) [18] investigate the behavior of a two-story building on a laboratory scale that is equipped with an orifice TLCD damper. In this study, the TLCD damper was placed on both the first and second floors, and the structural behavior was analyzed. The TLCD mass coefficient (the ratio of the fluid mass to the transfer mass of the vibrating table) and the cross-sectional area of the U-shaped tube were treated as variable at the orifice location, and the laboratory results show a good similarity with the results of the simulated models. The results show that there is no significant difference with the changes in the mass coefficient of TLCD in the behavior of the two-story structure (reduction of vibrations), but the changes in the cross-sectional area of the pipe at the orifice place create important differences in the behavior of the structure, so that by increasing the cross-sectional area of the pipe at the orifice place, it follows the reduction of the acceleration of the system movement [18,19].

Hokmabady et al. (2019) [20] increased the control of vibrations created on a platform and a three-dimensional marine structure (jacket platform) by adding MR fluid and a gas spring to the tuned liquid column damper (TLCD), which is under two types of lateral stimulation 1. Earthquake; 2. The sea waves are placed in two directions (one parallel to the x-axis and the other in a direction that makes an angle of 45 degrees with the x-axis). This system effectively combines the benefits of both components, resulting in a significant reduction in the structure's response to lateral movements. The researchers of this research, by changing the yield stress values of MR fluid and changing the gas pressure values of the u-shaped tube, different results for the structure response (RMS) found that the best and most desirable condition for the research marine platform case sample is achieved in the combination of gas pressure of 2 bar and yield stress of MR fluid equal to KPA 100. Of course, in the results of the mentioned research, the hybrid performance of MR-TLCGD results and a significant reduction in response compared to the performance shows TLCGD [20].

Hokmabady et al. (2019) [21] introduced a three-dimensional and developed passive control system called tuned liquid column ball gas damper (TLCBGD) whose performance is a combination of TLCGD and TLCBD. The initial model is a one degree of freedom model in the form of the installation of this mass (body) brake system, which is used in the continuation of the research to validate the new damper in a marine platform or jacket. In this research, the performance and efficiency of the TLCBGD system in the excitation of irregular waves is numerically analyzed and

with the changes of gas pressure and the parameter of the ratio of the size of the roller ball to the tube, the parametric evaluation of the new system is calculated, which shows the results of improving the performance of the TLCD system compared to the TLCD system, so that the reduction of the high level It shows displacement up to 61 and 62% under regular and irregular wave loads [21].

In this research, The U-shaped liquid column damper is chosen due to its suitable and more economical performance to control the vibration of the structure, and the dynamic balance equations governing its behavior are extracted using energy methods. In order to improve the performance of the system, the aforementioned damper is equipped with a variable orifice and a gas spring and the dynamic balance equations governing their behavior are obtained in separation and Combination state. Then, the analytical solution of the closed-form of the mentioned equations is presented in order to better understand the behavior of the system. In the following, the sensitivity analysis for the parameters affecting the performance and behavior of the system is examined and the design methodology of systems equipped with this damper is discussed with regard to system optimization. At the end, the method of applying semi-active control effects of Variable orifice is shown by presenting the necessary relationships.

2. Dynamic balance equations of TLCD with newtonian fluid

Using energy methods, the dynamic balance equations of the TLCD with Newtonian fluid are determined as follows:

According to Fig.1, if it is assumed that the liquid is incompressible and the columns are open (no pressure is created from the movement of the liquid y), assuming a constant cross-sectional area A_d , the resulting kinetic and potential energy is equal to:

$$E_k = \frac{1}{2} \rho_l A_d B_d \dot{x}^2 + \frac{1}{2} \rho_l A_d B_d \dot{y}^2 + \frac{2}{2} \rho_l A_d h \dot{x}^2 + \frac{2}{2} \rho_l A_d h \dot{y}^2 + \rho_l A_d B_d \dot{x} \dot{y} \tag{1}$$

$$E_k = \frac{1}{2} \rho_l A_d B_d (\dot{x} + \dot{y})^2 + \rho_l A_d h (\dot{x}^2 + \dot{y}^2) \tag{2}$$

$$E_p = \rho_l A_d g y \cdot y \rightarrow E_p = \rho_l A_d g y^2 \tag{3}$$

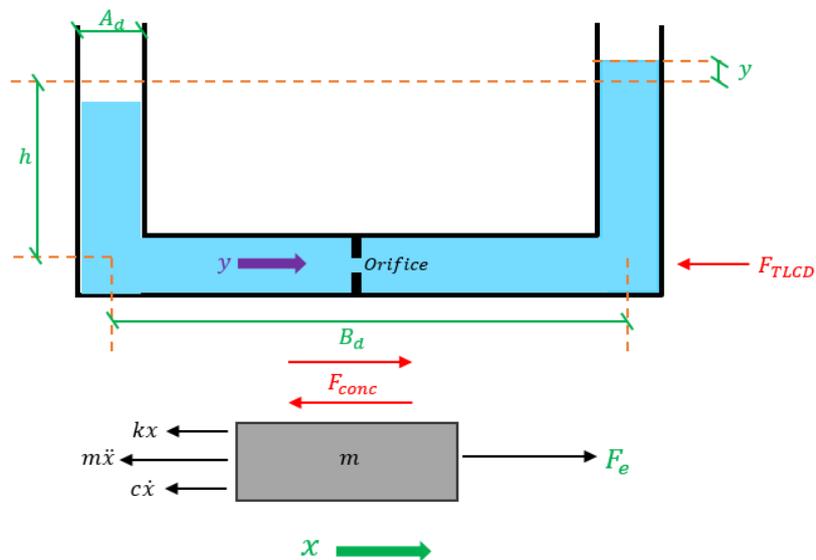


Fig. 1. Tuned liquid column damper.

ρ_l : Liquid density

g : Gravitational acceleration

Law of conservation of energy:

(Input power = $\frac{\text{work}}{\text{time}} = \frac{d}{dt}$ (kinetic and potential energy):

$$\frac{d}{dt}(E_k + E_p) = F_{\text{conc}}\dot{x} - F_{\text{TLCD}}\dot{y} \quad (4)$$

$$\frac{1}{2} \rho_l A_d B_d (2\ddot{x}\ddot{x} + 2\dot{y}\dot{y} + 2\ddot{x}\dot{y} + 2\dot{x}\ddot{y}) + \rho_l A_d h (2\ddot{x}\ddot{x} + 2\dot{y}\dot{y}) + 2\rho_l A_d g y \dot{y} = F_{\text{conc}}\dot{x} - F_{\text{TLCD}}\dot{y} \quad (5)$$

$$\rho_l A_d B_d \ddot{x}\ddot{x} + \rho_l A_d B_d \dot{y}\dot{y} + \rho_l A_d B_d \ddot{y}\dot{x} + \rho_l A_d B_d \ddot{x}\dot{y} + 2\rho_l A_d h \ddot{x}\ddot{x} + 2\rho_l A_d h \dot{y}\dot{y} + 2\rho_l A_d g y \dot{y} = F_{\text{conc}}\dot{x} - F_{\text{TLCD}}\dot{y} \quad (6)$$

$$(\rho_l A_d B_d \ddot{x} + 2\rho_l A_d h \ddot{x} + \rho_l A_d B_d \ddot{y})\dot{x} + (\rho_l A_d B_d \dot{y} + 2\rho_l A_d h \dot{y} + 2\rho_l A_d g y + \rho_l A_d B_d \ddot{x})\dot{y} = F_{\text{conc}}\dot{x} - F_{\text{TLCD}}\dot{y} \quad (7)$$

Using energy balances and the form for the structure of one degree of freedom, two relationships were calculated for F_{TLCD} and F_{conc} , which are:

$$F_{\text{conc}} = \rho_l A_d (B_d + 2h)\ddot{x} + \rho_l A_d B_d \dot{y} \quad (8)$$

$$F_{\text{TLCD}} = - [\rho_l A_d (B_d + 2h)\dot{y} + 2\rho_l g A_d y + \rho_l A_d B_d \ddot{x}] \quad (9)$$

If we write the equation of motion for the original mass, we will have:

$$m\ddot{x} + c\dot{x} + kx + F_{\text{conc}} = F_e \quad (10)$$

If we consider the change of the following variables, the form of the last equation is created as follows:

$$m_d = \rho_l A_d (B_d + 2h) \quad \text{and} \quad L_d = (B_d + 2h) \quad \text{and} \quad \beta = \frac{B_d}{L_d} = \frac{1}{1 + \frac{2h}{B_d}}$$

$$\Rightarrow (m + m_d)\ddot{x} + c\dot{x} + kx = F_e - \beta m_d \dot{y} \quad (11)$$

On the other hand, to establish the balance of TLCD, according to relation (9), we will have:

$$m_d \dot{y} + F_{\text{TLCD}} + k_d y = -\beta m_d \ddot{x} \quad \text{and} \quad k_d = 2\rho_l A_d g \quad (12)$$

which in equation (12), F_{TLCD} is the friction force or damping force caused by the TLCD damper, which if we consider it as the product of the damping coefficient in the speed of the damper ($F_{\text{TLCD}} = C \cdot \dot{y}$). We present the factors that can form this damping force separately, and in case of using the combination of factors, due to the fact that these forces are in the same direction, they can be algebraically added together [16][22][23][24][25][26].

3. Equations of dynamic balance of tuned liquid column damper system with newtonian fluid equipped with variable orifice

The system assumes a liquid column equipped with an orifice that can open and close. By Idelchik the following empirical relationship for the liquid head loss coefficient is proposed, where δ is known as the liquid head loss coefficient [27].

$$F_{TLCD} = \frac{1}{2} \rho_l A_d \delta |\dot{y}| \dot{y} = \frac{1}{2} \rho_l A_d \delta \dot{y}^2 \tag{13}$$

Here, λ is the percentage of the cross-sectional area closed with the orifice, and for example: $\lambda = 1$ corresponds to complete blockage.

$$\delta = (\lambda + 0.707\lambda^{0.375})^2 (1-\lambda)^{-2} \tag{14}$$

Wu et al. provide a better match for laboratory results using the following relationship [17,28]:

$$\delta = (-0.6\lambda + 2.1\lambda^{0.1})^{1.6} (1-\lambda)^{-2} \tag{15}$$

In order to prove the relationship (13), this analogy is used that the local losses resulting from the orifice are the same as the local losses resulting from sudden contraction in the pipe.

Finally, the dynamic balance equations of the system in this section will be:

$$(m + m_d)\ddot{x} + c\dot{x} + kx = F_e - \beta m_d \dot{y} \tag{16}$$

$$m_d \ddot{y} + \frac{1}{2} \rho_l A_d \delta \dot{y}^2 + 2\rho_l A_d g y = -\beta m_d \ddot{x} \tag{17}$$

4. Apply semi-active control effects in the dynamic balance equations of tuned liquid column damper system with newtonian fluid equipped with variable orifice

A variable orifice damper is a tunable liquid damper (TLCD) in which the orifice of the liquid column can be changed by applying an external voltage. This concept is known as semi-active TLCD [16][27][28][29][30]. Equations of motion:

$$(m + m_d)\ddot{x} + c\dot{x} + kx = F_e - \beta m_d \dot{y} \tag{18}$$

$$m_d \ddot{y} + \frac{1}{2} \rho_l A_d \delta |\dot{y}| \dot{y} + 2\rho_l A_d g y = -\beta m_d \ddot{x} \tag{19}$$

where the head loss (δ) can be variable with time.

$$m_d \ddot{y} + \frac{1}{2} \rho_l A_d \delta(t) |\dot{y}| \dot{y} + k_d y = -\beta m_d \ddot{x} \tag{20}$$

The above equation can be expressed in the state space by applying the nonlinear damping term as a scalar input force:

$$\dot{Z}(t) = AZ + B_p p + B_f f \tag{21}$$

in which we will have:

$$Z = [x \ y \ \dot{x} \ \dot{y}]^T$$

$$M = \begin{bmatrix} m + m_d & \beta m_d \\ \beta m_d & m_d \end{bmatrix}$$

$$M^{-1} = \frac{1}{\bar{m}} \begin{bmatrix} m_d & -\beta m_d \\ -\beta m_d & m + m_d \end{bmatrix}$$

$$C = \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \quad K = \begin{bmatrix} k & 0 \\ 0 & k_d \end{bmatrix}$$

$$\bar{m} = m_d(m + m_d) - \beta^2 m_d$$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad B_p = \frac{1}{\bar{m}} \begin{bmatrix} 0 \\ 0 \\ m_d \\ -\beta m_d \end{bmatrix}$$

Note that M is symmetric but not diagonal anymore. This is how the control force is considered:

$$f(t) = -\frac{1}{2} \rho_l A_d \delta(t) |\dot{y}| \dot{y}$$

5. The dynamic balance equations of the tuned liquid column damper system, in the case that the damper is equipped with a gas spring at both ends of the U-shaped tube

By using energy methods, the dynamic balance equations of the tuned liquid column damper system, in the case that the damper is equipped with a gas spring at both ends of the U-shaped tube (Fig. 2), is determined as follows:

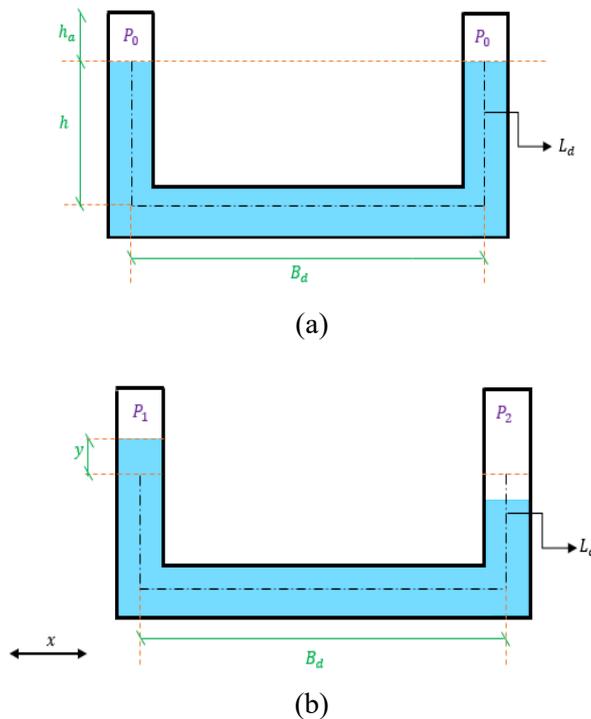


Fig. 2. TLCGD damper (a) before excitation (b) after excitation.

With the presence of gas in the TLCD damper, as a result of fluid movement, due to the compressibility of gases, it causes condensation in a U-shaped column, and on the other side, the expansion of the gas volume is used, and these changes act like a spring, and the stiffness of the gas spring is obtained from the following relation [16,21] [31–34]:

$$K_d = A_d \left(\frac{dp}{dy} \right)$$

A_d is the cross-sectional area of the liquid column, p_o is the initial pressure of the gas in the vertical columns which changes to p_1 and p_2 after excitation, h and h_a denote the vertical columns length and the part of the vertical length that is occupied by gas before excitation, respectively.

In gases: $p v^n = \text{constant}$ and $n =$ Polytropic index

Slow vibration $\rightarrow n = 1$

High frequency vibration $\rightarrow n = 1.4$

For TLCGD, That is recommended by references: $n = 1.4$

$$p v^n = p_0 v_0^n \rightarrow k_d = A_d p_0 v_0^n \frac{d}{dy} (v^{-n}) = -n A_d p_0 v_0^n v^{-(n+1)} \frac{dv}{dy} \tag{22}$$

We know the volume changes:

$$v = v_0 - A_d \cdot y \rightarrow \frac{dv}{dy} = -A_d$$

$$\Rightarrow K_d = (n A_d^2 \frac{p_0}{v_0}) (1 - \frac{A_d \cdot y}{v_0})^{-(n+1)} \tag{23}$$

According to the relationship obtained, the stiffness of the gas spring is non-linear and only if the displacement of the fluid $y(t)$ is small compared to the initial volume of the air column, the stiffness of the air spring can be obtained from the following relationship:

$$K_d \approx n A_d^2 \frac{p_0}{v_0} \tag{24}$$

p_0 and v_0 are the initial pressure and initial volume of the gas, respectively. Therefore, the governing equations of TLCGD will be as follows:

$$(m + m_d) \ddot{x} + c \dot{x} + kx = F_e(t) - \beta m_d \ddot{y} \tag{25}$$

$$m_d \ddot{y} + 2 \rho_1 A_d g y + 2 n A_d^2 \frac{p_0}{v_0} y = -\beta m_d \ddot{x} \tag{26}$$

6. The dynamic balance equations of the tuned liquid column damper system, in the case that the damper is equipped with a combination of variable orifice and gas spring at both ends of the U-shaped tube

The use of energy methods and combining the equations of parts 3 and 5, Dynamic balance equations of the tuned liquid column damper system, in the case that the damper is equipped with a combination of variable orifice and gas spring at both ends of the U-shaped tube, are obtained as follows:

$$(m + m_d) \ddot{x} + c \dot{x} + kx = F_e(t) - \beta m_d \ddot{y} \tag{27}$$

$$m_d \ddot{y} + \frac{1}{2} \rho_1 A_d \delta \dot{y}^2 + 2 \rho_1 A_d g y + 2 n A_d^2 \frac{p_0}{v_0} y = -\beta m_d \ddot{x} \tag{28}$$

7. Solving the dynamic balance equations of the problem

In this section, the closed-form analytical solution of the dynamic balance equations of the tuned liquid column damper system with Newtonian fluid is presented in the following cases [35]:

First; equipped with variable orifice and second; Equipped with a combination of variable orifice and gas spring.

In order to show the complete applicability of the responses, several problems in different states and conditions are presented to obtain useful results by displaying graphs and response curves, along with sensitivity analysis.

Also, due to the complexity of the equations, the numerical solution of the dynamic balance equations of the tuned liquid column damper system with Newtonian fluid equipped with a variable orifice, under the load of external earthquake excitation, is provided to the readers and researchers.

7.1. Analytical solution of the closed-form equations of tuned liquid column damper equipped with variable orifice

In part 3, the dynamic balance equations of the tuned liquid column damper system with a Newtonian fluid equipped with an orifice were obtained, and two equations (29) and (30) are the basic equations of the movement of the damper on the one degree of freedom structure:

$$(m+m_d)\ddot{x}+c\dot{x}+kx=F_e(t)-\beta m_d \ddot{y} \quad (29)$$

$$m_d \ddot{y} + \frac{1}{2} \rho_l A_d \delta \dot{y}^2 + 2p_l A_d g y = -\beta m_d \ddot{x} \quad (30)$$

By Idelchik, the following Experimental relationship is presented:

$$\delta = (\lambda + 0.707\lambda^{0.375})^2 (1-\lambda)^{-2} \quad (31)$$

Here, λ is the percentage of the cross-sectional area closed with the orifice.

The following relationship is presented by (Wu et al.) from the laboratory results:

$$\delta = (-0.6\lambda + 2.1\lambda^{0.1})^{1.6} (1-\lambda)^{-2} \quad (32)$$

It is noteworthy that if two equations (29) and (30) can be solved with harmonic excitation load, which is a periodic function, then by using Fourier series, it can be solved against any desired excitation load, according to the following points:

Note 1: Fourier has shown that a periodic function can be represented by an infinite set of sine and cosine functions, which is called the Fourier series.

Note 2: Any force that acts over a limited period of time can be assumed to be a periodic load (in other words, non-periodic loads in the frequency domain can be used instead of the usual solution in the time domain). For this, it is enough to add the effective time so that in this added distance, the force is zero.

Assuming $F_e(t) = f_0 e^{i\bar{\omega}t}$ and ignoring the transient response of the system, the response of the x and y variables, which are respectively the displacement of the structure despite the damper on it

and the vertical displacement of the fluid in the tube, after simplifying the two mentioned equations and changing the following variables and necessary mathematical operations, will have:

$$\omega^2 = \frac{k}{m} \text{ and } m_d = \rho_1 A_d L_d = \rho_1 A_d (B_d + 2h) \text{ and } \bar{m} = \frac{m_d}{m} \text{ and } r = \frac{\bar{\omega}}{\omega} \text{ and } c = 2m\omega\xi \text{ and } \beta = \frac{B_d}{L_d} \text{ and } k_d = 2\rho_1 A_d g \text{ and } q = \frac{\omega_d}{\omega}$$

$$(1 + \bar{m})\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x = \frac{F_e(t)}{m} - \beta\bar{m}\ddot{y} \tag{33}$$

$$m_d\ddot{y} + \frac{1}{2}\rho_1 A_d \delta \dot{y}^2 + k_d y = -\beta m_d \ddot{x} \tag{34}$$

Note: The energy dissipated by TLCD in one cycle can be calculated by considering the real part of the response (steady response) in the range of 0 to 2π.

By equating the work done by the TLCD in a complete cycle with the work done by the viscous damper and performing mathematical operations and calculations on the above set of nonlinear equations, the final response can be expressed as a closed-form solution below:

If $x = \bar{x}e^{i\omega t}$ and $y = \bar{y}e^{i\omega t}$, then:

$$D = [1 - (1 + \bar{m})r^2 + 2i\xi r] \tag{35}$$

$$\xi_d = \frac{\delta}{3L_d} \cdot \frac{e^{2i\pi} - 1}{e^{i\pi} - 1} \tag{36}$$

$$a_1 = r^2 \cdot \xi_d \text{ and } b_1 = \frac{\beta^2 \bar{m} r^4}{D} + r^2 - q^2 \text{ and } c_1 = \frac{f_0 \beta r^2}{KD} \tag{37}$$

$$\Delta = b_1^2 - 4 \cdot a_1 \cdot c_1 \tag{38}$$

$$\bar{y} = \frac{-b_1 \pm \sqrt{\Delta}}{2 \cdot a_1} \tag{39}$$

$$\bar{x} = \frac{f_0}{KD} + \frac{\beta \bar{m} r^2}{D} \bar{y} \tag{40}$$

7.1.1. Sensitivity analysis

It is assumed that a tuned liquid column damper equipped with a variable orifice of conventional dimensions is available with the following specifications and in two different cases, the amount of opening of the orifice $\delta = 9$ and $\delta = 50$ and each case with seven variable external loads should be analyzed. Due to the high volume of calculations for the obtained analytical relations, the programming code was prepared using MATLAB software, and the results are presented as follows:

$$B_d = 3(\text{m}); h = 1(\text{m}); \rho_1 = 1000(\text{Kg}/\text{m}^3);$$

$$g = 9.81(\text{m}/\text{s}^2); A_d = 0.8(\text{m}^2);$$

$$m = 8000(\text{kg}); k = 72000(\text{N}/\text{m});$$

$$c = 2400(\text{N} \cdot \text{s}/\text{m}); f_0 = 3500(\text{N});$$

For the case of $\delta=9$ and types of different angular frequency of external load ($\bar{\omega}$) will have:

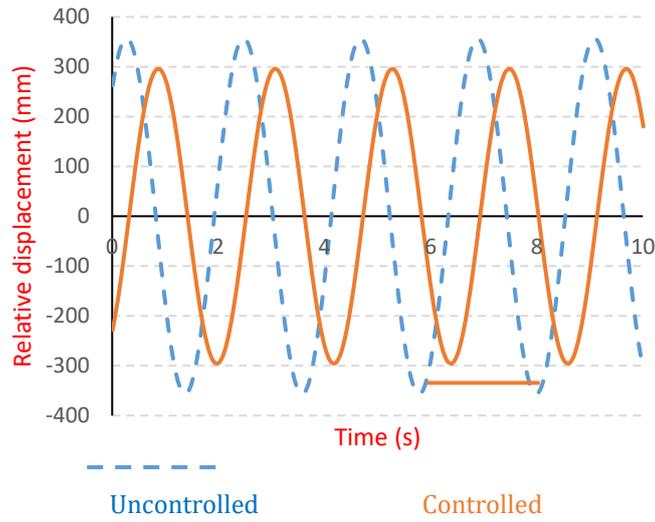


Fig. 3. Model response in case: $\delta = 9$, $\bar{\omega} = 2.85 \rightarrow$ Reduce displacement = 0.1716.

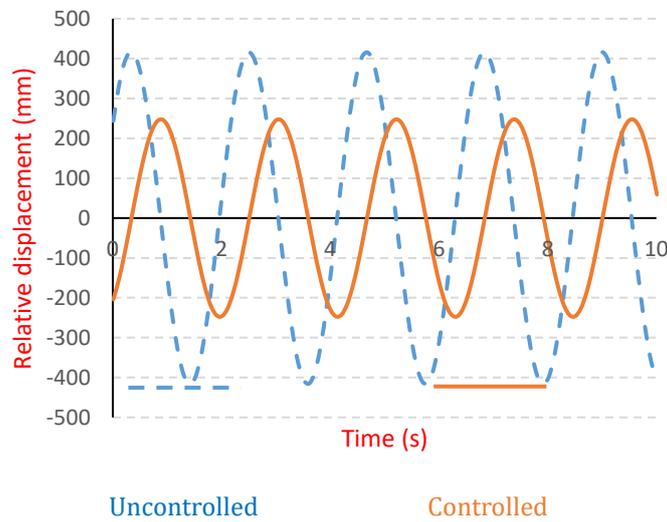


Fig. 4. Model response in case: $\delta = 9$, $\bar{\omega} = 2.9 \rightarrow$ Reduce displacement = 0.4049.

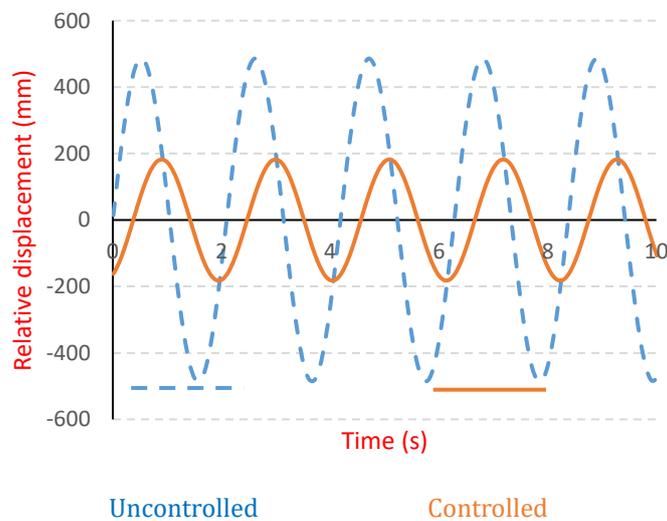


Fig. 5. Model response in case: $\delta = 9$, $\bar{\omega} = 3 \rightarrow$ Reduce displacement = 0.6248.

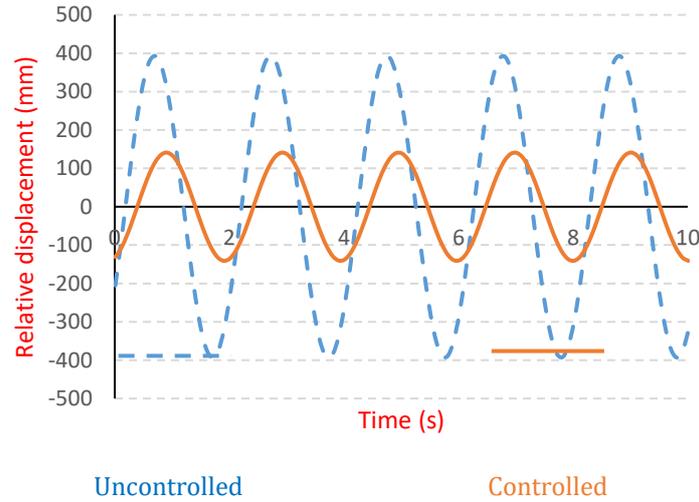


Fig. 6. Model response in case: $\delta = 9$, $\bar{\omega} = 3.1 \rightarrow$ Reduce displacement = 0.6413.

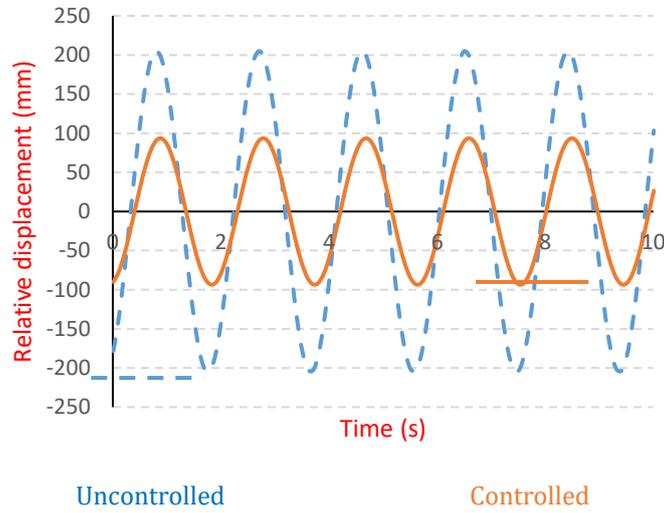


Fig. 7. Model response in case: $\delta = 9$, $\bar{\omega} = 3.3 \rightarrow$ Reduce displacement = 0.5436.

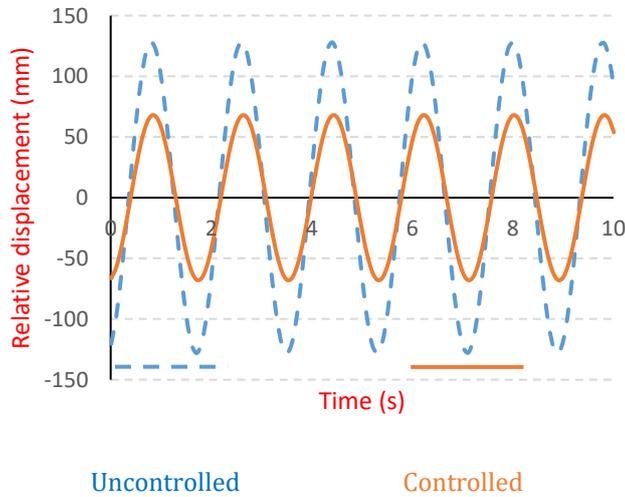


Fig. 8. Model response in case: $\delta = 9$, $\bar{\omega} = 3.5 \rightarrow$ Reduce displacement = 0.4683.

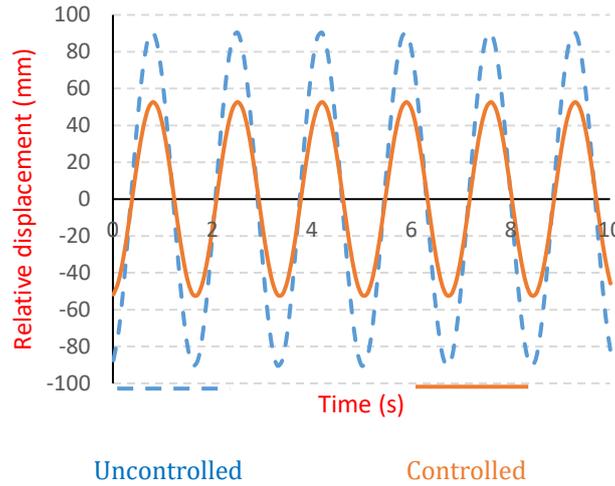


Fig. 9. Model response in case: $\delta = 9$, $\bar{\omega} = 3.7 \rightarrow$ Reduce displacement = 0.4199.

For a better comparison of different states of angular frequency types in non-controlled and controlled conditions by the damper, a three-dimensional diagram is drawn in Fig. 10.

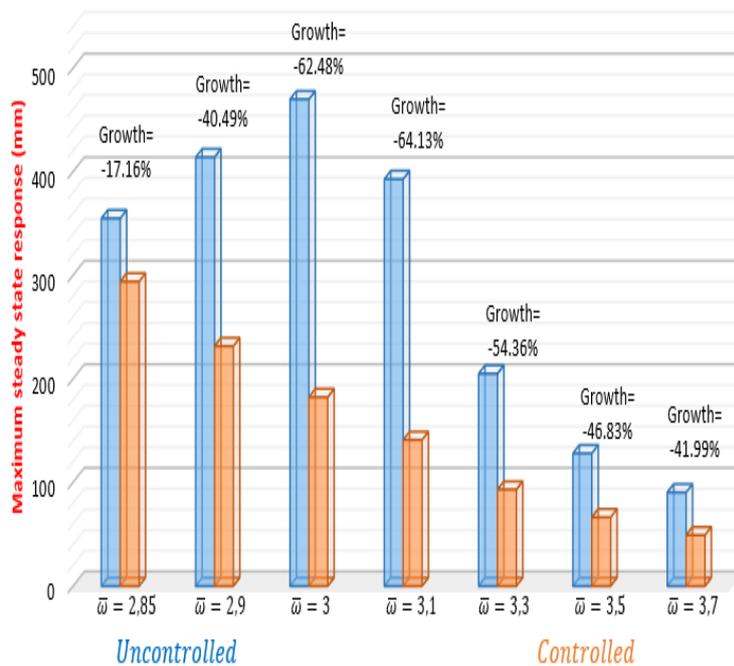


Fig. 10. Comparison between cases in uncontrolled and controlled conditions.

According to the obtained results, it is observed with changes in the angular frequency of the external lateral load, the amount of energy dissipation and displacement response curve changes and the results show As the angular frequency of the lateral load approaches the angular frequency of free vibration of the structure, that's mean “resonance event” We face a reduction in the percentage of energy dissipation and a reduction in the response curve.

In order to investigate the impact of the orifice changes, this time the types of changes in the angular frequency of the external lateral load in the case where $\delta=50$ (the orifice opens more) were investigated, and the results are presented as described in the following table:

For the case of $\delta=9$ and types of different angular frequency of external load (" $\bar{\omega}$ ") we will have:

Table 1. Displacement damping results.

Uncontrolled		Controlled-orifice	
Reduce displacement	δ	$\bar{\omega}$	Reduce displacement
1	50	2.85	0.4095
1	50	2.90	0.5598
1	50	3.00	0.7051
1	50	3.10	0.7042
1	50	3.30	0.5969
1	50	3.50	0.5102
1	50	3.70	0.4517

For a better comparison of different states of angular frequency types in non-controlled and controlled conditions by the damper, a three-dimensional diagram is drawn in Fig. 11.

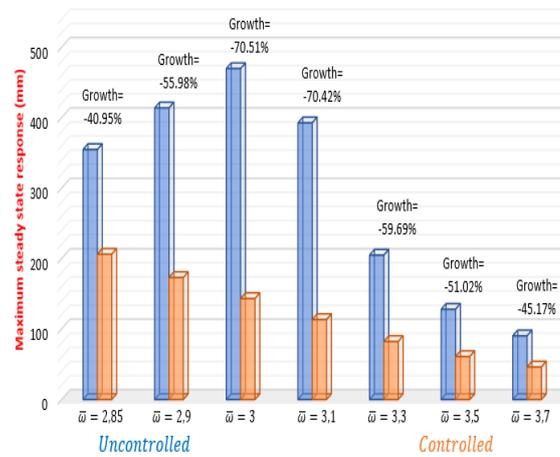


Fig. 11. Comparison between cases in uncontrolled and controlled conditions.

By comparing the results in the two states of $\delta=9$ and $\delta=50$ with the above assumed specifications, it can be seen that when the orifice opens more, the amount of energy dissipation and the reduction of the displacement response curve of the structure increases.

7.2. Analytical solution of the closed-form equations of tuned liquid column damper equipped with combination of variable orifice and gas spring

In part 6, the dynamic balance equations of the tuned liquid column damper system with Newtonian fluid were obtained in the case that the damper is equipped with a combination of variable orifice and gas spring at both ends of the U-shaped tube that the two equations (41) and (42) are the basic equations of the movement of this damper on the one degree of freedom structure:

$$(m+m_d)\ddot{x}+c\dot{x}+kx=F_e(t)-\beta m_d \ddot{y} \tag{41}$$

$$m_d \ddot{y} + \frac{1}{2} \rho_l A_d \delta \dot{y}^2 + 2p_1 A_d g y + 2n A_d^2 \frac{p_0}{v_0} y = -\beta m_d \ddot{x} \tag{42}$$

Assuming $F_e(t) = f_0 e^{i\bar{\omega}t}$ and ignoring the transient response of the system, the response of the x and y variables, which are respectively the displacement of the structure despite the damper on it and the vertical displacement of the fluid in the tube, after simplifying the two mentioned equations and changing the following variables and necessary mathematical operations, we will have:

$$\omega^2 = \frac{k}{m} \text{ and } m_d = \rho_1 A_d L_d = \rho_1 A_d (B_d + 2h) \text{ and } \bar{m} = \frac{m_d}{m} \text{ and } r = \frac{\bar{\omega}}{\omega} \text{ and } c = 2m\omega\xi \text{ and } \beta = \frac{B_d}{L_d} \text{ and } k_d = 2\rho_1 A_d g \text{ and } q = \frac{\omega_d}{\omega}$$

$$(1 + \bar{m})\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x = \frac{F_e(t)}{m} - \beta\bar{m}\dot{y} \quad (43)$$

$$m_d\ddot{y} + \frac{1}{2}\rho_1 A_d \delta\dot{y}^2 + k_d y + 2nA_d^2 \frac{p_0}{v_0} y = -\beta m_d \dot{x} \quad (44)$$

If we repeat the operations performed in part 7.1 for the nonlinear equations (43) and (44) and expand the mathematical calculations, the final response can be used in the form of the following closed-form:

If $x = \bar{x}e^{i\omega t}$ and $y = \bar{y}e^{i\omega t}$, we will have:

$$D = [1 - (1 + \bar{m})r^2 + 2i\xi r] \quad (45)$$

$$\xi_d = \frac{\delta}{3L_d} \cdot \frac{e^{2i\pi} - 1}{e^{i\pi} - 1} \quad (46)$$

$$a1 = r^2 \cdot \xi_d \quad (47)$$

$$b1 = \frac{\beta^2 \bar{m} r^4}{D} + r^2 - \frac{2g}{\omega^2 L_d} + \frac{2n}{\omega^2 m_d} A_d^2 \frac{p_0}{v_0} \quad (48)$$

$$c1 = \frac{f_0 \beta r^2}{KD} \quad (49)$$

$$\Delta = b1^2 - 4 \cdot a1 \cdot c1 \quad (50)$$

$$\bar{y} = \frac{-b1 \pm \sqrt{\Delta}}{2 \cdot a1} \quad (51)$$

$$\bar{x} = \frac{f_0}{KD} + \frac{\beta \bar{m} r^2}{D} \bar{y} \quad (52)$$

In order to analyze the sensitivity and investigate the effect of the gas spring on the damper equipped with a variable orifice and their combined performance, a case from part 7.1 with the following specifications is selected, Then the response of the system in the mentioned state with the states of gas spring addition in variable initial pressures (p_0) and a constant initial volume (v_0) are compared. Due to the high volume of calculations for the obtained analytical relations, the programming code was prepared using MATLAB software, the results of which are presented graphically in Fig. 12 and Table 2 as follows:

$$B_d = 3(\text{m}); h = 1(\text{m}); \rho_1 = 1000(\text{Kg}/\text{m}^3);$$

$$g = 9.81(\text{m}/\text{s}^2); A_d = 0.8(\text{m}^2); h_a = 0.3h$$

$$m = 80000(\text{kg}); k = 720000(\text{N}/\text{m});$$

$c = 24000(N \cdot s/m); f_0 = 35000(N);$

For the selected case; $\delta=9$ and $\bar{\omega} = 1.5$ and $n=1.4$ will have:

Table 2. Displacement damping results.

Controlled-orifice		Controlled-orifice-gas			
$\bar{\omega}$	δ	Reduce displacement	v_0 (m^3)	P_0 ($\frac{N}{m^2}$)	Reduce displacement
1.5	9	-0.0255	0.2400	0	-0.0255
1.5	9	-0.0255	0.2400	3000	0.1761
1.5	9	-0.0255	0.2400	4000	0.2392
1.5	9	-0.0255	0.2400	5400	0.2792
1.5	9	-0.0255	0.2400	5500	0.2797
1.5	9	-0.0255	0.2400	5550	0.2798
1.5	9	-0.0255	0.2400	5600	0.2798
1.5	9	-0.0255	0.2400	5700	0.2797
1.5	9	-0.0255	0.2400	7000	0.2480
1.5	9	-0.0255	0.2400	8000	0.1903
1.5	9	-0.0255	0.2400	9000	0.1095

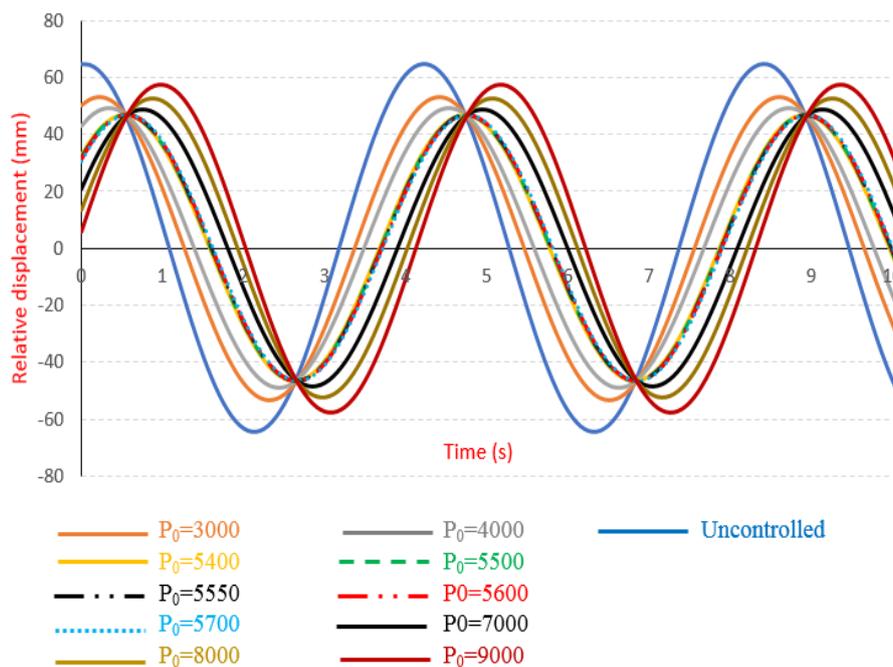


Fig. 12. Response in accordance with different P_0 .

Observing the results, the addition of a gas spring in a range of $p_0 = 5500$ (N/m^2) to $p_0 = 5700$ (N/m^2) leads to the greatest increase in displacement damping and improved damper performance, in other words, increasing the pressure beyond 5700 (N/m^2) or decreasing it below 5500 (N/m^2) reduces the damper's optimal performance, i.e. we are faced with a reduction in displacement damping. Of course, such a performance was predictable. At both low and high gas pressures beyond a certain limit, the system performance approaches that of a system without a gas spring and the state of no movement of the fluid in the U-shaped tube and becoming an additional mass on the system. Of course, the results confirm this performance.

7.3. Numerical solution of tuned liquid column damper equations equipped with variable orifice under external excitation load of earthquake

Following the contents mentioned in part 7.1, if the external excitation load entering the system is not a harmonic load, i.e. a periodic load, the complexity of the equation analysis is increased. Considering the scientific advances of the present era and the use of powerful computers and software such as the advanced MATLAB software, the analysis of heavy and complex mathematical equations is facilitated with high accuracy and speed. Therefore, using the powerful Simulink tool of MATLAB software, the numerical solution of the equations of tuned liquid column damper equipped with a variable orifice under the external excitation load of the elcentro earthquake in two states, uncontrolled and controlled with the said damper, It is presented along with the diagram of displacement response (Fig. 13) and their comparison assuming the values of the following parameters:

$$B_d = 3(\text{m}); h = 1(\text{m}); \rho_l = 1000(\text{Kg}/\text{m}^3);$$

$$g = 9.81(\text{m}/\text{s}^2); A_d = 0.8(\text{m}^2);$$

$$m = 8000(\text{kg}); k = 72000(\text{N}/\text{m});$$

$$c = 2400(\text{N} \cdot \text{s}/\text{m}); \delta=9;$$

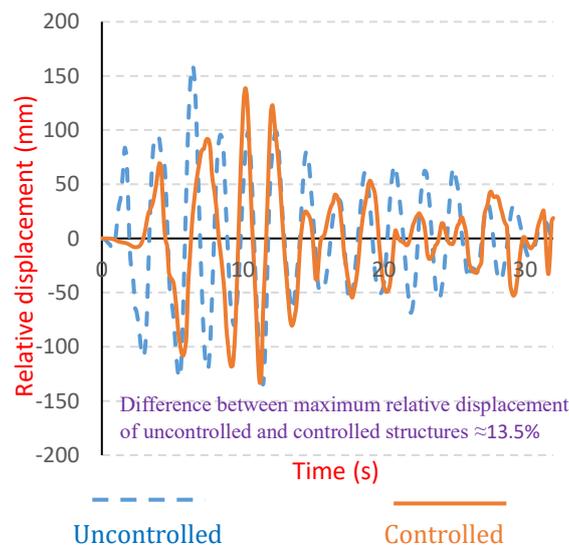


Fig. 13. The response of the structure equipped with variable orifice in the E-Icentro earthquake excitation loading case.

8. Methodology of designing a system equipped with tuned liquid column damper with Newtonian fluid and variable orifice

Having the characteristics and loading of the primary structure, the optimal corresponding values for q and ξ_d can be obtained by numerical modeling, and as a result, the optimal head loss coefficient is also obtained by numerical modeling [36,37]. In other words, optimization of the mentioned system can be achieved, and in order to do it, it is necessary to calculate the dynamic magnification of the system response according to the mentioned variables, as follows:

In part 3, the dynamic balance equations of the tuned liquid column damper system with Newtonian fluid equipped with an orifice were obtained.

$$(m+m_d)\ddot{x}+c\dot{x}+kx=F_e(t)-\beta m_d\ddot{y} \tag{53}$$

$$m_d\ddot{y}+\frac{1}{2}\rho_l A_d \delta|\dot{y}|\dot{y}+2\rho_l A_d g y=-\beta m_d\ddot{x} \tag{54}$$

In order to facilitate the introduction of the equations by changing the following variables, the equations (53) and (54) will be as follows:

$$\left\{ \begin{array}{l} \omega = \sqrt{\frac{k}{m}}, c=2m\omega\xi \quad . \quad K_d=2 \rho_l A_d g \\ \omega_d = \sqrt{\frac{k_d}{m_d}} = \sqrt{\frac{2g}{L}} \quad . \quad L_d=B_d+2h \\ q = \frac{\omega_d}{\omega} \quad . \quad \beta = \frac{B_d}{L_d} \\ m_d = \rho_l A_d (B_d+2h) = \rho_l A_d L_d \quad . \quad \bar{m} = \frac{m_d}{m} \\ \xi_d = \frac{c_d}{2\sqrt{k_d m_d}} \end{array} \right.$$

$$(m+m_d)\ddot{x}+c\dot{x}+kx+\rho_l A_d B_d \ddot{y}=F_e(t) \tag{55}$$

$$\rho_l A_d B_d \ddot{x}+m_d\ddot{y}+2\rho_l A_d g y+\frac{1}{2}\rho_l A_d \delta|\dot{y}|\dot{y}=0 \tag{56}$$

If in the relation (56), $c_d = \frac{1}{2}\rho_l A_d \delta|\dot{y}|$ the equation goes out of the nonlinear state and we will have:

$$\frac{1}{2}\rho_l A_d \delta|\dot{y}|\dot{y}=c_d\dot{y} \tag{57}$$

$$(m+m_d)\ddot{x}+c\dot{x}+kx+\rho_l A_d B_d \ddot{y}=F_e(t) \tag{58}$$

$$B_d \ddot{x}+(B_d+2h)\ddot{y}+\frac{c_d}{\rho_l A_d} \dot{y}+2gy=0 \tag{59}$$

$$(1+\bar{m})m\ddot{x}+2\xi\omega m\dot{x}+m\omega^2 x+\beta\bar{m}m\ddot{y}=F_e(t) \tag{60}$$

$$\beta\bar{m}m\ddot{x}+\bar{m}m\ddot{y}+2q\bar{m}\xi_d m\omega\dot{y}+\bar{m}q^2 m\omega^2 y=0 \tag{61}$$

Assuming an exponential harmonic external excitation load $F_e(t) = f_0 e^{i\bar{\omega}t}$ and considering the solution of the differential equation device and applying the assumptions $x(t) = x_0 e^{i\bar{\omega}t}$ and $y(t) = y_0 e^{i\bar{\omega}t}$ and $r = \frac{\bar{\omega}}{\omega}$ we will have:

$$(1+\bar{m})m(-x_0\bar{\omega}^2 e^{i\bar{\omega}t})+2\xi\omega m(x_0 i\bar{\omega} e^{i\bar{\omega}t})+m\omega^2 x_0 e^{i\bar{\omega}t}+\beta\bar{m}m(-y_0\bar{\omega}^2 e^{i\bar{\omega}t})=f_0 e^{i\bar{\omega}t} \tag{62}$$

$$\beta\bar{m}m(x_0 i\bar{\omega} e^{i\bar{\omega}t})+\bar{m}m(-y_0\bar{\omega}^2 e^{i\bar{\omega}t})+2q\bar{m}\xi_d \omega m(y_0 i\bar{\omega} e^{i\bar{\omega}t})+\bar{m}q^2 m\omega^2 (y_0 e^{i\bar{\omega}t})=0 \tag{63}$$

After performing mathematical operations on relations (62) and (63) and paying attention to this fact that the static response of the system is $\frac{f_0}{k}$, the dynamic magnification of the system response can be calculated as follows:

$$\frac{x_0}{f_0/k} = \frac{q^2 + 2iq\xi_d r - r^2}{(1 + \bar{m} - \beta^2 \bar{m})r^4 - 2i((1 + \bar{m})q\xi_d + \xi)r^3 - (1 + 4q\xi\xi_d + (1 + \bar{m})q^2)r^2 + 2i(q\xi_d + \xi q^2)r + q^2} \tag{64}$$

Also, the dynamic magnification factor of the fluid displacement response in the U-shaped pipe is calculated from the following equation:

$$\frac{y_0}{f_0/k} = \frac{\beta r^2}{(1 + \bar{m} - \beta^2 \bar{m})r^4 - 2i((1 + \bar{m})q\xi_d + \xi)r^3 - (1 + 4q\xi\xi_d + (1 + \bar{m})q^2)r^2 + 2i(q\xi_d + \xi q^2)r + q^2} \tag{65}$$

In the following, taking into account the coefficient of dissipation of the structure $\xi = 0.05$, $\bar{m} = 0.05$, $\beta = 0.06$, and $q=1$, the curve diagram of the dynamic magnification changes of the system response in relation to the parameter of dividing the angular frequency of the excitation load to the angular frequency of the structure without a damper (r), For the equivalent dissipation coefficients of tuned liquid column damper ($\xi_d = 0.02, 0.04, 0.06, 0.08, 0.10, 0.15, 0.20, 0.25$) are drawn (Fig. 14).

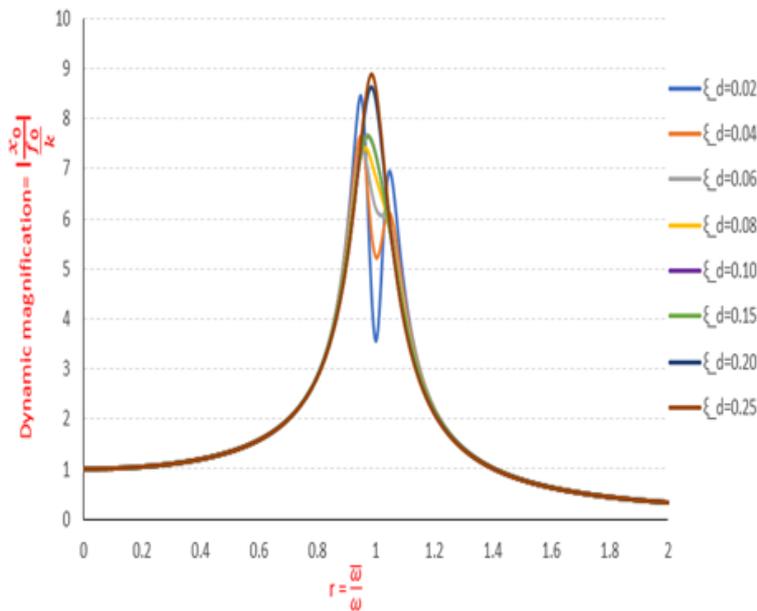


Fig. 14. The curve of changes in the dynamic magnification of the response of the system in relation to the parameter of dividing the angular frequency of the excitation load to the angular frequency of the structure without a damper (r).

By referring to the diagram in Fig. 14, we can see that at two points, the curves corresponding to different values of ξ_d are transformed into multiple branches equal to different values of ξ_d , in other words, before and after these two points, the curves coincide with each other. It means that at these two points and before and after that, the response dynamic magnification values are independent of the parameter and variable ξ_d . In other words, in a certain range of the angular frequency of the excitation load, the change in the characteristics of the damper will be effective in the behavior and dynamic response of the structure, and this range will have the greatest impact around the resonance event.

Due to the fact that the relation (65) is a relation in which the answer will be a complex number for giving value to its variables, therefore by plotting the real part of the resulting numbers with relative to their imaginary part, you can see the geometric location of the answer points for different ξ_d which can be seen for example in Fig. 15:

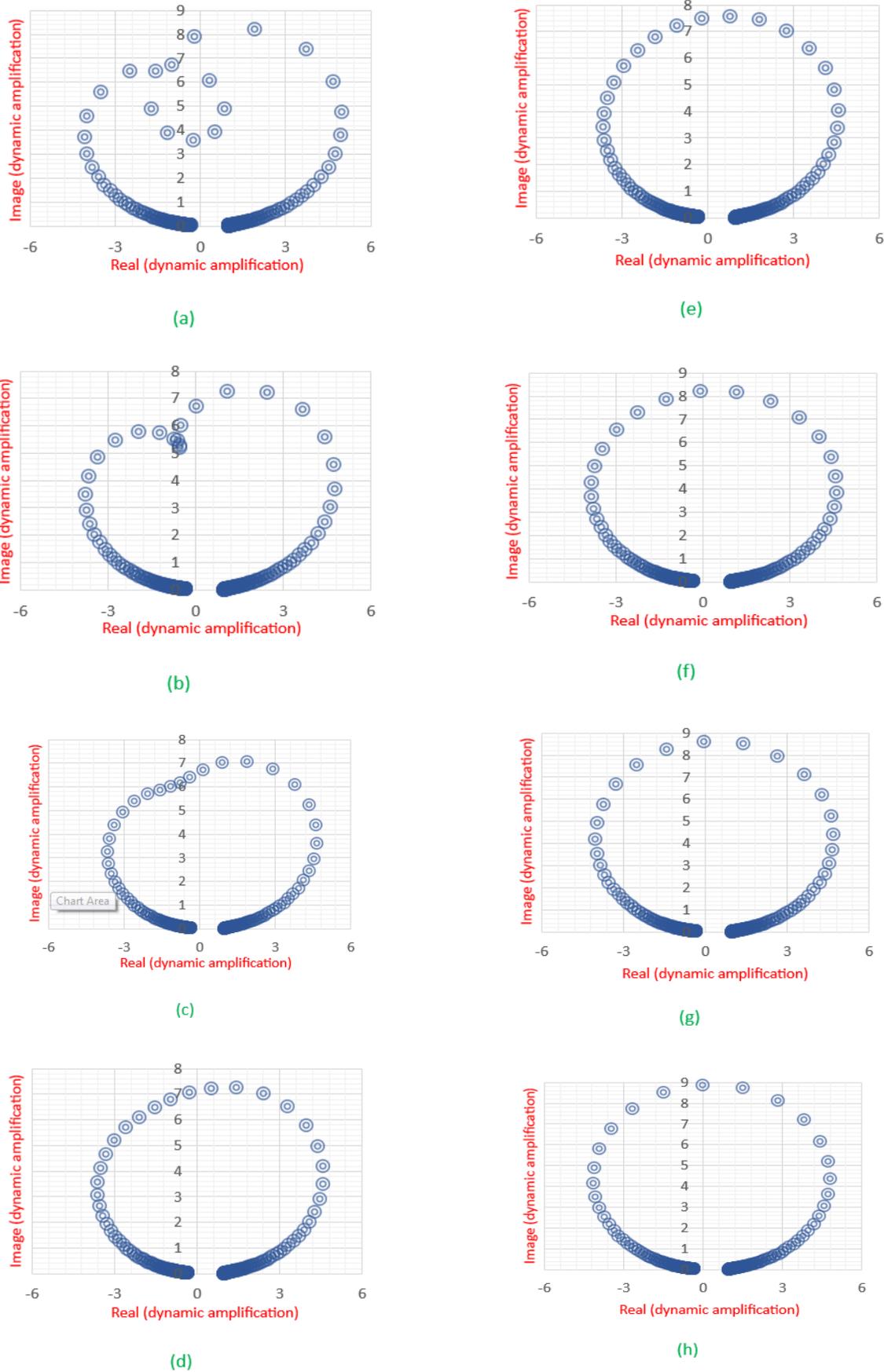


Fig. 15. Geometric location of response points in accordance with ξ_d : (a) $\xi_d=0.02$, (b) $\xi_d=0.04$, (c) $\xi_d=0.06$, (d) $\xi_d=0.08$, (e) $\xi_d=0.10$, (f) $\xi_d=0.15$, (g) $\xi_d=0.20$, (h) $\xi_d=0.25$.

9. Applying semi-active control effects and optimization in the dynamic balance equations of tuned liquid column damper system with Newtonian fluid equipped with variable orifice

In Fig. 15 the curve of dynamic magnification changes of the system response to the parameter of dividing the angular frequency of the excitation load to the angular frequency of the structure without a damper, that is, the r parameter for the equivalent Dissipation coefficients of different tuned liquid column dampers ($\xi_d=0.02, 0.04, 0.06, 0.08, 0.10, 0.15, 0.20, 0.25$) were drawn. By numbering with smaller intervals for the parameter ξ_d , it is possible to obtain the limits of ξ_d that result in the lowest values of the curve of dynamic magnification changes of the system response. In Fig. 16, the diagram of Changes in maximum dynamic response magnification due to changes in ξ_d is drawn in the conditions where $\xi = 0.05$, $\bar{m}=0.02$, $\beta = 0.6$ and $q=1$ which clearly shows that ξ_d which results in the lowest response, can be achieved. Of course, by changing the conditions, different results are obtained.

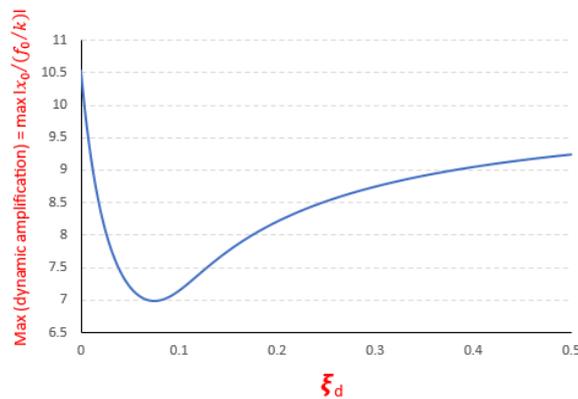


Fig. 16. The diagram of changes in maximum dynamic response magnification due to changes in ξ_d .

According to the equations obtained in part 3 and changing the variables in part 8, the relationship between ξ_d and the liquid head loss coefficient (δ) is as follows:

$$\xi_d = \frac{c_d}{2\sqrt{k_d m_d}} \rightarrow c_d = 2\sqrt{k_d m_d} \xi_d \quad (66)$$

$$c_d = \frac{1}{2} \rho_l A_d \delta |\dot{y}| = 2\rho_l A_d \sqrt{2gL_d} \xi_d \rightarrow \delta = \frac{4\sqrt{2gL_d}}{|\dot{y}|} \xi_d \quad (67)$$

Considering that $y = y_0 e^{i\bar{\omega}t}$, the relationship for calculating the liquid head loss coefficient, which is a function of time, is as follows:

$$\delta(t) = \frac{\sqrt{32gL_d}}{|y_0 i \bar{\omega} e^{i\bar{\omega}t}|} \xi_d \quad (68)$$

On the other hand, in part 3, the relationship between the parameter of the liquid head loss coefficient (δ) and the parameter of the percentage of closed cross-sectional area of flow channels with orifice (λ) was stated by Wu et al.:

$$\delta = (-0.6\lambda + 2.1\lambda^{0.1})^{1.6} (1-\lambda)^{-2} \quad (69)$$

Therefore, according to the relation (68), the opening and closing of the flow passage will be a function of time and can be controlled by a weak energy source:

$$\delta(t) = (-0.6\lambda(t) + 2.1\lambda(t)^{0.1})^{1.6} (1 - \lambda(t))^{-2} \quad (70)$$

From the above contents and relationships, it is easy to simulate the semi-active control of the structure by the new damper.

In the following, in a sample condition where $k=16000(\text{N/m})$, $m=4000(\text{kg})$, $f_0=1000(\text{N})$, $\bar{\omega}=1.5$, $\xi=0.05$, $\bar{m}=0.02$, $\beta=0.6$ and $q=1$ assuming exponential harmonic external excitation load $F_e(t) = f_0 e^{i\bar{\omega}t}$, the changes of liquid head loss coefficient parameter (δ) with relative to time are drawn by numerical values in MATLAB software (Fig. 17)

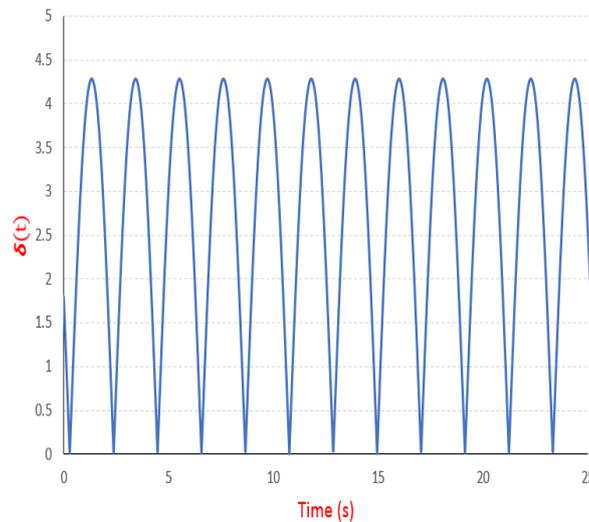


Fig. 17. The diagram of changes in the liquid head loss coefficient (δ) with relative to time.

10. Conclusion

In this research, using energy methods with the help of structural dynamics, hydrodynamics and structural control, the behavior of the new hybrid damper is discussed in detail, and finally, the following useful and fully practical results are provided to researchers, engineers and readers in its use:

- 1- The differential equations governing the combined performance of tuned liquid column damper in the modes equipped with variable orifice and gas spring and their combination were obtained.
- 2- The analytical solution of the mentioned equations was presented in a closed-form for the state of the system equipped with a variable orifice, and the state of its combination with gas spring, which can be seen by referring to them; the sensitivity analysis of the influencing variables on this new hybrid damper and the investigation of the behavior of the system equipped with this damper is greatly facilitated. Also, the optimization of each of the influencing variables is easily possible.
- 3- The numerical solution of the above system of equations, using the powerful Simulink MATLAB tool and having the convergence of the answer, is also a confirmation of the correctness of the obtained set of equations.
- 4- Near the angular frequency of the external lateral load, with the angular frequency of the structure in free vibration, the performance of the variable orifice is very effective in the displacement response of the structure, and its positive and desirable performance, means increasing

the damping of the displacement of the structure, it can easily be controlled and accessed, in order to solve the closed-form of equations.

5- The results show, when the angular frequency of the external lateral load is significantly lower than the angular frequency of the structure in the free vibration of the structure, the combined use of a gas spring in the case of using the optimal pressure (which, of course, according to the relations can be achieved), will be helpful and increasing the damping response of the displacement of the structure can be controlled by it.

6- Due to the advantages of low energy demand and permanent stability of using semi-active control systems, taking advantage of the semi-active performance of variable orifice and combining it with passive function, in the form of relationships, with the aim of improving the performance of the system, it is accessible in this article., So that researchers and engineers have the ability and necessary knowledge tools to design systems equipped with this new damper and optimize it.

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

Yousef Shiri: Roles/Writing – original draft; Data curation; Formal analysis; Investigation; Software.

Jafar Keyvani: Methodology; Writing – review & editing.

Seyed Hossein Hosseini Lavassani: Conceptualization; Project administration; Resources; Supervision; Validation; Visualization; Writing – review & editing.

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